

# Simulation of a 2D cavity under Qucs<sup>[1]</sup>

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*This paper deals with the simulation of a two-dimensional cavity under an open source integrated electrical simulator: Qucs<sup>[1]</sup>. We use here an electrical equivalent diagram introduced by S.Bazzoli<sup>[2]</sup> to describe the behaviour of the electric field inside the cavity. In order to validate this model, we compare the results with those obtained with the analytical equations of D.A.Hill<sup>[3]</sup> and also with the solutions of the eigen value problem of the propagation inside the cavity. This paper offers a way to model an unloaded mode stirred reverberation chamber in a circuit approach.*

## 1. INTRODUCTION

We want to introduce the modelling and simulation of an unloaded mode stirred reverberation chamber by using a circuit network formalism as an example of electromagnetic compatibility problem (EMC).

Reverberation chambers are screened rooms with a minimum absorption of electromagnetic energy. They are usually used with a mechanical stirrer to generate a high homogeneous electrical field by feeding the cavity with a relatively low level power.

Actually, mode stirred reverberation chambers involve both propagation phenomena and electromagnetic coupling.

If we study the transverse magnetic modes  $TM_{m,n,0}$  inside the cavity, the electric field is independent to the  $z$  variable. Hence, we model the chamber in a 2D framework as shown in figure 1.

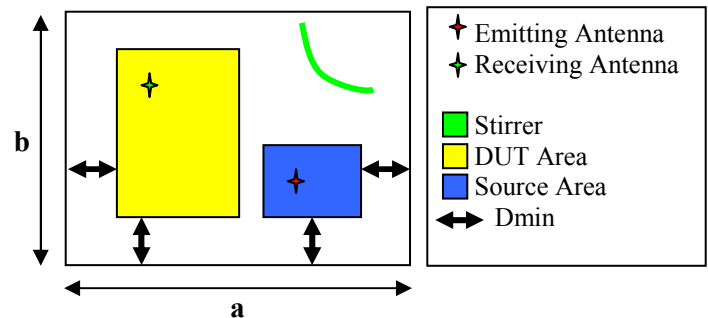


Figure 1: 2D modelling of the mode stirred reverberation chamber

First, we study the modelling of a single resonance mode of the electric field inside the cavity. Then, we combine each equivalent circuit together as a serial network to simulate the behaviour of the 2D cavity and we explain how to figure out the value of the passive elements of this circuit.

Finally, we conclude by a comparison between theory, eigen value solutions of the propagation inside the cavity and simulations under Qucs<sup>[1]</sup>.

## 2. MODELLING ONE RESONANCE MODE WITH THE ELECTRICAL EQUIVALENT CIRCUIT.

The resonance of the electric field inside a 2D cavity can be modelled by using a simple RLC circuit resonator as introduced by S.Bazzoli<sup>[2]</sup>. In the same way, the coupling of a resonance mode to the cavity can be expressed by ideal transformers in each emission or reception plan.

Figure 2 displays the circuit equivalent to a single resonance mode of the cavity; it is based on the theory presented by “Ragan”<sup>[4]</sup>.

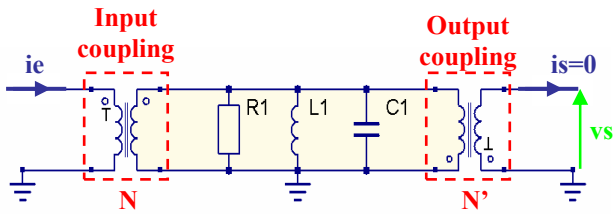


Figure 2: Diagram equivalent to a single resonance mode of a rectangular cavity

According to the ABCD matrix theory, we can express the transfer function of the circuit as follows :

$$\frac{vs}{ie} = \frac{j\omega NN' L_1}{1 - \frac{\omega^2}{\omega_0^2} \left(1 - \frac{j}{Q_1}\right)} \quad (1)$$

with:  $\omega_0 = \frac{1}{\sqrt{L_1 C_1}}$  angular frequency of the mode.  $Q_1 = R_1 C_1 \omega_0$  quality factor

When we simulate the response of the diagram equivalent to a single resonance mode, we obtain, as expected, a maximum resonance at the first resonance frequency 267.86 MHz as evidenced by figure 3 (left).

Furthermore, the polar diagram (right) shows a perfect circle corresponding to the same resonance. Finally, we can observe that the magnitude of the output depends directly on the product  $NN'$  of the equation 1 because in the case of a RLC circuit, the output is equal to the voltage drop across the resistor at the

resonance. Here, with an input current of 1A and  $R1 = 185.74 \text{M}\Omega$  we should have 185.74MV. Instead, we have 19.24 MV which is exactly the product of the voltage across the resistor by the coupling coefficients  $N=0.1154$  and  $N'=0.8978$ .

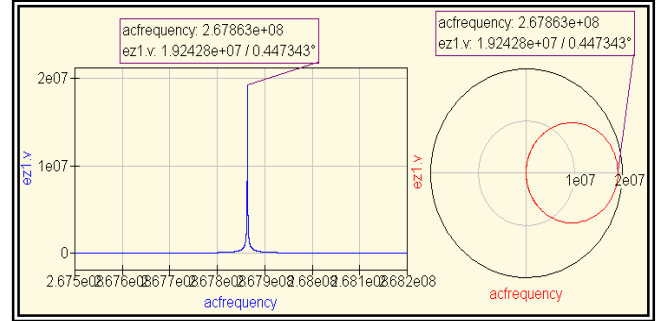


Figure 3: Simulation of the first mode of a 2D cavity under Qucs<sup>[1]</sup>

Now, we want to describe the behaviour of the cavity using a finite number of equivalent diagrams to resonance modes.

## 3. SERIAL NETWORK OF EQUIVALENT CIRCUIT AND PASSIVE ELEMENTS VALUE

If we consider the cavity as a sum of a finite number of resonance modes, we can create a network of serial quadrupoles composed by the diagram equivalent to each mode as shown in the figure 4.

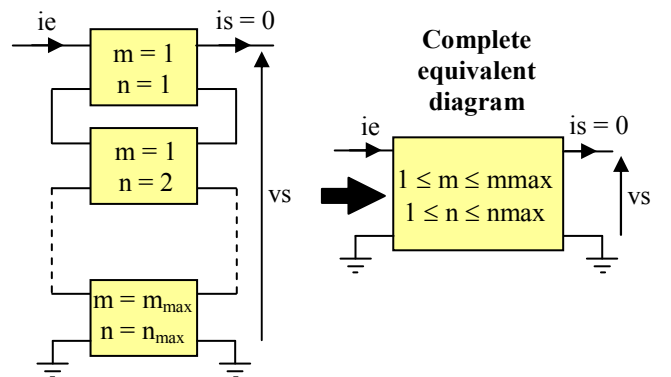


Figure 4: Serial network of equivalent diagrams

Actually, the cavity is excited by the current circulating through the emitting antenna and, thus, we can assume that the same current flows between each equivalent quadrupole.

Finally we can generalize the transfer function to the whole network using impedance matrices. Hence, we have:

$$vs = ie \sum_{m=1}^{m \max} \sum_{n=1}^{n \max} \frac{j \omega NN' L}{1 - \frac{\omega^2}{\omega_0^2} \left(1 - \frac{j}{Q}\right)} \quad (2)$$

Now, we want to find the value of each passive component of our equivalent diagram. To achieve this goal, we express the analytical equation for the electrical field inside a rectangular cavity excited by a specific antenna.

When we solve the propagation equations inside the cavity, we find the analytical expression introduced by D.A. Hill [3]:

$$Ez(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} K_{m,n} \sin\left(\frac{m\pi x_p}{a}\right) \sin\left(\frac{n\pi y_p}{b}\right) \quad (3)$$

$$K_{m,n} = \frac{4j\omega\mu_0 I_0}{ab} \frac{1}{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} * \sin\left(\frac{m\pi x_s}{a}\right) \sin\left(\frac{n\pi y_s}{b}\right) \quad (4)$$

Here,  $\mu_0$  is the magnetic permeability of vacuum,  $I_0$  is the direct component of the excitation current,  $a$  and  $b$  are the dimensions of the cavity,  $k$  is the wave number, and  $(x_s, y_s)$  and  $(x_p, y_p)$  are respectively the emitting and receiving antenna coordinates.

Therefore, we are able to identify the value of each passive component of the equivalent diagram by comparing expressions (2), (3) and (4). We obtain equations (5), (6) and (7) for the input current and the output voltage, the coupling factors  $N$  and  $N'$  and finally the resistor, inductor and capacitor components, respectively.

$$Ie = I_0 \cdot \sin(\omega t) \quad vs = Ez \cdot \sin(\omega t) * \Delta_z \quad (5)$$

$$N = \sin\left(\frac{m\pi x_s}{a}\right) \cdot \sin\left(\frac{n\pi y_s}{b}\right) \quad N' = \sin\left(\frac{m\pi x_p}{a}\right) \cdot \sin\left(\frac{n\pi y_p}{b}\right) \quad (6)$$

$$R_1 = \frac{Q_1}{C_1 \omega_0} \quad L_1 = \frac{4\mu_0}{abk_0^2} * \Delta_z \quad C_1 = \frac{ab}{4\mu_0 c^2} * \frac{1}{\Delta_z} \quad (7)$$

With  $\Delta_z$ , the dimension of a section of the cavity in the direction of the electric field and “ $c$ ” the speed of light.

Here, values of the coupling coefficients are given by the normalized value of the electrical field at the location of the antenna.

#### 4. NORM FOR MODE STIRRED CHAMBER (MSC)

It is important to note that there are normalized standards that describe techniques of test and measure in the MSC like the norm EN 61000-4-21<sup>[5]</sup>. This norm presents the minimal characteristics that MSC should verify in order to proceed in EMC tests.

Hence, to be able to use a chamber, the density of mode should be sufficiently high. Usually, the minimal operating frequency must be, at least, between 4 and 6 times the fundamental frequency so that the stirring of modes can be efficient.

In figure 5 we can see how the density of mode increases with frequency.

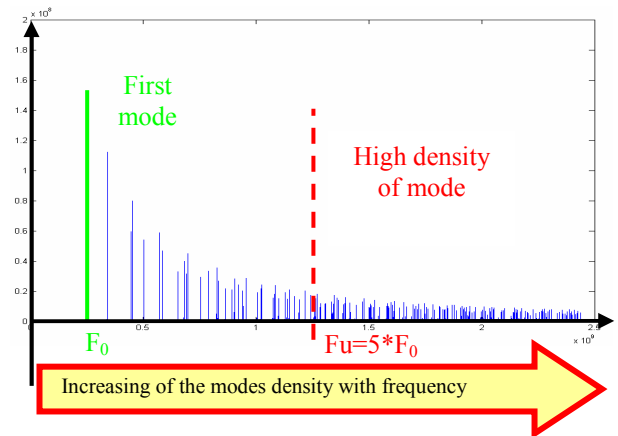


Figure 5: Modes density vs. frequency

## 5. COMPARISON BETWEEN SIMULATION RESULTS AND EIGEN VALUE SOLUTIONS

In order to validate the analogism between electrical field and output voltage of the equivalent diagram, we have to compare the results from simulation under Qucs<sup>[1]</sup> with those obtained from analytical equations and eigen value solutions.

We can use the analytical equations (3) and (4) to find the eigen vector and, thus, the resonance frequency according to it:

$$k_{m,n} = \sqrt{\left(\frac{m\Pi}{a}\right)^2 + \left(\frac{n\Pi}{b}\right)^2} \quad f_{m,n} = \frac{c_0 \times k_{m,n}}{2\Pi} \quad (8)$$

Then, in addition to the solution of the propagation equation, we can use equations (5) to (8) to calculate the electrical field inside the cavity and all the values of the passive components needed for simulations. Each computation has been done under a free open source partial differential equation solver, (PDE solver) Freefem++<sup>[6]</sup>.

Figure 6 displays the mesh of the stirred mode chamber realised with Freefem++.

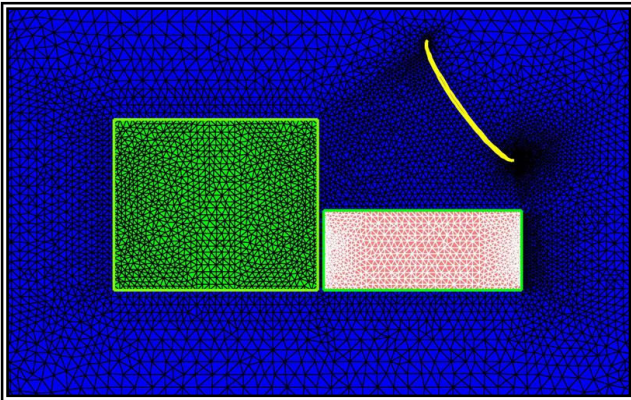


Figure 6: Mesh of the chamber by Freefem++

For example, we have for the first resonance mode  $R_1 = 185.75\text{M}\Omega$ ,  $L_1 = 0.2323\mu\text{H}$ ,  $C_1 = 1.5196\text{nF}$ ,  $N_1 = 0.12$ ,  $N_1' = 0.9$  and the resonance frequency  $f_1 = 267.86\text{MHz}$ .

We choose to simulate only the first fifteen modes in order to get a non-overloaded figure as shown below:

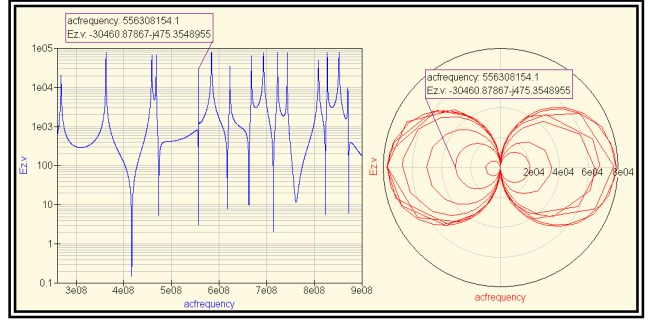


Figure 7: Simulation of the first fifteen modes of a 2D cavity under Qucs<sup>[1]</sup>

The left graph displays the semi-logarithmic representation of the output voltage as a function of the frequency; it shows band gaps that are due to the filtering function realized by the serial network.

On the right graph, we can see the polar representation of the same output that yields a visual information about the quantification step of the simulation. Indeed, the more simulation points we have the better the shapes of resonance circles are.

Finally, we compare the frequencies of the first fifty resonance modes with those from the PDE solver by calculating the relative variation. Results are displayed in the figure 8.

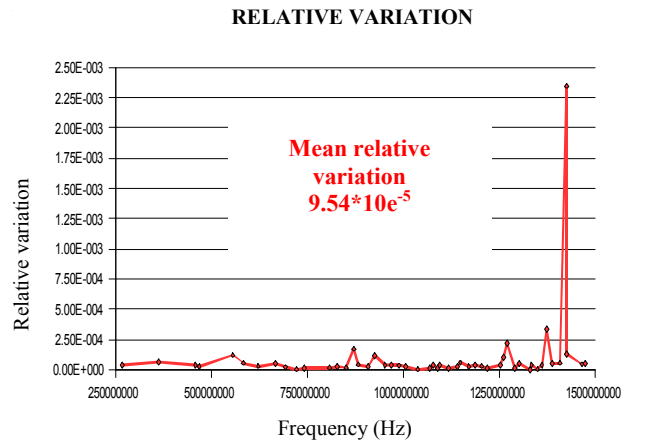


Figure 8: Relative variation between frequencies from simulation and frequencies from PDE solver

It appears that the mean relative variation value is very low, (about  $9.54 \cdot 10^{-5}$ ), except for one singular point.

## 6. CONCLUSION

The simulation results obtained under Qucs<sup>[1]</sup>, compared with those from PDE solver, are quite good with a mean relative variation below  $10^{-4}$ . It is important to note that both computing and exploitation are done under free open source softwares.

Work is going on with the integration of the emitting and receiving antennas in the equivalent diagram by determining the input and output complex impedance  $Z_e$  and  $Z_s$  and the direct coupling between them (figure 9).

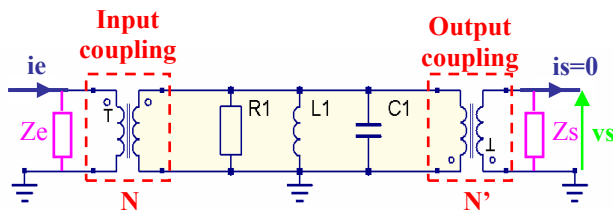


Figure 9: Equivalent diagram to a single resonance mode of the MSC with the modelling of the antennas

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