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Brief Paper

Hopf bifurcation in indirect field-oriented control of induction motors☆

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Abstract

The appearance of self-sustained oscillations in high performance AC drives, and in particular, in induction motors whose speed is regulated with the industry standard field-oriented control, is a well-documented but little understood phenomenon. It is well known that the oscillations may be quenched by returning the outer-loop PI speed control, but no precise rules to carry out this task, which may be time consuming, are known. In this paper, we show that these oscillations may arise due to the existence of Hopf bifurcations. Some simple rules for selecting the gains of the PI controller are obtained as a result of our analysis.

Keywords: Induction motors; Hopf bifurcation; Harmonic balance analysis

1. Introduction

Due to its high reliability, field-oriented control (FOC) is the standard for high dynamic performance induction motor drives. Historically, this remarkable controller was derived as a result of physical intuition and a deep understanding of the machine operation, with little concern about a rigorous analytical study of its stability and performance. An approximate analysis (based on steady-state behavior, time-scale assumptions, and linearizations, e.g., Bose, 1986; Chhaya, 1995; Leonhard, 1985) can be combined with the designer expertise to commission the controller in simple applications. However, to meet large bandwidth requirements, or other tight specifications, this ad hoc commissioning stage may be time-consuming and expensive. To simplify the off-line tuning of FOC, and eventually come to terms with its achievable performance, a better theoretical understanding of the dynamic behavior of FOC is unquestionably needed. Such an analysis is unfortunately stymied by the fact that the dynamic behavior of the closed loop is described by complex nonlinear relationships.

Realizing the practical importance of FOC, and motivated by the need to clarify its theoretical underpinnings, in the last few years a series of studies on indirect FOC for current-fed induction machines have been carried out by the third author of this paper. The outcome of this research is summarized in the recent book (Ortega, Loria, Nicklasson, & Sira Ramirez, 1998). In this paper, we continue with this line of research and concentrate on the practically important problem of *oscillation quenching* through suitable tuning of the gains of the PI velocity loop. The appearance of self-sustained oscillations in high performance AC drives, and in particular in FOC of induction motors, is a well documented — but little understood — phenomenon. It is well known that the oscillations may be quenched by retuning the outer-loop PI speed control, but no precise rules to carry out this task are known. This tuning procedure is particularly difficult due to the high uncertainty on the rotor time-constant. In this short paper, we apply some standard techniques of dynamical systems (in particular harmonic balance and bifurcation analysis) to show that these oscillations may arise due to the existence of Hopf bifurcations. As a result of our analysis, we obtain some simple rules to quench the oscillations via a suitable PI tuning.

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The present study was motivated by the interesting paper (Bazanella & Reginatto, 2000), where the robustness results of FOC reported in de Wit, Ortega and Mareels (1996) are generalized. In particular, in Bazanella & Reginatto, (2000) an explicit construction of a Lyapunov function, which allows for the evaluation of stability margins, is given. Furthermore, the authors also study the existence of a saddle-node bifurcation. Our study is complementary to the latter in so far as we are interested here in Hopf bifurcations, which as our study conclusively proves, are at the core of the oscillation phenomenon often observed in practice (Chhaya, 1995). A recent paper (Bazanella & Reginatto, 2001) deals with the same problem but with a different and complementary approach.

The rest of the paper is organized as follows. In Section 2, the Hopf bifurcation is detected in a current driven induction motor with indirect FOC. Simulations that corroborate the theoretical predictions are given in Section 3. Section 4 is devoted to a complementary analysis of bifurcation detection via harmonic balance. We end the paper with Section 5, which contains some concluding remarks.

2. Detection of Hopf bifurcations in induction motors with indirect FOC

In this section, the above procedure is applied to detect Hopf bifurcations in a current-driven induction motor with indirect FOC. The equations of the system are Ortega et al. (1998):

$$\dot{x}_1 = -c_1 x_1 + \frac{\kappa c_1}{u_2^0} x_2 x_4 + c_2 u_2^0, \tag{1}$$

$$\dot{x}_2 = -c_1 x_2 - \frac{\kappa c_1}{u_2^0} x_1 x_4 + c_2 x_4, \tag{2}$$

$$\dot{x}_3 = -c_4[c_5(x_1 x_4 - x_2 u_2^0) - \tau_L], \tag{3}$$

$$\dot{x}_4 = k_i x_3 - k_p c_4[c_5(x_1 x_4 - x_2 u_2^0) - \tau_L], \tag{4}$$

where x_1 and x_2 which are the coordinates of the rotor flux expressed in a rotating reference frame, x_3 which is the angular speed of the rotor, and x_4 which comes from the outer-loop PI, are the state variables. All the parameters are related with the physical characteristic of the motor except u_2^0 which is a design parameter. All of them can be considered positive. Assuming that the rotor inductance is known, parameter κ is equal to the ratio between the estimated and the true values of the rotor resistance, i.e., $\kappa = \hat{R}_r/R_r$. The rotor resistance is really unknown due to the fact that it changes with time. Therefore, κ has been chosen as the bifurcation parameter. Notice that only positive values of κ are of physical interest. The analysis in this paper is directed towards the causes of the loss of stability. Thus, the desired equilibrium point is assumed to be locally stable in the case of exact estimation of the rotor resistance $\kappa = 1$. The aim of the study is to find the range of values for κ, around $\kappa = 1$, for which the equilibrium preserves its stability, by looking for the emergence of bifurcations. It must be pointed out that other complex phenomena due to the emergence of more bifurcations may occur beyond that range but these phenomena are not the subject of this paper. Two cases will be examined here: the case when there is no load torque ($\tau_L = 0$) and the general case with $\tau_L \neq 0$.

Suppose that the dynamical system $\dot{x} = f(x, \mu)$ with $x \in \mathbb{R}^n$ and $\mu \in \mathbb{R}$ has an equilibrium point at x^0, for some $\mu = \mu^0$; that is, $f(x^0, \mu^0) = 0$. Let $A(\mu) = D_x f(x^0(\mu), \mu)$ be the Jacobian matrix of the system at the equilibrium point. Assume that $A(\mu^0)$ has a single pair of purely imaginary eigenvalues $\lambda(\mu^0) = \pm j\omega$, and no other eigenvalue with zero real part; and furthermore, $\mathrm{d}\Re(\lambda(\mu))/\mathrm{d}\mu|_{\mu=\mu^0} \neq 0$. This last condition is known as the transversality hypothesis. Under these conditions, the Hopf bifurcation theorem states that a limit cycle is born at (x^0, μ^0) (Moiola & Chen, 1996).

The conditions for the existence of a pair of pure imaginary eigenvalues for $A(\mu)$ can be formulated for any dimension n in the following way (Liu, 1994):

$$H_{n-1}(\mu) = 0, \tag{5}$$

$$H_{n-2}(\mu) > 0, \quad H_{n-3} > 0, \ldots, H_1(\mu), \quad p_0(\mu) > 0, \tag{6}$$

where $H_i(\mu)$ stands for the i principal minor of the Hurwitz matrix of the characteristic polynomial of $A(\mu)$ and $p_0(\mu)$ is the zero-order term of this polynomial.

2.1. Zero load torque case

When $\tau_L = 0$ the analysis is simpler because in this case the system has only one equilibrium point, which is independent of the rest of parameters (de Wit et al., 1996). Indeed, the equilibrium is $x^0 = (x_1^0, x_2^0, x_3^0, x_4^0) = ((c_2/c_1)u_2^0, 0, 0, 0)$ and can be obtained by equating the time derivatives of x_i to zero in (1)–(4) and solving the resulting system of equations.

The Jacobian matrix of system (1)–(4) at the equilibrium point has the following expression:

$$A(x^0) = \begin{bmatrix} -c_1 & 0 & 0 & 0 \\ 0 & -c_1 & 0 & c_2 - \kappa c_2 \\ 0 & c_4 c_5 u_2^0 & 0 & -\dfrac{u_2^0 c_4 c_5 c_2}{c_1} \\ 0 & k_p c_4 c_5 u_2^0 & k_i & -\dfrac{u_2^0 k_p c_4 c_5 c_2}{c_1} \end{bmatrix}. \tag{7}$$

Looking at the first row of the Jacobian matrix, it is obvious that $\det(c_1 I - A(x^0)) = 0$. Therefore, $-c_1$ is an eigenvalue of $A(x^0)$ and, then, its characteristic polynomial must be divisible by $\lambda + c_1$. Indeed:

$$\det(\lambda I - A(x^0)) = (\lambda + c_1)(\lambda^3 + \alpha_2 \lambda^2 + \alpha_1 \lambda + \alpha_0)$$

with

$$\alpha_2 = \frac{c_1^2 + u_2^0 k_p c_4 c_5 c_2}{c_1}, \tag{8}$$

$$\alpha_1 = \frac{u_2^0 k_p c_4 c_5 c_1 \kappa c_2 + c_4 c_5 k_i c_2 u_2^0}{c_1}, \tag{9}$$

$$\alpha_0 = u_2^0 c_4 c_5 k_i \kappa c_2. \tag{10}$$

Conditions (5) and (6) for the particular case of $n = 3$ can be written as

$$\alpha_0 - \alpha_1 \alpha_2 = 0, \tag{11}$$

$$\alpha_1 > 0, \tag{12}$$

$$\alpha_0 > 0. \tag{13}$$

Since all the parameters in (9) and (10) are positive, Eqs. (12) and (13) hold. Denoting the value of κ corresponding to the Hopf bifurcation by k^*, Eq. (11) leads to

$$\frac{(c_1^2 + u_2^0 k_p c_4 c_5 c_2)(u_2^0 k_p c_4 c_5 c_1 \kappa^* c_2 + c_4 c_5 k_i c_2 u_2^0)}{c_1^2} - u_2^0 c_4 c_5 k_i \kappa^* c_2 = 0$$

which yields

$$\kappa^* = -\frac{(c_1^2 + u_2^0 k_p c_4 c_5 c_2) k_i}{c_1 (k_p c_1^2 + u_2^0 k_p^2 c_4 c_5 c_2 - c_1 k_i)}. \tag{14}$$

In order to test the transversality condition, the behavior of the eigenvalues of $A(x^0)$ in a neighborhood of κ^* should be analyzed. Thus, κ may be written as $\kappa^* + \varepsilon$, that is:

$$\kappa = -\frac{(c_1^2 + u_2^0 k_p c_4 c_5 c_2) k_i}{c_1 (k_p c_1^2 + u_2^0 k_p^2 c_4 c_5 c_2 - c_1 k_i)} + \varepsilon. \tag{15}$$

The transversality condition will be fulfilled if the sign of the left-hand side of Eq. (11) changes when the sign of ε changes. Using Eq. (15), and after some direct manipulations, the left-hand side of (11) becomes:

$$\alpha_0 - \alpha_1 \alpha_2 = -\frac{(k_p c_1^2 + u_2^0 k_p^2 c_4 c_5 c_2 - k_i c_1) u_2^0 c_4 c_5 c_2}{c_1} \varepsilon,$$

where it is obvious that $\alpha_0 - \alpha_1 \alpha_2$ changes sign as does ε.

Expression (14) is of great practical interest. It has been assumed that the desired equilibrium is stable for $\kappa = 1$. In that case, no limit cycles will appear in the neighborhood of this point provided that this equilibrium does not suffer a Hopf bifurcation. Expression (14) gives the value of $\kappa = \hat{R}_r / R_r$ corresponding to a Hopf bifurcation in the case of $\tau_L = 0$. In other words, Eq. (14) gives an upper limit on the error of the estimation of R_r when $\tau_L = 0$, in order to prevent the existence of limit cycles. In de Wit et al. (1996) the authors prove that for $\kappa < 3$ there is only one equilibrium point, and necessary and sufficient conditions are given to assure the stability of this equilibrium. Here, we look at the cause of the loss of stability: the emergence of a limit cycle by a Hopf bifurcation.

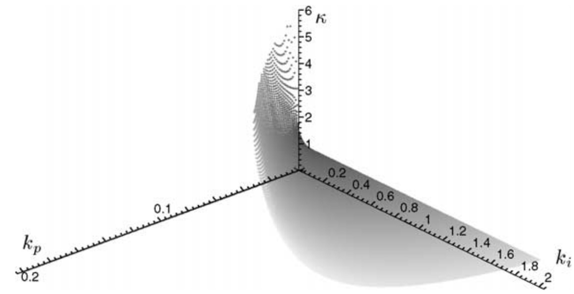

Fig. 1. Representation of Eq. (14) for $c_1 = 4$, $c_2 = 4$, $c_4 = 1$, $c_5 = 1$, $u_2^0 = 1$.

In Fig. 1, the values of κ given by Eq. (14) are plotted against k_p and k_i for $c_1 = 4$, $c_2 = 4$, $c_4 = 1$, $c_5 = 1$, $u_2^0 = 1$. For each pair of values of k_p and k_i the system will not present limit cycles provided that κ is below the curve. As can be seen from Eq. (14), the curve corresponding to the above values goes to infinity for $k_i = k_p^2 + 4k_p$. For $k_i < k_p^2 + 4k_p$, the critical value of κ is less than zero and has no practical meaning. In general, the curve goes to infinity for $k_i = k_i^*$ with $k_i^* = k_p c_1 + (u_2^0 c_4 c_5 c_2 / c_1) k_p^2$ and k^* is negative for $k_i < k_i^*$ $k^* < 0$ and, therefore, has no practical meaning and no Hopf bifurcation can occur.

A practical conclusion from Eq. (14) and Fig. 1 is that increasing k_p allows for larger values of κ^* (larger admissible estimation errors), and increasing k_i yields the opposite effect. The first statement can be verified computing the sign of the partial derivative of κ^* with respect to k_p:

$$\frac{\partial \kappa^*}{\partial k_p} = \frac{k_i (2\alpha k_p c_1^2 + k_p^2 \alpha^2 + \alpha c_1 k_i + c_1^4)}{c_1 (k_p c_1^2 + k_p^2 \alpha - c_1 k_i)^2} > 0.$$

The effect of k_i depends on

$$\frac{\partial \kappa^*}{\partial k_i} = -\frac{(c_1^2 + k_p \alpha)^2 k_p}{c_1 (k_p c_1^2 + k_p^2 \alpha - c_1 k_i)^2}$$

which is, obviously, negative.

2.2. Nonzero load torque case

The case with $\tau_L \neq 0$ is more involved and a single expression similar to (14) is difficult to obtain. The reason is that, for this case, the equilibrium point is obtained by solving a third-order polynomial equation de Wit et al. (1996). Substitution of the value of the equilibrium in the Jacobian matrix leads to complex expressions. Nevertheless, for this case conclusions can be drawn for concrete values of the parameters. In fact, for particular values of c_1, c_2, c_4, c_5, u_2^0, k_p, k_i and τ_L, the critical value of κ may be obtained following the same procedure as above. For example, making $c_1 = 4$, $c_2 = 4$, $c_4 = 1$, $c_5 = 1$, $u_2^0 = 1$, $k_p = 0.1$, $k_i = 1$ and $\tau_L = 0.3$ the eigenvalues of the Jacobian matrix at the corresponding equilibrium point can be computed for each

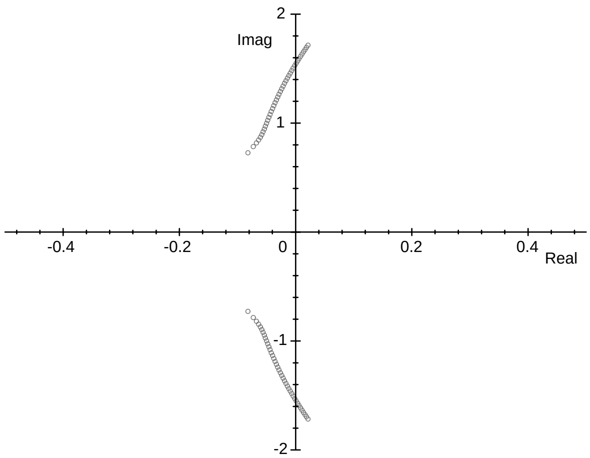

Fig. 2. Root locus for $c_1 = 4$, $c_2 = 4$, $c_4 = 1$, $c_5 = 1$, $u_2^0 = 1$, $k_p = 0.1$, $k_i = 1$ and $\tau_L = 0.3$.

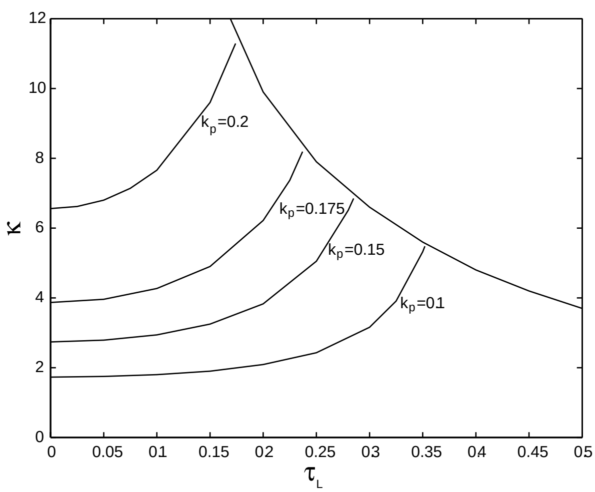

Fig. 3. Values of κ corresponding to a Hopf bifurcation vs. τ_L.

value of κ, giving the root locus of Fig. 2 (in this figure only two branches are plotted since the other two branches are far away to the left). It can be seen in this figure that the condition of transversality is fulfilled, and a Hopf bifurcation is expected to be produced.

Fig. 3 shows the values of the critical κ for several values of τ_L, and k_p (the rest of parameters have the same values as above).

The most critical value of κ corresponding to the Hopf bifurcation will be that which is closest to one since it has been assumed that the desired equilibrium is stable for $\kappa=1$. As can be seen in Fig. 3, this case corresponds to $\tau_L = 0$. If this fact were general, it would give more practical value to Eq. (14), but is difficult to be proven for the general case. Nevertheless, it seems to be true in accordance with all the cases tested.

Fig. 3 also shows that as τ_L increases so does the value of κ corresponding to a Hopf bifurcation. It could be thought that, for high values of τ_L, the estimation error would not affect the stability of the system. Nevertheless, this stability is lost in another bifurcation, a saddle-node bifurcation, due to the emergence of two new equilibria, as has been reported in Bazanella and Reginatto (2000) and de Wit et al. (1996). The upper right curve in Fig. 3 represents the values of κ corresponding to this bifurcation. It must be pointed out that the Hopf bifurcation exists above the curve of the saddle-node bifurcation but it has not been plotted because it has no practical meaning since the local stability of the equilibrium has already been lost at this point. Furthermore, Fig. 3 does not represent the full bifurcation diagram of the system, but only the curves associated with both bifurcations. Specifically, around the intersection of the two kind of curves in Fig. 3 more complicated bifurcations will probably exist. The Hopf bifurcation studied in this paper is of a more generic character that the possible bifurcations associated with the intersection of those curves.

The practical application of this graph is the following: for a given set of parameters the graph can be constructed. The admissible values of κ (so the equilibrium point is stable) as a function of τ_L are under two curves: the one corresponding to the given value of k_p and the one corresponding to the saddle-node bifurcation. Parameter κ must be less than the minimum of these curves (for admissible τ_L). In all the situations we have tested this minimum corresponds to $\tau_L=0$ if excessively large values of τ_L are not considered. The value of the critical κ for $\tau_L = 0$ is given by Eq. (14).

Curiously, there are situations where very large estimation errors may be admissible: for small values of the load torque ($\tau_L \to 0$) and when k_p and k_i are such that Eq. (14) yields negative values. In this case, the saddle-node bifurcation is not possible (the upper curve of Fig. 3 goes to infinity as $\tau_L \to 0$) and the Hopf bifurcation would occur for $\kappa < 0$, which has no practical meaning.

3. Simulations

In order to corroborate the above results, the system has been simulated with the value of the parameters given in Section 2.2 (including $k_i = 1$) but with different values for k_p, τ_L and κ which are shown in Table 1.

It can be seen in this table that simulations A and B correspond to the case of $\tau_L = 0$. Introducing the values of the parameters in Eq. (14), this gives $\kappa = 1.73$ as the limit for the emergence of a limit cycle. Therefore, simulation A must correspond to a stable behavior and simulation B has to present a limit cycle. On the other hand, simulations C and

Table 1
Values for k_p, τ_L and κ corresponding to the four simulations

Simulation	k_p	τ_L	κ	Predicted behaviour
A	0.1	0	1.65	Stable
B	0.1	0	1.8	Limit cycle
C	0.15	0.2	3.7	Stable
D	0.15	0.2	3.9	Limit cycle

D correspond to $\tau_L = 0.2 \neq 0$ and, therefore, the bifurcation point has to be detected using Fig. 3 instead of Eq. (14). In this case, the bifurcation point is $\kappa = 3.83$ and, thus, simulation C should correspond to a stable behavior and simulation B must present a limit cycle.

In performing all these simulations, the predicted behaviors are corroborated as it is shown in Fig. 4. In this figure, the time evolution of x_3 (the angular velocity of the rotor) is plotted for each simulation. In the figures the evolution of amplitude of the oscillations of x_3 can be observed. It can be seen that the amplitudes of simulations A and C go to zero while in simulations B and D the oscillations tend to a limit cycle. These behaviors correspond to the predictions.

4. Approximate study of limit cycles using the harmonic balance method

Although, as has been stated in previous sections, the Hopf bifurcation is of a local nature, in this section, we will apply a method that is not restricted to the bifurcation point, but has a more global nature. This method, called harmonic balance, not only allows us to detect limit cycles at their birth, as in local methods, but also to evaluate an approximation to their frequency and amplitude (Mees, 1981).

In order to apply the method to the fourth order system (1)–(4), we assume that each state variable x_i has a self-sustained oscillation of the same frequency ω:

$$x_i = a_{i0} + a_{i1}\cos\omega t + a_{i2}\sin\omega t \quad i = 1,\ldots,4, \tag{16}$$

where a_{12} is assumed to be zero, since the time origin can be defined arbitrarily. The rest of the sine and cosine coefficients are not null because a phase shift between the oscillations of the state variables is considered. By substituting Eqs. (16) and their derivatives into Eqs. (1)–(4), grouping bias, sine and cosine terms, and ignoring harmonics higher than one, a nonlinear system of 12 equations with 12 unknown variables, which are a_{ij} and ω, is obtained.

The solutions to the system of equations for $a_{i1} = 0$, and $a_{i2} = 0$ $(i = 1,\ldots,4)$ will be the equilibrium points, and those with any $a_{ij} \neq 0$, with $j \neq 0$ are periodic solutions. These periodic solutions will give an approximation to the frequency and amplitude of the self-sustained oscillations of the state variables. There will be several solutions, real and imaginary, but only the real ones are of interest and thus will be retained.

The study is again divided into two cases: the case $\tau_L = 0$ and the general case $\tau_L \neq 0$.

4.1. Zero load torque case ($\tau_L = 0$)

As was stated in Section 2, when $\tau_L = 0$ the analysis is simpler than in the case $\tau_L \neq 0$, because in this case the system has only one equilibrium point, which is independent of the rest of the parameters. An analytical solution of the system of equations is too complex to be given as a function of parameters (c_1, c_2, c_4, c_5, u_2^0), but for particular values of

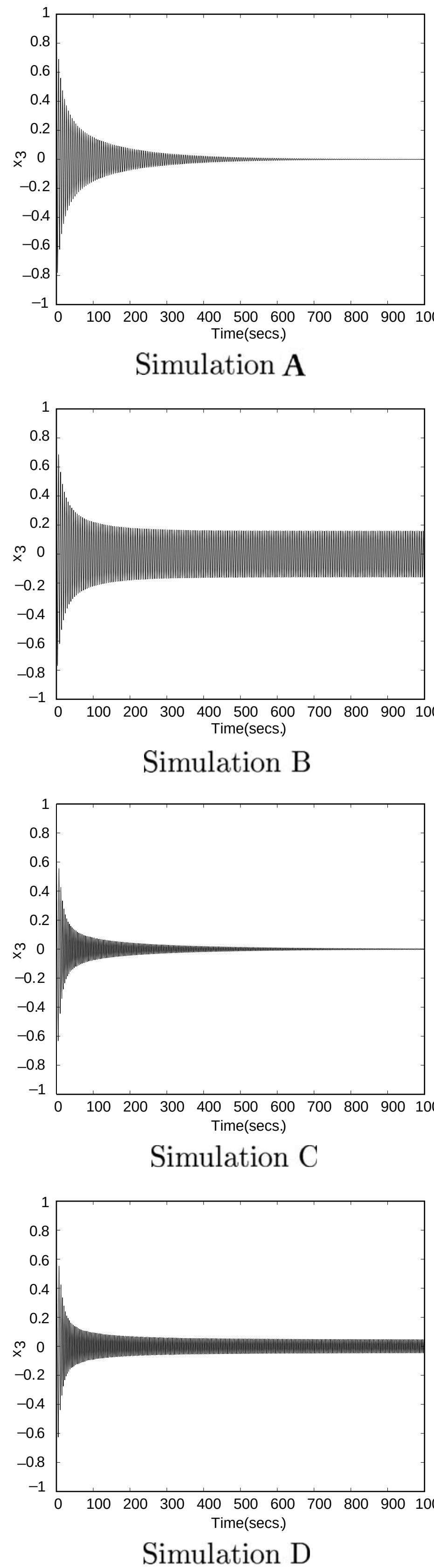

Fig. 4. Evolution of x_3 in the four simulations.

these parameters, a set of solutions can be given for any κ, k_p, k_i. Using the values of the parameters given in Section 2.2 and the values of κ, k_p, k_i given in Table 1, the solutions to the system can be obtained numerically:

- For $\kappa = 1.6$, $k_p = 0.1$, $k_i = 1$ only an equilibrium solution is obtained,
- but for $\kappa = 1.8$, $k_p = 0.1$, $k_i = 1$ two solutions are obtained:
  (1) ($a_{10} = 1$, $a_{11} = 0$, $a_{20} = 0$, $a_{21} = 0$, $a_{22} = 0$, $a_{30} = 0$, $a_{31} = 0$, $a_{32} = 0$, $a_{40} = 0$, $a_{41} = 0$, a_{42}) for any ω. This solution corresponds to the equilibrium point, and
  (2) ($a_{10} = 0.937$, $a_{11} = 0$, $a_{20} = 0$, $a_{21} = 0.2157$, $a_{22} = 0.0391$, $a_{30} = 0$, $a_{31} = 0$, $a_{32} = 0.4183$, $a_{40} = 0$, $a_{41} = -0.3321$, $a_{42} = 0.0418$, $\omega = 1.2595$).

  If these solutions are now compared with the simulations presented in Section 3 it can be stated that the frequency of the limit cycle predicted above ($\omega = 1.2595$ rad/s) is very similar to the one of simulation B of Fig. 4 ($\omega = 1.309$ rad/s). On the other hand, if the evolution of x_1 in simulation A were plotted it could be seen that x_1 does not oscillate as is predicted in this study. The predicted amplitudes of the limit cycles are not exactly the ones obtained in the simulations because of the approximate nature of the method. These differences would be smaller if more harmonics were used, but that would make the problem computationally more involved (Bergen, Chua, Mees, & Szeto, 1982; Bonani & Gilli, 1999; Moiola & Chen, 1996). From a qualitative point of view the first-harmonic balance method captures the essence of the phenomenon.

4.2. Nonzero load torque case ($\tau_L \neq 0$)

In this case, numerical solution of the system of equations for the parameter values given in Table 1 gives:

- For the values of parameters ($c_1 = 4$, $c_2 = 4$, $c_4 = 1$, $c_5 = 1$, $u_2^0 = 1$, $\kappa = 3.7$, $k_p = 0.15$, $k_i = 1$, $\tau_L = 0.2$) only one solution can be obtained which corresponds to the equilibrium point: ($a_{10} = 0.9657$, $a_{11} = 0$, $a_{20} = -0.1455$, $a_{21} = 0$, $a_{22} = 0$, $a_{30} = 0$, $a_{31} = 0$, $a_{32} = 0$, $a_{40} = 0.0562$, $a_{41} = 0$, a_{42}) for any ω.
- For the values ($c_1 = 4$, $c_2 = 4$, $c_4 = 1$, $c_5 = 1$, $u_2^0 = 1$, $\kappa = 3.9$, $k_p = 0.15$, $k_i = 1$, $\tau_L = 0.2$) there must be a limit cycle in the state variables (Section 2), and, indeed, there are two solutions to the system of equations:
  (1) ($a_{10}=0.9691$, $a_{11}=0$, $a_{20}=-0.1483$, $a_{21}=0$, $a_{22}=0$, $a_{30}=0$, $a_{31}=0$, $a_{32}=0$, $a_{40}=0.0534$, $a_{41}=0$, $a_{42}=0$) for any ω. This solution is the equilibrium point.
  (2) ($a_{10} = 0.8867$, $a_{11} = 0.1428$, $a_{20} = -0.1576$, $a_{21} = 0.2833$, $a_{22} = -0.0750$, $a_{30} = 0$, $a_{31} = 0.067$, $a_{32} = 0.1806$, $a_{40} = 0.0545$, $a_{41} = -0.0833$, $a_{42} = 0.0617$, $\omega = 1.9348$). This solution corresponds to the limit cycle of simulation D in Fig. 4 (in the simulation $\omega = 1.848$ rad/s).

Notice that in the case $\tau_L \neq 0$ all the state variables oscillate.

5. Conclusions

In this paper, we have shown that self-sustained oscillations in indirect FOC for induction motors may be due to the appearance of a Hopf bifurcation. Other causes of oscillations may exist but, for these cases, the local stability of the desired equilibrium point would be preserved. Therefore, the Hopf bifurcation is the most interesting phenomenon regarding oscillations from the practical point of view. We have given some simple rules for tuning the PI gains of the velocity loop in order to prevent Hopf bifurcation and quench these oscillations. Our theoretical results were validated with some simulation evidence. Current research is under way to test our theoretical predictions in an experimental facility.

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