

# A Robustly Stable Output Feedback Saturated Controller for the Boost DC-to-DC Converter<sup>2</sup>

Hugo Rodriguez<sup>a</sup> Romeo Ortega<sup>a,1</sup> Gerardo Escobar<sup>b</sup>  
Nikita Barabanov<sup>c</sup>

<sup>a</sup> *Laboratoire des Signaux et Systèmes, Supelec, 91192, Gif-sur-Yvette, FRANCE*

<sup>b</sup> *Department of Electrical and Computer Engineering, Northeastern University, Boston, USA*

<sup>c</sup> *Department of Software Engineering, Electrotechnical University, St. Petersburg, RUSSIA*

---

## Abstract

The problem of regulating the output voltage of the Boost DC-to-DC power converter has attracted the attention of many control researchers for several years now. Besides its practical relevance, the system is an interesting theoretical case study because it is a switched device whose averaged dynamics are described by a bilinear second order nonminimum phase system with saturated input, partial state measurement and a highly uncertain parameter –the load resistance–. In this paper we propose an output-feedback saturated controller which ensures regulation of the desired output voltage and is, at the same time, insensitive to uncertainty in the load resistance. Furthermore, bounds on this parameter can be used to tune the controller so as to (locally) ensure robust performance, e.g., that the transient has no (under)over-shoot. The controller, which is designed following the passivity-based interconnection and damping assignment methodology recently proposed in the literature, is a static nonlinear output feedback. This allows us to invoke simple phase-plane techniques to determine the exact domain of attraction as well as the admissible initial conditions that ensure the control objectives. One final advantage of our controller is that it is computationally less demanding than the industry standard lead-lag filters.

---

<sup>1</sup> Author to whom all correspondence should be addressed.

<sup>2</sup> This work has been partially supported by the CONACyT of Mexico

## 1 Problem formulation

The averaged model of the DC-to-DC Boost converter depicted in Fig. 1 is given by [3], [4]

$$\dot{x}_1 = -\frac{1}{C}ux_2 + E \quad (1)$$

$$\dot{x}_2 = -\frac{1}{RC}x_2 + \frac{1}{L}ux_1 \quad (2)$$

$$x(0) = (x_1(0), x_2(0)) \in \mathcal{R}_{>0}^2 \quad (3)$$

where  $x_1$  is the inductance flux,  $x_2$  is the charge in the capacitor voltage, and  $u$  is the continuous control signal, which represents the slew rate of a PWM circuit controlling the switch position in the converter. The positive constants  $C$ ,  $L$ ,  $R$ ,  $E$  are the capacitance, inductance, load resistance, and voltage source, respectively, and  $\mathcal{R}_{>0}^2$  denotes the open first quadrant.

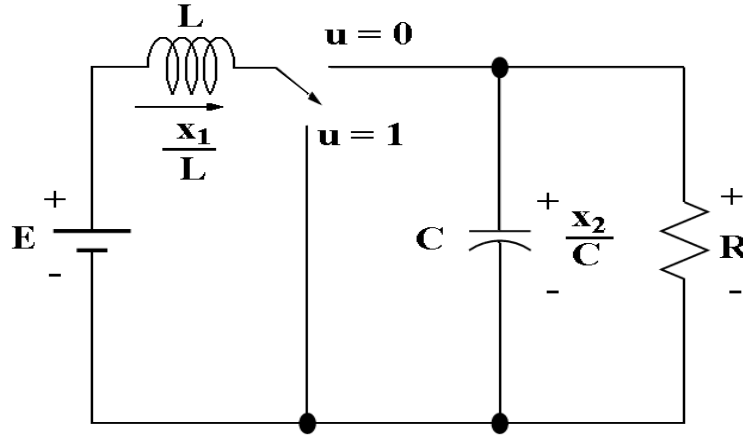


Fig. 1. Boost converter circuit.

The following conditions of operation of the device are imposed by technological considerations:

- C.1** The only signal available for measurement is  $x_2$ .
- C.2** The control signal  $u$  ranges in the set  $(0, 1)$ .
- C.3** The state vector  $x$  lives in  $\mathcal{R}_{>0}^2$ .
- C.4** The load resistance  $R$  is unknown.

The *control objective* is to regulate the output capacitor voltage  $\frac{1}{C}x_2$  to a desired constant value  $V_* > E$ , verifying the conditions **C.1**–**C.4**.

The *main contribution* of this paper is to show that the problem can be solved following the interconnection and damping assignment passivity-based controller (IDA-PBC) design methodology proposed in [5]. Furthermore, the re-

sulting controller is a simple static nonlinear output feedback, hence it is computationally less demanding than (even) the industry standard lead-lag filters.

### Remarks

1. If we fix  $u$  to a constant value, the equilibria of (1) (denoted  $\bar{x}$ ), verify the algebraic relation

$$\bar{x}_1 = \frac{L}{REC^2} \bar{x}_2^2 \quad (4)$$

Hence, if we set  $\frac{1}{C}x_2$  at the desired output voltage  $V_*$ , we get the equilibrium point we want to stabilize  $x_*$  and the corresponding constant control  $u_*$  as

$$x_* = (x_{1*}, x_{2*}) = \left( \frac{L}{RE} V_*^2, CV_* \right), \quad u_* = \frac{E}{V_*} \quad (5)$$

2. It is interesting to note that the *open-loop* control  $u = u_*$  solves the problem above. However, the absence of feedback renders this “solution” highly sensitive to unavoidable parasitic effects and noise. The feedback controller proposed in this paper adds a suitable “state-modulation” to the open-loop control. This type of feedback, explained in more detail in [5],<sup>3</sup> does not rely on the generation of error signals (i.e.,  $x_2 - x_{2*}$ ), which although natural for linear systems, are not necessarily the best choice in nonlinear design. In general, state-modulated feedbacks –as opposed to error signal-based feedbacks– lead to *non-quadratic* Energy–Lyapunov functions, which may provide a more accurate description of the system dynamics.

3. It can be shown, see e.g. [4], that (1) defines a non–minimum phase system with respect to the output  $x_2$ . On the other hand, it is minimum phase with respect to  $x_1$ . Due to this property,  $x_2$  is usually controlled indirectly via regulation of  $x_1$ . This indirect approach is further motivated by the fact that (in the operating region  $\mathcal{R}_{>0}^2$ ) there is a one-to-one correspondence between  $\bar{x}_1$  and  $\bar{x}_2$ , see (4). It is however clear that indirect regulation is extremely sensitive to uncertainty in  $R$ . Adaptation schemes, which incorporate an integral action, have been used to alleviate this drawback, at the expense of computational complexity. An important feature of our scheme is that, in spite of the non–minimum phase obstacle,  $x_2$  is regulated directly. This feature is intrinsic to the IDA–PBC designs of [5], which do not rely on systems inversion.

---

<sup>3</sup> See also [6] for an application of state-modulated feedback to mass balance systems.

4. Many different techniques have been used to address the control problem posed above. They range from the classical linear lead–lag designs exposed in [3], to complicated nonlinear dynamic state feedbacks using linearization [8] or passivation [9] ideas. In [5] we derived a stabilizing controller using the same energy–balancing ideas exploited here. Unfortunately, the resulting controller does not satisfy conditions **C.1–C.4**, in particular, it requires measurement of the full state and exact knowledge of  $R$ . See [4] for a list of references and [1] for a comparative experimental study. In spite of this intensive research, to the best of our knowledge, the control problem with all the specifications mentioned above remained open.

## 2 Main result

This section presents our main result whose proof is established in Section 4.

**Proposition 1** Consider the DC–to–DC Boost converter model (1) in closed–loop with the static output feedback controller

$$u = u_* \left( \frac{x_2}{x_{2*}} \right)^\alpha, \quad 0 < \alpha < 1 \quad (6)$$

where  $u_*$ ,  $x_{2*}$  are defined in (5), with  $V_* > E$ . Then, for all load resistances  $R > 0$ :

- (i)  $x_*$  is an *asymptotically stable* equilibrium with a proper Lyapunov function (defined in  $\mathcal{R}_{>0}^2$ )

$$H_d(x) = \frac{1}{2L}x_1^2 + \frac{1}{2C}x_2^2 + \kappa_1 x_2^{2(1-\alpha)} - (\kappa_2 + \kappa_3 x_1)x_2^{1-\alpha} \quad (7)$$

where  $\kappa_i$ ,  $i = 1, 2, 3$  are some positive constants given in (19).

- (ii) An estimate of the domain of attraction, which guarantees the state trajectories remain in  $\mathcal{R}_{>0}^2$ , is given by<sup>4</sup>

$$\Xi_\alpha \triangleq \{x | x \in \mathcal{R}_{>0}^2 \text{ and } H_d(x) \leq H_d(0, x_{2*})\}$$

That is, for all  $x(0) \in \Xi_\alpha$  we have  $x(t) \in \Xi_\alpha$ ,  $\forall t \geq 0$  and  $\lim_{t \rightarrow \infty} x(t) = x_*$ .

- (iii) For all  $x_*$  we can define a value of  $0 < \alpha < 1$  such that, if  $x(0) \in \Xi_\alpha$  then  $0 < u(t) < 1$ ,  $\forall t \geq 0$ .
- (iv) Assume  $0 < R \leq R_M$ , and let  $0 < \alpha \leq \alpha_M$ , where

---

<sup>4</sup> In Section 4 we determine the *exact* domain of attraction, which is much larger than our estimate. However, this set is given in terms of trajectories thus can only be determined numerically.

$$\alpha_M \triangleq 1 + \frac{2}{x_{1*}} \left[ RCE - \sqrt{2LV_*x_{2*} + R^2C^2E^2} \right] < 1$$

$$R_M \triangleq \frac{L}{RC}(1 - \alpha_M)$$

Then, there exists a (sufficiently small) constant  $\delta > 0$  such that, if  $|x(0) - x_*| \leq \delta$ , then the output voltage  $x_2(t)$  exponentially converges to  $x_{2*}$  *without (under)overshoot*.

### 3 Interconnection and damping assignment passivity-based control

Before presenting the proof of Proposition 1 we briefly review here the IDA-PBC methodology of [5] which was used to derive the controller. We refer the reader to that paper for further details and proofs of the statements.

Towards this end, we write the dynamics of the Boost converter in the form of port-controlled Hamiltonian (PCH) system as<sup>5</sup>

$$\dot{x} = [J(u) - \mathcal{R}] \frac{\partial H}{\partial x}(x) + gE \quad (8)$$

where

$$H(x) = \frac{1}{2L}x_1^2 + \frac{1}{2C}x_2^2$$

is the total energy of the device, and

$$J(u) = \begin{bmatrix} 0 & -u \\ u & 0 \end{bmatrix}, \quad \mathcal{R} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{bmatrix}, \quad g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

represent the internal interconnection, damping and external interconnection matrices, respectively. Notice that,  $J(u) = -J^T(u)$  and  $\mathcal{R} = \mathcal{R}^T \geq 0$ , reflecting the energy conservation and dissipation features of PCH dynamics.

The design proceeds in three steps:

---

<sup>5</sup> As pointed out in [2], in switching devices like the power converters, the control signal is not a port signal, it affects the dynamics via the interconnection matrix.

1) (*Interconnection and damping assignment*) From physical considerations we *fix* the desired interconnection and damping matrices. We leave here  $J$  untouched and only modify the damping structure as<sup>6</sup>

$$\mathcal{R}_d \triangleq \mathcal{R} + \mathcal{R}_a \geq 0 \quad (9)$$

2) (*Integrability conditions*) We *compute* a family of energy functions –and corresponding controls  $u = \beta(x)$ – that can be assigned preserving the PCH structure. That is, we want our closed–loop dynamics to be of the form

$$\dot{x} = [J(\beta(x)) - \mathcal{R}_d] \frac{\partial H_d}{\partial x}(x) \quad (10)$$

with the closed–loop energy function

$$H_d(x) \triangleq H(x) + H_a(x) \quad (11)$$

It is easy to see that this is achieved if and only if the partial differential equation (PDE)

$$[J(\beta(x)) - \mathcal{R}_d] K(x) = \mathcal{R}_a \frac{\partial H}{\partial x}(x) + gE \quad (12)$$

$$K(x) \triangleq \frac{\partial H_a}{\partial x}(x) \quad (13)$$

is solvable for  $H_a$ . This in its turn is equivalent (under the assumption that  $J(\beta(x)) - \mathcal{R}_d$  is invertible) to

$$\frac{\partial K}{\partial x}(x) = \left[ \frac{\partial K}{\partial x}(x) \right]^T$$

The latter is a PDE to be solved directly for  $\beta$ , which (because of **C.1**) should be a function only of  $x_2$ .

3) (*Energy–shaping*) From (10) we have

$$\dot{H}_d = - \left[ \frac{\partial H_d}{\partial x}(x) \right]^T \mathcal{R}_d \frac{\partial H_d}{\partial x}(x) \leq 0 \quad (14)$$

---

<sup>6</sup> In standard applications of PBC damping is injected to achieve asymptotic convergence, see e.g. [4]. As explained in [5], in IDA–PBC the choice of  $\mathcal{R}_a$  is guided by other considerations; in the present case it is motivated by the solvability of the output–feedback problem.

Hence,  $H_d$  is a Lyapunov function if it has a minimum at the desired equilibrium  $x_*$ . Furthermore, to ensure asymptotic stability, the following implication should also be true

$$\left[ \frac{\partial H_d}{\partial x}(x) \right]^T \mathcal{R}_d \frac{\partial H_d}{\partial x}(x) = 0 \Rightarrow x = x_*$$

In the third step of the procedure we choose, among the family of controls obtained in step 2) above, one that ensures these two properties.

It will be shown below that all this is possible if we remove the damping from the voltage coordinate  $x_2$  and inject it into the current  $x_1$ .<sup>7</sup> Interestingly enough, this can be done *without knowledge* of  $R$ . A circuit-theoretic interpretation of the control action is given in Section 6.

#### 4 Proof of Proposition 1

The proof is divided into four parts corresponding to points (i)–(iv).

(i) (*Asymptotic stability*) Following the design methodology outlined above, we select the injected damping matrix as  $\mathcal{R}_a = \text{diag}\{R_a, -\frac{1}{R}\}$ ,  $R_a > 0$ . According to (9), this yields the closed-loop damping  $\mathcal{R}_d = \text{diag}\{R_a, 0\}$ . Replacing in (12), and *assuming* the matrix  $J(\beta(x)) - \mathcal{R}_d$  is invertible, we obtain

$$K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \frac{1}{\beta(x)} \begin{bmatrix} -\frac{1}{RC}x_2 \\ -\frac{1}{L}R_ax_1 - E + \frac{R_a}{RC}\frac{x_2}{\beta(x_2)} \end{bmatrix} \quad (15)$$

A necessary and sufficient condition for this vector function to be a gradient function (13) is given by  $\frac{\partial K_2}{\partial x_1}(x) = \frac{\partial K_1}{\partial x_2}(x)$ . Now, since  $u = \beta(x_2)$  (see **C.1**), this condition reduces to the simple ODE

$$\frac{d\beta}{dx_2}(x_2) = \frac{\alpha}{x_2}\beta(x_2)$$

where we have defined  $\alpha \triangleq 1 - \frac{R_a RC}{L}$ . This ODE can be easily solved by the separation of variables method to get  $u = c_1 x_2^\alpha$ , where  $c_1$  is a constant which we choose to assign the equilibrium, that is, such that<sup>8</sup>  $\frac{\partial H_d}{\partial x}(x_*) =$

<sup>7</sup> A similar idea was used in [6] to solve an output-feedback problem for a class of mass-balance systems.

<sup>8</sup> Recall from (13) that  $K = \frac{\partial H_a}{\partial x}(x)$ .

$\frac{\partial H}{\partial x}(x_*) + \frac{\partial H_a}{\partial x}(x_*) = 0$ . This yields  $c_1 = \frac{u_*}{x_2^2}$ . Now, to ensure that  $x_*$  is not just an extremum but a minimum of  $H_d$  we look at its Hessian (evaluated at  $x_*$ )

$$\frac{\partial^2 H_d}{\partial x^2}(x_*) = \begin{bmatrix} \frac{1}{L} & -\frac{R_a}{u_* L} \\ -\frac{R_a}{u_* L} \frac{1}{C} + \frac{(R_a x_{1*} + EL)\alpha}{u_* L x_{2*}} & \frac{(1-2\alpha)R_a}{u_*^2 RC} \end{bmatrix}$$

This matrix is positive definite if and only if  $-1 < \alpha < 1$ .

Evaluating  $\dot{H}_d$  along the closed-loop dynamics (1), (6), which –for future reference– we write explicitly as

$$\dot{x}_1 = -R_a \frac{\partial H_d}{\partial x_1} - \beta(x_2) \frac{\partial H_d}{\partial x_2} \quad (16)$$

$$\dot{x}_2 = \beta(x_2) \frac{\partial H_d}{\partial x_1} \quad (17)$$

we get

$$\dot{H}_d = -R_a \left( \frac{\partial H_d}{\partial x_1} \right)^2 \quad (18)$$

We have thus established that  $x_*$  is a stable equilibrium point *for all*  $-1 < \alpha < 1$ .

We will now invoke La Salle's invariance principle to prove *asymptotic* stability and define the domain of attraction. We have the following chain of implications

$$\begin{aligned} \frac{\partial H_d}{\partial x_1} = 0 &\Rightarrow \dot{x}_2 = 0 && (\Leftarrow (17)) \\ &\Rightarrow x_2 = \text{const} \\ &\Rightarrow u = \text{const} && (\Leftarrow (6)) \\ &\Rightarrow x_1 = \text{const} && (\Leftarrow (1)) \\ &\Rightarrow x = \bar{x} && (\text{i.e., an equilibrium point (4)}) \end{aligned}$$

Now, it is possible to show that  $\{x | x_1 = \frac{L}{REC^2} x_2^2\} \cap \{x | \frac{\partial H_d}{\partial x_1}(x) = 0\} = x_*$ , which completes the proof of asymptotic stability.

It is interesting to note that to prove this point we did not need to actually evaluate the Lyapunov function  $H_d$ . It can be, however, easily derived

integrating  $\frac{\partial H_a}{\partial x}$  to get (7) with the positive constants

$$\kappa_1 \triangleq \frac{1}{2} \frac{Lx_{2*}^{2\alpha}}{u_*^2 R^2 C^2}, \quad \kappa_2 \triangleq \frac{ELx_{2*}^\alpha}{RCR_a u_*}, \quad \kappa_3 \triangleq \frac{x_{2*}^\alpha}{u_* RC} \quad (19)$$

(ii) (*Domain of attraction*) Notice that, up to this point, we have not proved that the trajectories of the closed-loop system remain in  $\mathcal{R}_{>0}^2$  (see **C.3** in Section 1). Furthermore, our analysis is only valid in the *open* right hand plane, where  $J(u) - \mathcal{R}_d$  is invertible. Actually, it can be shown that trajectories may leave the first quadrant and even leave the open right-hand plane. A first modification that is needed to avoid this situation is to restrict ourselves to  $0 < \alpha < 1$ .<sup>9</sup> In this case, it is easy to see that  $x_2(t) \equiv 0$  is a trajectory of the closed loop dynamics (1), (6). Unfortunately, this does not mean that  $x(t)$  will live in the half plane  $x_2 > 0$ , because the closed-loop vector field is not continuous at  $x_2 = 0$  and uniqueness of solutions is no more guaranteed! It is clear that a more detailed analysis, that we carry out below, is needed to complete our proof.

We will study then the closed-loop dynamics obtained replacing (6) in (1). To get a first qualitative estimate of its behaviour we introduce a partition of the right-half plane defined by the curves  $\dot{x}_1 = 0$  and  $\dot{x}_2 = 0$ . Notice that the first graph is the line  $x_2^{\alpha+1} = \frac{CEx_{2*}^\alpha}{u_*}$ , which of course coincides with  $x_2 = x_{2*}$ . The second graph is defined only on the first quadrant, and is given by  $x_1 = \frac{Lx_{2*}^\alpha}{RCu_*} x_2^{1-\alpha}$ . These graphs, which intersect in  $x_*$ , are shown in Fig. 2 together with two representative trajectories (with initial conditions (0, 15) and (9, 120), respectively) and plots of the vector field. We see from this figure that trajectories starting from the positive vertical axis turn (clockwise) around  $x_*$ , with some of them converging to that point, while others (not shown) hit the negative vertical axis.<sup>10</sup> The *exact* domain of attraction of  $x_*$  is then defined by the trajectory that crosses at the zero point, and can be determined numerically integrating back from the origin. The result is shown in Fig. 3, together with a zoom around the equilibrium point. Remark that there are trajectories starting in this set that temporarily leave  $\mathcal{R}_{>0}^2$  (eventually crossing back to converge to  $x_*$ ). To comply with the condition **C.3** we have to restrict the set of admissible initial conditions. Noticing that on the interval of the half line  $x_1 = 0$ ,  $x_{2*} > x_2 > 0$ , the vector field points towards  $\mathcal{R}_{>0}^2$ , it is easy to see that the admissible set is then limited by the trajectory which is tangent to the axes  $x_1 = 0$  at the point  $x_2 = x_{2*}$ .

Even though the derivations above provide us with an exact characterization of

<sup>9</sup> In Section 5 we will show, via simulations, that the transient responses for  $0 < \alpha < 1$  are “faster” than those for  $-1 < \alpha < 0$ . Notice that  $\alpha = 0$  corresponds to the *open-loop* control  $u = u_*$ .

<sup>10</sup> Remember that there are no unique solutions for trajectories starting in  $x_2 = 0$ .

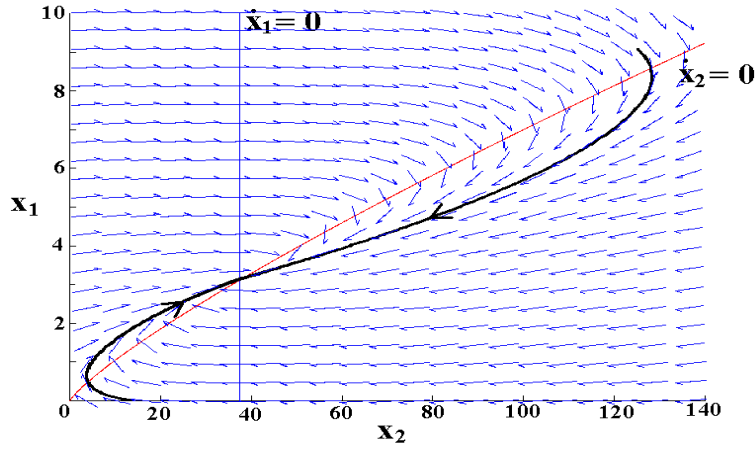


Fig. 2. Partition of the state space, two trajectories and vector fields.

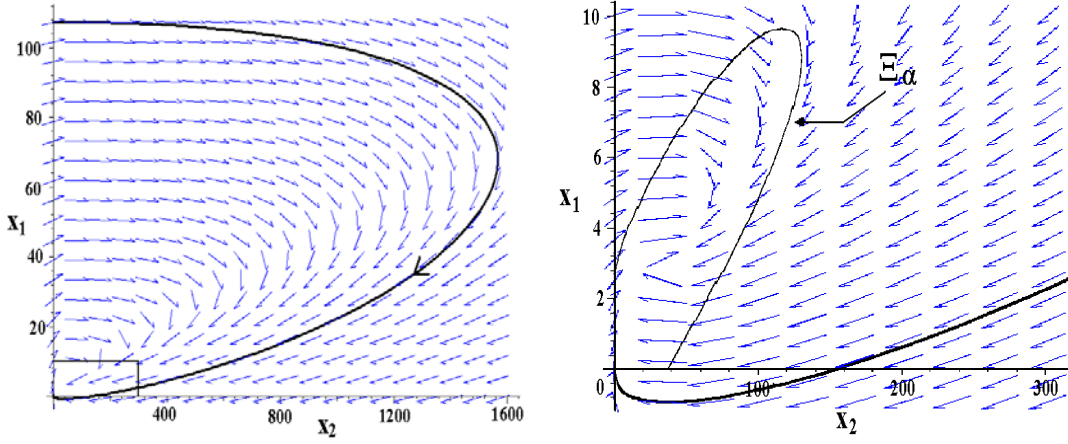


Fig. 3. Exact domain of attraction, zoom around the equilibrium point and set  $\Xi_\alpha$ .

the set of initial conditions of interest, this can only be determined numerically, and we would like to dispose of an analytic expression. Towards this end, we use our Lyapunov function (7). First, we will prove that  $H_d$  is strictly convex on the first quadrant, by looking at the behaviour of the Hessian  $\frac{\partial^2 H_d}{\partial x^2}(x)$  along the right half plane rays  $x_1 = \beta x_2$ ,  $x_2 > 0$ , with  $0 < \beta < \infty$ , and  $x_2 \rightarrow +\infty$ . Using the facts that the constants  $\kappa_i$  are positive and that  $\max(2, 2 - 2\alpha) > \max(2 - \alpha, 1 - \alpha)$ ,  $\forall \alpha \in (0, 1)$ , it can be shown that the Hessian remains positive definite as  $x_1 \rightarrow +\infty, x_2 \rightarrow +\infty$ . Consequently,  $H_d$  grows unbounded and the level sets  $\{H_d(x) \leq c\}$  cover the first quadrant  $\mathcal{R}_{>0}^2$ . We should recall here that these level sets provide estimates of the domain of attraction only for trajectories that remain bounded away from  $x_2 = 0$ . From the analysis above we know that trajectories that stay inside  $\mathcal{R}_{>0}^2$  remain bounded away from  $x_2 = 0$ . Therefore, as our estimate of domain of attraction we choose the level set that intersects  $x_1 = 0$  at  $x_{2*}$ , see Fig. 4.<sup>11</sup> This is precisely the set

<sup>11</sup> To underscore the conservativeness of our estimate we have plotted also this level set in Fig. 3, where the exact domain of attraction is depicted.

$\Xi_\alpha$ , hence completing the proof of (ii).

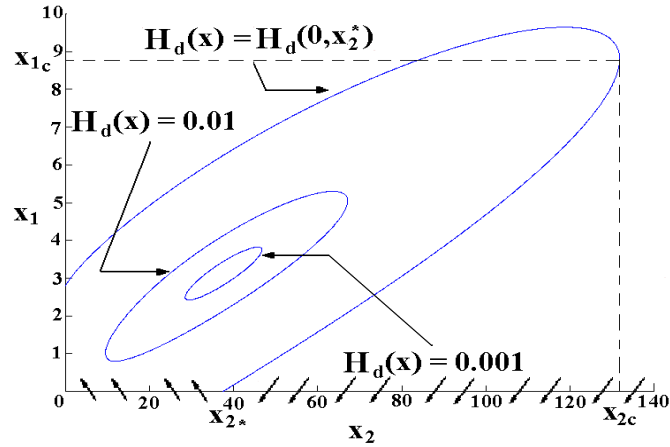


Fig. 4. Estimated of the domain of attraction and closed-loop vector fields on the axis  $x_1 = 0$ .

(iii) (*Saturated control*) To prove the *saturated control* property we notice from (6) that  $\beta(x_2) < 1 \Leftrightarrow x_2 < \left(\frac{1}{u_*}\right)^{\frac{1}{\alpha}} x_{2*}$ . Let us define a line  $l_\alpha \triangleq \{x | x_2 = \left(\frac{1}{u_*}\right)^{\frac{1}{\alpha}} x_{2*}\}$ , parametrized in  $\alpha$ . Given that  $u_* < 1$ , this line shifts to the right as  $\alpha \rightarrow 0$ . The question is now whether we can find a value for  $\alpha$  such that the set  $\Xi_\alpha$  does not intersect with  $l_\alpha$ . (Notice that  $\Xi_\alpha$  also depends on  $\alpha$ .) If this is the case, the trajectories starting inside the level sets cannot cross the line and consequently will satisfy  $u(t) < 1, \forall t \geq 0$ . We need to compute the maximum value of  $x_2$  on the level set  $H_d(x) \leq H_d(0, x_{2*})$ , which we denote in Fig. 4 as  $x_{2c}$ . Towards this end, we evaluate the derivative of  $H_d(x)$  with respect  $x_1$  for a fixed  $x_2$  to get  $x_{1c} = L\kappa_3 x_{2c}^{1-\alpha}$ . Replacing this value in  $H_d$  gives the algebraic equation

$$x_{2c}^2 + \frac{2ELx_{2*}^\alpha}{RR_a u_*} x_{2c}^{1-\alpha} - \left(1 + \frac{L}{u_*^2 R^2 C}\right) x_{2*}^2 + \frac{2EL}{RR_a u_*} x_{2*} = 0$$

Now, as  $\alpha \rightarrow 0$  the positive real root of this equation converges to  $CV_* + \frac{V_*^2}{RE} \sqrt{LC}$ , while –as pointed out above– the line shifts towards  $+\infty$ . Therefore, for all  $x_*$ , we can always find a (sufficiently small) value of  $\alpha$  such that the set and the line do not intersect. This completes the proof of (iii).

(iv) (*Performance*) This part of the proof is established linearizing the closed-loop dynamics (16), (17) at the equilibrium point and verifying that, under the conditions of the proposition, the equilibrium is a stable node. The details of the calculations, being quite standard, are left to the interested reader.

## 5 Simulation results

Computer simulations were performed to evaluate the performance of the derived nonlinear output feedback controller. The parameter values are  $L = 20mH$ ,  $C = 20\mu F$ ,  $E = 15V$ ,  $R = 30\Omega$ , the desired voltage is  $V_* = 37.5V$ , the initial conditions in all simulations are  $x_1(0) = 0A$ ,  $x_2(0) = 15V$ .

In Fig. 5, we show the phase portrait corresponding to these initial conditions and the controller parameter  $\alpha = \alpha_M = 0.1767$ .

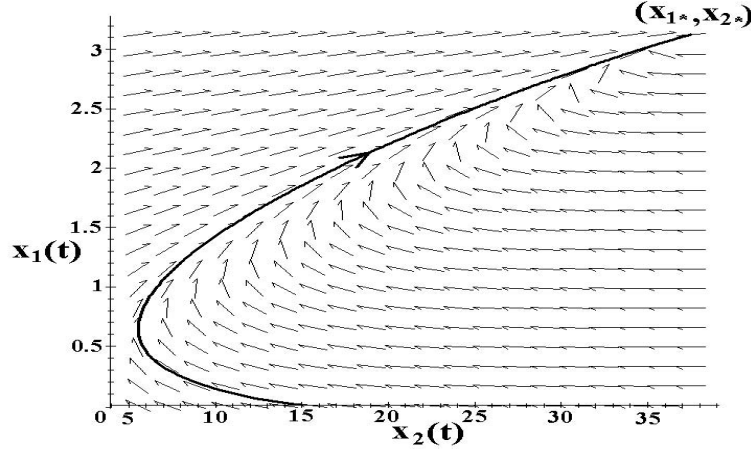
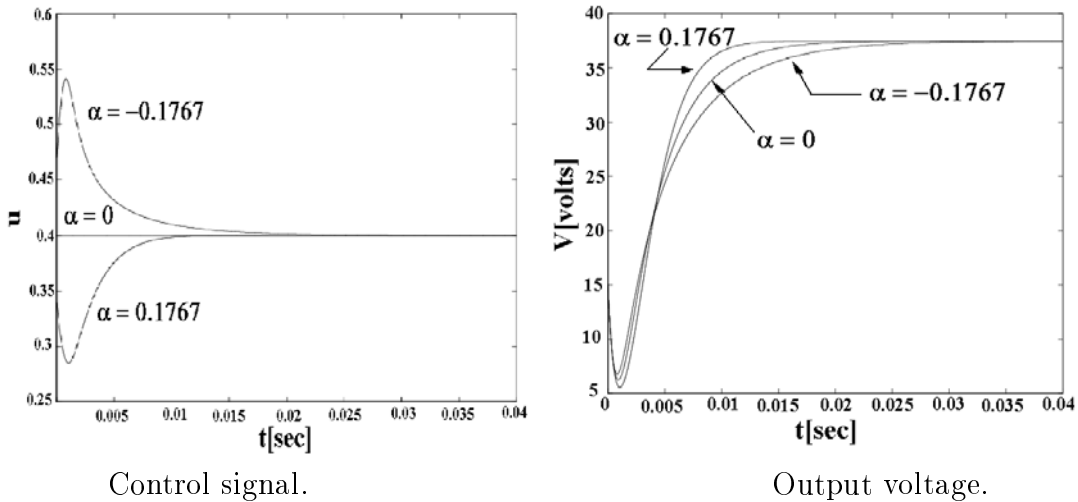


Fig. 5. Phase portrait for  $\alpha = \alpha_M$ .

In Fig. 6 we present the transient responses of the voltage and the control law for different values of  $\alpha$ , including the open-loop control corresponding to  $\alpha = 0$ . Notice that, as stated above, the response is *faster* for positive  $\alpha$ . Also, remark the undershoot due to the non-minimum phase nature of the system.



Control signal.

Output voltage.

Fig. 6. Transient responses for  $\alpha = -\alpha_M, 0, \alpha_M$ .

## 6 Concluding remarks

1. We have presented an extremely simple solution to the problem of output-feedback regulation with saturated input of the Boost DC-to-DC converter which is insensitive to uncertainty in the load resistance. These remarkable properties of the controller stem from the effective exploitation of the systems physical structure. This feature, intrinsic to all passivity-based designs, is particularly clear in the new IDA-PBC methodology proposed in [5]. As explained in that paper, the control law may be viewed as a state-modulated interconnection between a source system (in this case the open-loop control  $u_*$ ) and the plant.

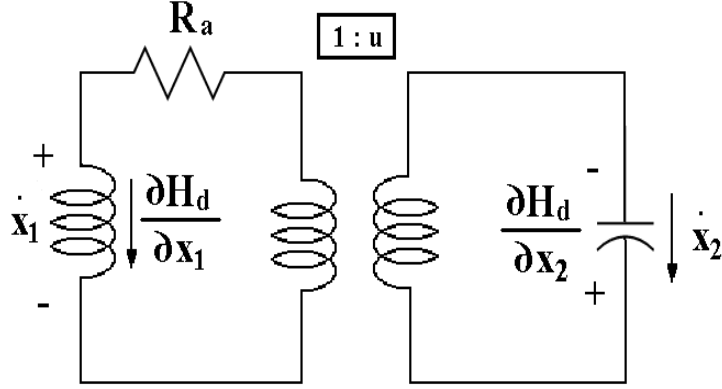


Fig. 7. Circuit interpretation of the closed-loop dynamics.

2. The action of the controller has a nice *circuit-theoretic* interpretation. A circuit description of the closed-loop dynamics (16), (17) is given in Fig. 7. Comparing with<sup>12</sup> Fig. 1 we observe the following modifications:

- i) The load resistance  $R$  and the voltage source  $E$  have been removed, being absorbed by the energy function  $H_d$ . Further, the injected damping  $R_a$  appears now in *series* with the inductance, and not in parallel with the capacitor as the original load resistance  $R$ .
- ii) The current of the “new” inductance element, whose voltage is  $\dot{x}_1$ , is now  $\frac{\partial H_d}{\partial x_1}$ , instead of  $\frac{x_1}{L}$ . Similarly, the voltage of the “new” capacitance, whose current is  $\dot{x}_2$ , has been changed from  $\frac{x_2}{C}$  to  $\frac{\partial H_d}{\partial x_2}$ .
- iii) The actual capacitor voltage  $\frac{x_2}{C}$  is controlled via *regulation of the current*  $\dot{x}_2$ . Remark that (in switching operating mode) the circuit commutes between a series RLC-circuit and an LR-circuit with an open-ended capacitor. See Fig. 8.

<sup>12</sup> As explained in [4] the action of the control in the averaged dynamics may be regarded as an ideal power-preserving transformer with transformation ratio  $1 : u$ .

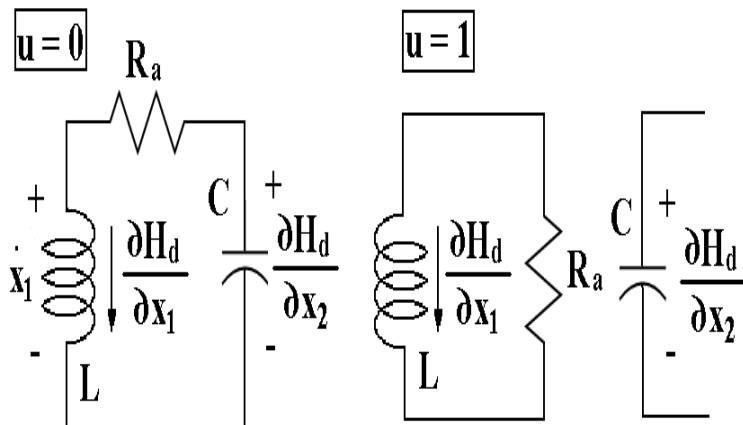


Fig. 8. RCL-circuit and RL-circuit corresponding to  $u = 1$  and  $u = 0$ , respectively.

**3** It is possible to show that the closed-loop system (6), (16), (17) may have two equilibrium points, with one of them on the fourth quadrant, hence of no physical interest. However, if  $\alpha$  *cannot* be expressed in the form  $\alpha = \frac{2k+1}{2l+1}$  for some positive integers  $k, l$ , then we can prove that the equilibrium is unique.<sup>13</sup>

**4.** From (18) one might be led to believe that larger values of  $R_a$  will yield faster convergence rates. However, our simulations have shown exactly the opposite effect. This is also consistent with the circuit interpretation given above, where it is clear that larger  $R_a$  make the circuit more sluggish. It is clear from this example that we cannot hope to build-up a *performance theory* solely from considerations on the negativity of the derivative of the Lyapunov function. Attempts to achieve exponential stability will also, in general, be doomed to failure or –if at all possible– extremely local to be of practical use. The design reported here ensures passivity of the closed-loop, hence possesses some (quantifiable) robust stability (*vis á vis* dissipative uncertainty) and performance properties (that stem from inverse optimality). See [5] for further details on the general theory and [7] for this particular application.

**5.** Although the result is presented for the case of a special kind of converter, we have shown in [7] that IDA-PBC is applicable to a very broad class of devices, including rectifiers, and the Čuk and series-resonant converters. Current research is under way on the development of a general theory out of which the various circuit topologies would follow as particular cases. Also, experimental work of the present controller has been recently terminated and will be reported elsewhere.

<sup>13</sup> We thank Françoise Lamnabhi-Lagarrigue for the proof of this fact.

## References

- [1] G. Escobar, R. Ortega, H. Sira-Ramírez, J-P. Vilain and I. Zein, An experimental comparison of several nonlinear controllers for power converters, *IEEE Control Systems Magazine*, 66-82, 1999.
- [2] G. Escobar, A. J. van der Schaft and R. Ortega, A Hamiltonian viewpoint in the modeling of switching power converters, *Automatica*, 35, 445-452, 1999.
- [3] J. G. Kassakian, M. Schelecht and G. C. Verghese, **Principles of Power Electronics**, Reading, Mass., Addison-Wesley, 1991.
- [4] R. Ortega, A. Loría, P. J. Nicklasson and H. Sira-Ramírez, **Passivity-based Control of Euler-Lagrange Systems**, Eds. Springer-Verlag, 1998.
- [5] R. Ortega, A. J. van der Schaft, B. Maschke and G. Escobar, Stabilization of Port-Controlled Hamiltonian Systems: Passivity and Energy-Balancing, *Proc IEEE CDC*, Phoenix, AZ, USA, Dec. 1999.
- [6] R. Ortega, A. Astolfi, G. Bastin and H. Rodríguez, Output feedback stabilization of mass-balance systems, in the book **Output feedback stabilization of nonlinear systems**, Eds. H. Nijmeijer and T. Fossen, Springer-Verlag, 1999.
- [7] H. Rodríguez, G. Escobar, R. Ortega, A new family of energy-based Non-linear controllers for DC to DC power converters, *Journées Doctorales d'Automatique 99*, Nancy, France, 1999.
- [8] S. R. Sanders, G. C. Verghese and D. F. Cameron, Nonlinear control laws for switching power converters, *Proc. 25th. IEEE Conf. Decision Contr.*, Athens, Greece, 1986.
- [9] H. Sira-Ramírez, R. Perez, R. Ortega and M. Garcia, Passivity-based controllers for the stabilization of DC-to-DC power converters, *Automatica*, Vol. 39, No. 4, pp. 499-513, May 1997.