

# Averaging Level Control of Multiple Tanks: A Passivity Based Approach

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**Abstract**—Level control plays a fundamental role in the economic operation of flotation circuits. This paper shows that Interconnection and Damping Assignment Passivity-Based Control, well-known for electrical and mechanical applications, can be used to design high performance averaging level controllers for flotation circuits using physically motivated mass-balance principles. Using this method we can deal with multiple tanks described by nonlinear dynamics with soft state constraints and rigorously establish the stability of the closed loop system. Furthermore, the transient behavior can be easily regulated selecting the mass storage function assigned to the system. As far as we know, such performance specifications are unattainable with existing methods. The simulation of a three tank flotation circuit illustrates the main issues raised in this paper.

## I. INTRODUCTION

Control of liquid inventory in many industrial processes, and specifically in flotation plants, is an important basic problem. In this context, there are two competing objectives. First, the rate of change of flow from one vessel to another should be reasonably smooth to avoid upsetting downstream equipment. Second the minimum and maximum vessel level constraints must not be exceeded. Another important objective in a cascade of tanks is to minimize the outlet flow overshoot since this can be amplified as it moves down the cascade.

The traditional approach to tackle this control problem has involved the use of averaging controllers with proportional and integral modes or simple intuition based nonlinear controllers [1]. These averaging level controllers are deliberately detuned to produce slow outlet flow responses. However, they must be tuned tight enough to insure that the maximum and minimum vessel level constraints are not violated.

Recent developments have formulated the problem as an optimization one, where the objective is quantified by the maximum rate of change of outlet flow for a given inlet flow disturbance subject to level constraints [2], [3]. These works consider only one tank processes,

and a closed-loop stability analysis is conspicuous by its absence.

In this work a different approach to averaging level control is taken. The problem is formulated and solved using the approach of [4], where energy functions of the closed-loop system are properly shaped to satisfy the control objectives. In particular, in the spirit of [6], we adapt the Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) method reported in [5] to design high performance averaging level controllers for flotation circuits that shape the *total mass function*.<sup>1</sup>

As an illustrative example, the level control in a flotation circuit is considered. This is a very important control problem, since cell levels can be used effectively for controlling either the concentrate or the tailing grade from a specific flotation circuit. The main problems faced by conventional controllers stem from the couplings that exist in plants with connected stages and recycle streams. The instability of the upstream process; i.e. grinding circuit, and changes in the ore properties represent the major disturbances for the normal operation of flotation circuits. In practice, the grinding circuit frequently delivers a flow rate having big variations, which are transmitted through the flotation circuit. This effect is undesirable since it causes oscillations and affects the overall performance of the circuit [7]. Many authors, [7], [8], have discussed the use of linear controllers for this process but, to the best of our knowledge, no one has explored the use of tank capacities to filter the flow disturbance. The IDA-PBC method proposed here allows us to deal with multiple tanks described by nonlinear dynamics with hard state constraints and rigorously establish the stability of the closed loop system. Furthermore, the transient behavior can be easily regulated selecting the mass storage function—hence the tank capacity—assigned to the system.

This paper is organized as follows: Section 2 describes the averaging level problem. Section 3 provides a brief description of the mathematical model for the

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<sup>1</sup>Rigorously speaking, we don't deal with mass but with *volume*, but keep the former wording for ease of understanding.

hydraulic behavior of the flotation cells. Section 4 presents the IDA–PBC approach. Section 5 applies IDA–PBC to a simple flotation circuit. Some simulation results are presented in Section 6 and finally, in Section 7, some closing remarks are given.

## II. THE AVERAGING LEVEL CONTROL PROBLEM

Standard averaging level control applications consider the surge tank shown in figure 1. The model of the tank is

$$\frac{dV_1}{dt} = f_0 - f_1,$$

where incoming flow  $f_0$  acts a disturbance and  $f_1$  must be adjusted so as not to perturb the process downstream and keep the liquid volume in the tank constrained in a given range; i.e.

$$V_{min} \leq V_1 \leq V_{max}. \quad (1)$$

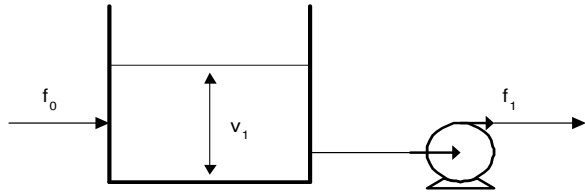


Fig. 1. Surge Tank

The averaging level control problem is to find a smooth outlet flow so that the inequalities (1) are satisfied. In this way, the downstream effect of the inlet flow disturbances is minimized.

This problem does not require tight control around a specific level set point, which itself is contrary to the inlet flow rate filtering objective. It is desirable to eventually return the tank level to a nominal value in order to have a capacity for filtering future inlet flow rate disturbances.

It is worth mentioning that in flotation circuits the tanks are interconnected and the outlet flow will be a function of two adjacent cell levels and the position of the valve's plug.

## III. FLOTATION CIRCUIT MODEL

In flotation circuits several flotation cells are connected in series, as depicted in Fig. 2. The equations describing the circuit can be obtained by doing a mass balance in each of them.

For the first cell the equation is:

$$\frac{dV_1}{dt} = f_0 - f_1,$$

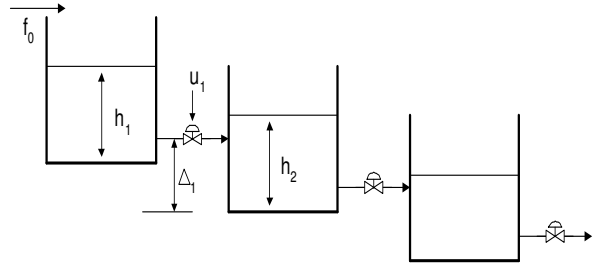


Fig. 2. Flotation circuit

where  $f_0$  is the feedrate to the first cell and  $f_1$  is given by:

$$f_1 = k_1(u_1)\sqrt{h_1 - h_2 + \Delta_1}.$$

In general,  $h_i \in \mathbb{R}_+$  is the pulp level associated to cell  $i$ ,  $\Delta_i$  is the physical difference in the height between cells  $i$  and  $i + 1$ , and  $k_i$  is a function of the valve opening  $u_i$ , then for any cell downstream in between cells the equations are:

$$\frac{dV_i}{dt} = f_{i-1} - f_i,$$

where

$$f_i = k_i(u_i)\sqrt{h_i - h_{i+1} + \Delta_i}. \quad (2)$$

For the last cell:

$$f_n = k_n(u_n)\sqrt{h_n + \Delta_n}.$$

In order to illustrate the method, and without any loss of generality, we consider a very simply circuit, with just three ideal cells; i.e. with constant cross-sections. In addition, we will also assume valves with linear characteristics,  $k_i(u_i) = u_i$ . The equations describing each cell in terms of volume variables are:

$$\begin{aligned} \dot{x}_1 &= f_0 - u_1\sqrt{\frac{x_1}{A_1} - \frac{x_2}{A_2} + \Delta_1} \\ \dot{x}_2 &= u_1\sqrt{\frac{x_1}{A_1} - \frac{x_2}{A_2} + \Delta_1} - u_2\sqrt{\frac{x_2}{A_2} - \frac{x_3}{A_3} + \Delta_2} \\ \dot{x}_3 &= u_2\sqrt{\frac{x_2}{A_2} - \frac{x_3}{A_3} + \Delta_2} - u_3\sqrt{\frac{x_3}{A_3} + \Delta_3} \end{aligned}$$

To apply IDA–PBC it is convenient to represent the system in port–controlled Hamiltonian form [9]. For, we define a total mass (volume) function

$$H(x) = x_1 + x_2 + x_3 \geq 0,$$

and write the system dynamics as:

$$\dot{x} = [J(x, u) - R(x, u)]\nabla H(x) + gd, \quad (3)$$

where  $x = \text{col}(x_1, x_2, x_3)$ ,  $u = \text{col}(u_1, u_2, u_3)$ ,  $g$  is a real constant,  $d = f_0$ , the skew-symmetric interconnection matrix  $J(x, u)$  is

$$J_1 u_1 \sqrt{\frac{x_1}{A_1} - \frac{x_2}{A_2} + \Delta_1} + J_2 u_2 \sqrt{\frac{x_2}{A_2} - \frac{x_3}{A_3} + \Delta_2}, \quad (4)$$

the matrices  $J_1$  and  $J_2$  are defined as:

$$J_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad J_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

The damping and input matrices are:

$$R(x, u) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & u_3 \sqrt{\frac{x_3}{A_3} + \Delta_3} \end{bmatrix}, \quad g = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

and  $d = f_0$  represents an *unknown constant disturbance*.

It is important to remark that the control signals must be positive, since they represent valve openings. If this basic requirement is satisfied by the controller design, then  $R(x, u) \geq 0$ .

#### IV. CONTROLLER DESIGN USING IDA-PBC

The control objective in IDA-PBC is achieved by assigning to the closed-loop a desired total mass function that captures the desired performance. In some applications, see [5], to shape the mass function it is necessary to modify the interconnection and/or damping matrices—hence the name IDA. Interestingly, in the present application we can assign arbitrary mass functions without modifying these matrices. Therefore, we fix as desired closed-loop dynamics

$$\dot{x} = [J(x, u) - R(x, u)] \nabla H_d \quad (5)$$

where  $H_d(x)$  is the desired total mass function fixed by the designer. Evaluating the time derivative of the mass function along the closed-loop dynamics and if  $R(x, u) \geq 0$  we get

$$\dot{H}_d = -(\nabla H_d)^\top R(x, u) \nabla H_d \leq 0;$$

Hence, to stabilize a given equilibrium  $x^*$ , it suffices to select  $H_d(x)$  with an isolated minimum at  $x^*$ . That is, such that

- *Necessary extremum assignment condition:*

$$\nabla H_d(x^*) = 0.$$

- *Sufficient minimum assignment condition:*

$$\nabla^2 H_d(x^*) > 0.$$

Defining  $H_d(x) = H(x) + H_a(x)$ , and equating (3) and (5) we obtain the key matching equation

$$[J(x, u) - R(x, u)] \nabla H_a = -gd. \quad (6)$$

For a given function  $H_a(x)$ , solving this equation for  $u$  defines the state-feedback control that achieves the desired mass shaping objective, and consequently stabilizes  $x^*$ .

From (6) it is clear that the computation of the control requires knowledge of the disturbance  $d$ . In the next section we show that, for the particular problem at hand, the control depends linearly on  $d$ , and standard adaptive control techniques can be used to estimate the disturbance.

#### V. APPLICATION TO A FLOTATION CIRCUIT

The basic idea is to shape the total mass functions associated to each tank. For smooth outlet flow a flat energy function around a nominal level,  $H_{d1}$ , will be required, as illustrated in Fig. 3. However, to keep a tight control around the set point, a energy function, like function  $H_{d2}$ , which penalizes big errors should be designed.

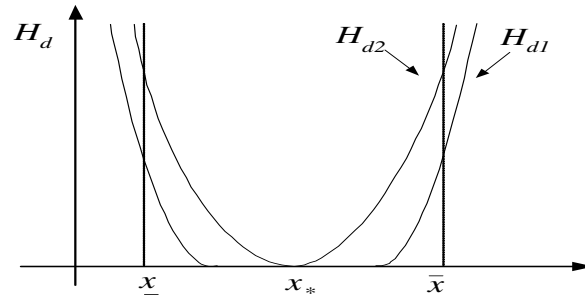


Fig. 3. Total mass functions

In our application we will use the first tank to damp the inflow disturbances; i.e. we will allow its level to move within certain boundaries. However, the levels of the downstream tanks must be kept constant to obtain a desired concentrate grade. Towards this end, the following desired total mass function is proposed:

$$H_d(x) = H(x) + H_a(x) = \sum_{i=1}^3 x_i + \phi_i(x_i, x_{i*}), \quad (7)$$

where the functions  $\phi_i(x_i, x_{i*})$  must assign the equilibrium point to the closed loop system and capture the objectives described above. The functions  $\phi_i(x_i, x_{i*})$  can be chosen as:

$$\phi_i(x_i, x_{i*}) = -x_{i*} \ln(x_i). \quad (8)$$

The rationale behind the choice of (8) lies in the fact, that it provides simple control laws, which take into account the non-negativity characteristics of the manipulated variable.

In addition, we will also explore if these functions can be designed to constrain the states to stay within certain boundaries  $[\underline{x}_i, \bar{x}_i]$ , for instance one of these functions could be:

$$\phi_i(x_i, x_{i*}) = -x_i + \alpha \left[ \frac{(x_{i*} - \underline{x}_i)^2}{x_i - \underline{x}_i} - \frac{(\bar{x}_i - x_{i*})^2}{\bar{x}_i - x_i} \right], \quad (9)$$

where the positive real constant  $\alpha$  defines the flatness of the mass function. Given the desired closed loop energy function  $H_d(x)$ , the control law can be calculated from (6) as

$$\begin{bmatrix} 0 & -\hat{u}_1 & 0 \\ \hat{u}_1 & 0 & -\hat{u}_2 \\ 0 & \hat{u}_2 & -\hat{u}_3 \end{bmatrix} \begin{bmatrix} \frac{\partial H_d}{\partial x_1} \\ \frac{\partial H_d}{\partial x_2} \\ \frac{\partial H_d}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \hat{d} \quad (10)$$

where, for notational convenience, we have defined

$$\begin{aligned} \hat{u}_1 &= u_1 \sqrt{\frac{x_1}{A_1} - \frac{x_2}{A_2} + \Delta_1}, \\ \hat{u}_2 &= u_2 \sqrt{\frac{x_2}{A_2} - \frac{x_3}{A_3} + \Delta_2}, \\ \hat{u}_3 &= u_3 \sqrt{\frac{x_3}{A_3} + \Delta_3}, \end{aligned} \quad (11)$$

and we notice that we have replaced the unknown disturbance  $d$  by its estimate denoted  $\hat{d}$ —whose update law will be given below. Solving (10) we get

$$\begin{aligned} \hat{u}_1 &= -\frac{1}{\frac{\partial H_d}{\partial x_2}} \hat{d} \\ \hat{u}_2 &= -\frac{\frac{\partial H_d}{\partial x_1}}{\frac{\partial H_d}{\partial x_2} \frac{\partial H_d}{\partial x_3}} \hat{d} \\ \hat{u}_3 &= -\frac{\frac{\partial H_d}{\partial x_1}}{\frac{\partial H_d}{\partial x_3}} \hat{d}. \end{aligned}$$

Replacing this equation into (3) we obtain

$$\dot{x} = [J(x, u) - R(x, u)] \nabla H_d + g \tilde{d}$$

where  $\tilde{d} = \hat{d} - d$ . Now,

$$\dot{H}_d = -(\nabla H_d)^\top R(x, u) \nabla H_d + (\nabla H_d)^\top g \tilde{d}. \quad (12)$$

Since  $d$  is a flowrate, it make sense to constrain its estimate to be positive. Following the standard procedure of adaptive control we define the Lyapunov function candidate as in [10]

$$W(x, \tilde{d}) = H_d(x) + \frac{1}{2\gamma} \ln \left[ \frac{(\hat{d} - \bar{d})^\tau}{(\hat{d} - \underline{d})^{\tau+1}} \right],$$

where  $\gamma$  is a positive real constant that, as will be shown below, defines the adaptation speed,  $\tau$  is defined as  $\tau = \frac{\underline{d} - \bar{d}}{\bar{d} - \underline{d}} < 0$ ,  $\tau + 1 > 0$  and the variables  $\underline{d}$  and  $\bar{d}$  define the upper and lower bounds for the flowrate estimate. Selecting

$$\dot{\hat{d}} = \gamma(\hat{d} - \bar{d})(\hat{d} - \underline{d}) \nabla H_d^\top g,$$

we cancel in  $\dot{W}$  the second right hand term of (12) yielding

$$\dot{W} = -(\nabla H_d)^\top R(x, u) \nabla H_d \leq 0$$

which establishes stability of the equilibrium for all  $x_1$  satisfying

$$\frac{\partial \phi_1}{\partial x_1} \leq 0. \quad (13)$$

Notice that for functions defined as equation (8), this condition is always meet; however, for the hard constrained case, equation (9), the condition (13) is only satisfied for all  $x_1$  within the interval  $[\underline{x}_1, x_\alpha]$ , where  $x_\alpha$  satisfies

$$1 + \alpha \frac{(x_{1*} - \underline{x}_1)^2}{(x_\alpha - \underline{x}_1)^2} = \alpha \frac{(\bar{x}_i - x_{1*})^2}{(\bar{x}_1 - x_\alpha)^2}. \quad (14)$$

We close this section noting that the parameter update law takes the simple form

$$\dot{\hat{d}} = \gamma(\hat{d} - \bar{d})(\hat{d} - \underline{d}) \left( 1 + \frac{\partial \phi_1}{\partial x_1} \right),$$

and all resulting control signals are positive; i.e.  $u_i \in \mathbb{R}_+$ , which agree with their physical meaning.

## VI. SOME SIMULATION RESULTS

To simulate the system we have used function (8) to shape the mass function. Fig. 4 shows the performance of the algorithm for step changes in the set-point of the second tank and in the input flow rate. In this experiment  $\gamma = 0.1$  and  $\alpha = 0.01$ .

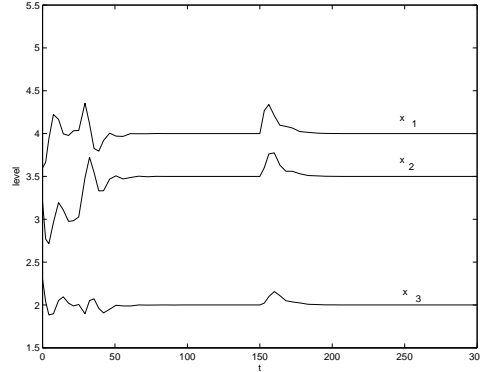


Fig. 4. Step responses :Same energy function for all cells.

In the second experiment, the energy function of the first tank is assigned to be like equation (9), which allows its level to move within  $[3.3, 5.5]$ . As seen in Fig. 4, the direct effect of this approach is to attenuate the effects of the disturbance on the downstream cells. Some improvements on the transient responses for step changes in the references are also obtained.

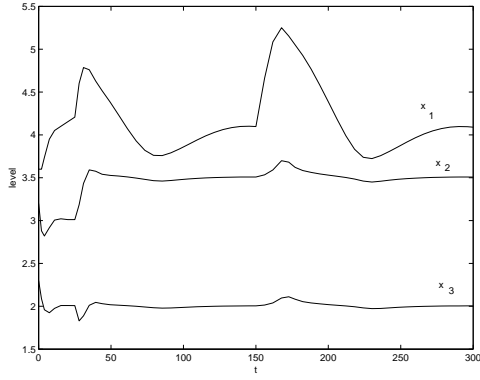


Fig. 5. Step responses: Energy function associated to the first cell as (9)

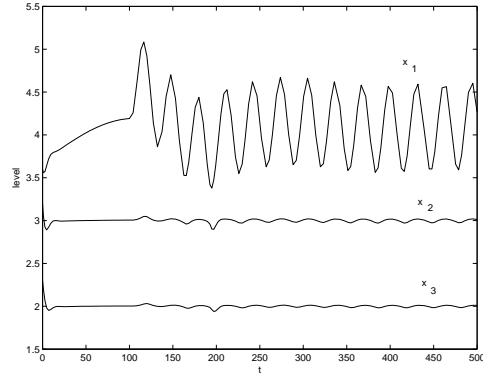


Fig. 7. Periodic disturbance: Energy function associated to the first cell as (9)

The effect of filtering provided by the first cell can be illustrated by considering a periodic disturbance. Fig. 6 shows the propagation effect of the disturbance, when all the energy functions are selected to maintain a given set-point. However, if the first tank uses its capacity for filtering, the effect of the disturbance is attenuated in the downstream stages, as seen in Fig. 7.

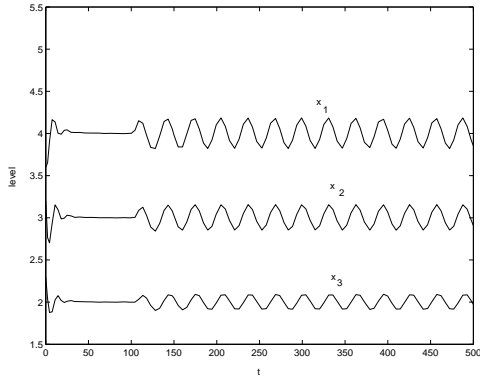


Fig. 6. Periodic disturbance : Same energy function for all cells.

The control signal for all the valves are smooth for both schemes, as seen in figures 8 and 9. We can also see in figure 9 the effect of the energy function associated to the first tank, which reduce the magnitude of the control signal. Notice that around  $t = 200$  the controller is able to maintain the level of the first tank with the defined boundaries.

For sake of a fair comparison, three PI controllers were designed, considering their outputs as  $\hat{u}_i$  in equation 11. Even though the simulation results, figure 10, shows a bigger attenuation of the disturbance than the one obtained with passivity-based method with the same type of energy function for each tank; the disturbance is propagated down up to the last tank.

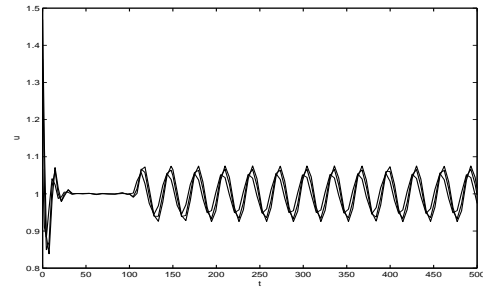


Fig. 8. Control signals for periodic disturbance : Same energy function for all cells.

We also notice a slight amplification of the oscillation in the last tank, this result is consistent with the ones reported in [11]. Figure 11 shows well behaved control signals for all three tanks.

## VII. FINAL REMARKS

We have proposed a new approach based on shaping the total mass function to extend the systematic design of averaging controller to multi-tanks systems. A simple example, considering a three tanks flotation circuit, has illustrated the main features of the proposed approach. Future work will consider the nonlinear characteristics of the actuator, the effect of adding damping to the controller, as well as the presence of nonconstant disturbances, in the feed rate of the first tank.

## VIII. ACKNOWLEDGMENTS

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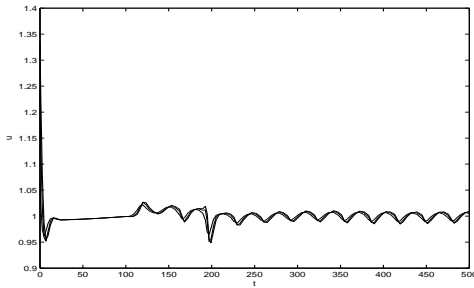


Fig. 9. Control signals for periodic disturbance: Energy function associated to the first cell as (9).

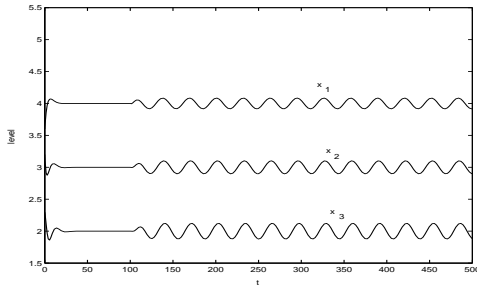


Fig. 10. Periodic disturbance: PI controllers.

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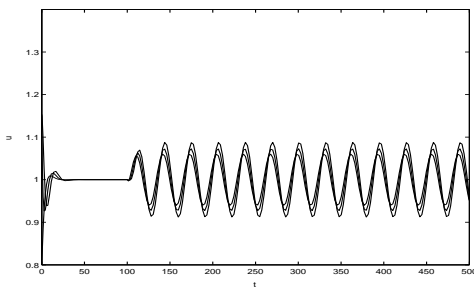


Fig. 11. Control signals for periodic disturbance: PI controllers.