

# An Energy–Shaping Approach to the Design of Excitation Control of Synchronous Generators

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## Abstract

In this paper we discuss the estimation of the domain of attraction of equilibria in power systems and propose a new passivity-based controller design methodology for excitation control of synchronous generators. The methodology goes beyond the widely popular damping injection ( $L_g V$ ) schemes, to actually shape the total energy function via modification of the energy transfer between the mechanical and electrical components of the system. Applying the procedure it is shown that a, properly tuned, linear state feedback enlarges both the estimates and the actual domain of attraction, thus increasing critical clearing time for faults. This is illustrated in two case studies, including a benchmark comparison with the classical control scheme.

*Key words:* Power systems stability, passivity-based control, nonlinear control, energy-shaping.

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(**Caveat** This is an abridged version of the full paper which is available upon request to the authors.)

## 1 Introduction

The electric utility industry is undergoing unprecedented changes in its structure worldwide. The emergence of separate entities for generation, transmission and distribution have given rise to new issues in power system operation and planning. (See the Special Issue (2000) of Proceedings of the IEEE for an excellent tutorial account.) It has been suggested that to ensure reliable and secure planning under the new deregulated market conditions the problem of transient stability should be revisited. This topic has recently attracted the attention of both the power systems and the control community. New term definitions and classifications for power systems stability studies have been proposed (Kundur, 1994) and a lot of discussion on transient stability assessment methods has been raised. In particular, the general validity of the currently known direct methods of estimation of the domain of attraction of the

post-fault equilibrium has been questioned. It is worth recalling also that some of the claims of the classical reference Tsolas, Arapostathis & Varaiya (1985)—that until now is systematically cited—were proven to be incorrect in Zaborsky, Huang & Zheng (1988). See also Paganini & Lesieutre (1998) for a related discussion.

A better understanding of these issues is fundamental not only for the development of software tools for transient stability assessment, but also for the design of devices aiming at enhancing fault critical clearing times. Critical clearing times can be increased basically in two ways: either reducing the (excursions of the) trajectory of the faulted system, which is achieved by currently used devices such as fast-acting high-ceiling voltage regulators and break resistors, or enlarging the size of the region of attraction of the post-fault equilibrium through control. The application of nonlinear control methods to enhance transient stability has been given much attention in the literature since the late eighties. Both turbine control (Lu & Sun (1989)) and the more challenging and widely applicable excitation control have been explored. These excitation controllers are proposed to replace the traditional Automatic Voltage Regulator and the Power System Stabilizer (AVR+PSS) control structure, and questions about the benefits of this replacement have not yet been answered. On the other hand,

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with new policies for provision of ancillary services and the increasing number of small sized distributed generators competing for voltage regulation, it is not clear which role AVR's are to play in the future.

Feedback linearization was one of the early strategies to be explored, with applications proposed to both single and multimachine systems, even with output feedback and state observers (King, Chapman & Ilic (1994), Wang, Hill, Middleton & Gao (1993), Mielczarsky & Zajczkowski (1994)). Robustness issues, both against parameter uncertainties and unmodeled dynamics, are still open, as well as stability analysis in the output feedback case. The fragility of these methods, which are based on nonlinearity cancellations, has motivated the more recent works on energy related design techniques, usually developed for the well-known single machine third-order flux-decaying model (just as the large majority of other works) and its associated energy-like Lyapunov function (Pai, 1989). Most results are based on the application of damping injection (also called  $L_gV$ ) controllers to replace the AVR and PSS (see Machowski, Bialek & Bumby (1997), Moon, Choi & Roh (2000), Sun, Song & Li (2000), Shen, Ortega, Lu & Tamura (2000), Ghandhari, Andersson, Pavella & Ernst (2001)). In Bazanella, Kototovic & e Silva (1999) a dynamic damping injection controller is proposed which is proven to enlarge the estimate of the region of attraction and is shown to enhance critical clearing times in simulation studies. See also Ortega, Stankovic & Stefanov (1998) for an alternative passivation approach.

In this paper we propose to apply a new methodology to design excitation controllers that—going beyond the widely popular damping injection ( $L_gV$ ) schemes—actually shapes the total energy function via modification of the energy transfer between the mechanical and electrical components of the system. It is our contention that this broader framework is more suitable to revisit the transient stability problem from a control design perspective—which is the long term objective of our research.

The remaining of the paper is organized as follows. The problem is formulated in Section 2. In Section 3 estimation of domains of attraction is analyzed, shedding new light into the open issues regarding this topic. Then, in Section 4, the interconnection and damping assignment (IDA) passivity-based methodology proposed in Ortega, van der Schaft, Maschke & Escobar (2002), is applied to excitation control design. It is shown that the resulting controller, which is a simple linear state feedback, enlarges the domain of attraction. Further, we prove that earlier proposed  $L_gV$  control designs represent a particular case of this methodology. The stabilization mechanism of this control is analyzed in Section 5, giving insight into the choice of the control parameters and assessing their impact on the domain of attraction of the

desired equilibrium. Two case studies are presented in Section 6 showing the enlargement of the domains of attraction and its effect on the enhancement of critical clearing times. A comparison, rarely seen in previous papers, with the usual AVR+PSS excitation control structure in a benchmark example is also provided. We conclude with some final remarks and comments on future work in Section 7.

## 2 Problem Formulation

We consider a simplified dynamical model of a power system. This model consists of a single generator connected to an infinite bus through a transmission line namely the single machine infinite bus power system (SMIB). The machine model considered here is the classical flux-decay model, see Pai (1989) and Anderson & Fouad (1977). After making standard considerations, including the assumption that the mechanical input power is constant, a model for the generator can be written as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -b_1 x_3 \sin x_1 - b_2 x_2 + P \\ \dot{x}_3 &= b_3 \cos x_1 - b_4 x_3 + E + u. \end{aligned} \quad (1)$$

The state variables of this system are the load angle ( $x_1$ ), the shaft speed deviation from the synchronous speed ( $x_2$ ), and the quadrature axis internal voltage ( $x_3$ ). The coefficients  $b_i$ ,  $i = 1, \dots, 4$  are positive, the inputs  $P$  and  $E$  which represent the mechanical power and field voltage, respectively, are held constant, and a supplementary signal  $u$  is added to the latter, which represents the control input. Due to physical considerations we would like to restrict the operation of the system to the closed set

$$\mathcal{D}_\delta = \{x \in \mathbb{R}^3 \mid 0 \leq x_1 \leq \frac{\pi}{2} - \delta_1, \delta_1 \leq x_3\} \quad (2)$$

where  $\delta_1$  and  $\delta_2$  are some small positive numbers.

The open-loop equilibria of the system (1) are given by  $\bar{x} = [\bar{x}_1, 0, \bar{x}_3]^\top$ , where  $\bar{x}_1$  and  $\bar{x}_3$  are the solutions of

$$E = b_4 \bar{x}_3 - b_3 \cos \bar{x}_1, \quad P = b_1 \bar{x}_3 \sin \bar{x}_1.$$

It is easy to see that a necessary condition for existence of equilibria is  $E > \frac{b_4 P}{b_1} - b_3$ . If this inequality holds, there exists an asymptotically stable equilibrium in  $\mathcal{D}_\delta$ , which we denote with the subindex  $(\cdot)_*$ , i.e.,  $x_* = [x_{1*}, 0, x_{3*}]^\top$ , to underscore the fact that it is the equilibrium point to be stabilized. Besides  $x_*$ , the open-loop system has an additional equilibrium (that we denote  $\bar{x}_u$ ) which may be shown to be unstable (see Pai (1989)). Notice that, if  $\frac{b_4 P}{b_1 E} > 1$ , this unstable equilibrium is inside  $\mathcal{D}_\delta$ , severely restricting the size of the domain of attraction of  $x_*$ , denoted in the sequel  $\mathcal{R}$ .

As the system becomes more heavily loaded, the domain of attraction of the stable equilibrium can become too small, which can result in unbearably small fault critical clearing times. We shall discuss the estimation of the domain of attraction and design a control  $u$  aiming at the enlargement of  $\mathcal{R}$ . We will show that with a simple *linear* state feedback it is possible to generate estimates of  $\mathcal{R}$  that are strictly larger than the ones known for the open-loop system. An additional feature of our controller is that it pushes away from  $\mathcal{D}_\delta$  the unstable equilibrium. Open issues of practical interest which are not addressed in this paper are the uncertain knowledge of the equilibrium and/or the system parameters, and the inability to measure the full state, these problems have been studied in, e.g., Bazanella et al. (1999) & De Leon-Morales, Busawon & Acha-Daza (2001), respectively.

### 3 Domain of Attraction of the Open-loop System

It is well-known (Pai, 1989) that the equilibrium  $x_*$  of the SMIB model (1) is locally stable with Lyapunov function

$$H(x) = \frac{1}{2}x_2^2 + b_1x_3(\cos x_{1*} - \cos x_1) - P\tilde{x}_1 + \frac{b_1b_4}{2b_3}\tilde{x}_3^2 \quad (3)$$

where we have defined  $\tilde{x}_i \triangleq x_i - x_{i*}$ ,  $i = 1, 3$ .<sup>1</sup> Indeed,  $H(x_*) = 0$  and  $H$  has an isolated local minimum at  $x_*$ , hence it is a positive definite function in some neighborhood of  $x_*$ . Further, its derivative along trajectories of (1) with  $u = 0$  is non-positive. To establish asymptotic stability we can invoke La Salle's invariance principle, see e.g. (Sastry, 1999). We stress the fact that the invariance principle requires that the trajectories are *bounded* and remain inside the set where  $\dot{H} \leq 0$ . In typical applications of this theorem these two properties are ensured postulating the existence of a *compact invariant* set, which is usually assumed to be a sublevel set of the Lyapunov function, that is the set,  $\Omega_c \triangleq \{x \mid H(x) \leq c\}$ . Boundedness of the sublevel sets is often tested verifying that  $H$  is radially unbounded. In our case  $H$  is *not radially unbounded* and we can ensure that  $\Omega_c$  is bounded only for sufficiently small  $c$ . Thus, the best estimate of the domain of attraction is given by  $\Omega_c$  with the largest  $c$  such that its connected subset containing the equilibrium is bounded.

In view of the discussion above, a more careful analysis is needed to estimate  $\mathcal{R}$ . In the proposition below we exploit the convexity properties of  $H$ .

<sup>1</sup> To simplify the notation, but with an obvious abuse of notation, throughout the paper we will mix  $\tilde{x}$  and  $x$  in the definition of some functions.

**Proposition 1** Fix a small  $\delta > 0$ . Estimates of the domain of attraction of the stable equilibrium  $x_*$  of (1) are the sublevel sets  $\Omega_c$  that are contained in  $\mathcal{S}_\epsilon \cap \mathcal{D}_\delta$ , where

$$\mathcal{S}_\epsilon \triangleq \left\{x \mid \left(\frac{b_1b_4}{b_3} - \epsilon\right)x_3 > \frac{1}{\cos x_1}(b_1 \sin^2 x_1 + \frac{\epsilon}{b_1})\right\} \quad (4)$$

for all  $0 < \epsilon < \min\left\{\frac{b_1b_4}{b_3}, 1, b_1\delta \cos\left(\frac{\pi}{2} - \delta\right)\right\}$ . See Fig. 1.

**Proof (sketch)** The proof relies on the fact that if  $H$  is a *strongly convex* function on some set  $\mathcal{A} \subset \mathbb{R}^3$  (i.e.,  $\frac{\partial^2 H}{\partial x^2}(x) \geq \epsilon I$  for some  $\epsilon > 0$  and all  $x \in \mathcal{A}$ ), then the sublevel sets of  $H$  are *bounded* (Boyd & Vandenberghe, 1999).

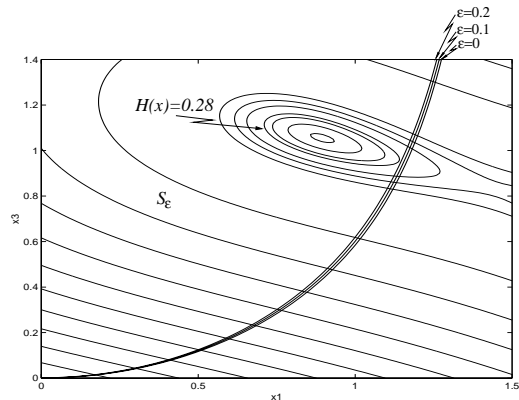


Fig. 1. Estimates of  $\mathcal{R}$  of the open loop system

**Remark 1** Some authors have reported estimates of  $\mathcal{R}$  for this problem investigating the region of *strict* convexity of  $H$ , that is, the region defined as  $\{x \mid \frac{\partial^2 H}{\partial x^2}(x) > 0\}$ , which is not an established sufficient condition for boundedness of sublevel sets. Other reported estimates, on which currently applied transient stability assessment algorithms are based, assume that sublevel sets remain bounded until they intersect the unstable equilibrium point. Neither of these assumptions is proven to be correct in general and may result in incorrect estimates of the domain of attraction (see, e.g., Zaborsky et al., 1988).

**Remark 2** Strong convexity is a sufficient but not a necessary condition for boundedness of sublevel sets, which results in more conservative estimates, yet guaranteed to be correct. Characterizing the largest closed sublevel set of a non strongly convex Lyapunov function and its relationship with the boundary of the real domain of attraction is a very hard problem, particularly for nonconservative systems. In our control design we will construct a Lyapunov function for the closed-loop system which is strongly convex in the *whole* set  $\mathcal{D}_\delta$ .

## 4 Control Design

Our excitation controller is derived following the IDA procedure proposed in Ortega et al. (2002)—we refer the reader to this paper for further details on IDA control. To apply the technique we need an alternative representation of the SMIB system (1), called port-controlled Hamiltonian model

$$\dot{x} = (\mathbf{J} - \mathbf{R}) \frac{\partial H}{\partial x} + \mathbf{g}u$$

where the damping ( $\mathbf{R} = \mathbf{R}^\top \geq 0$ ), interconnection ( $\mathbf{J} = -\mathbf{J}^\top$ ), and input ( $\mathbf{g}$ ) matrices are given by

$$\mathbf{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & \frac{b_3}{b_1} \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Stabilization in IDA control is achieved aiming at the closed-loop dynamics

$$\dot{x} = (\mathbf{J}_d - \mathbf{R}_d) \frac{\partial H_d}{\partial x}(x) \quad (5)$$

where  $H_d(x)$  is the desired total energy function, which has a *minimum* at  $x_*$ ,  $\mathbf{J}_d = -\mathbf{J}_d^\top$  and  $\mathbf{R}_d = \mathbf{R}_d^\top \geq 0$  are some *desired* interconnection and damping matrices, respectively. As explained in Ortega et al. (2002) (and references therein), with the IDA method it is possible to design a controller based on physical considerations.

Let us describe our design choices in our particular example. Injecting damping into the electrical variable ( $x_3$ ) is easily achieved feeding back  $\frac{\partial H}{\partial x_3}$ , this is the action of the well-known  $L_gV$  controllers. On the other hand, it can be seen that the damping in the mechanical coordinates ( $x_1, x_2$ ) is weak, since  $b_2$  is usually very small. Furthermore, since the interconnection matrix  $\mathbf{J}$  does not contain any coupling between the electrical and the mechanical dynamics, the propagation of the damping injected in the mechanical coordinate is far from obvious. This suggests the new interconnection and damping matrices

$$\mathbf{J}_d = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & \alpha_1 \\ 0 & -\alpha_1 & 0 \end{bmatrix}, \quad \mathbf{R}_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & \frac{b_3}{b_1} + k_v \end{bmatrix} \quad (6)$$

where  $k_v \geq 0$  and  $\alpha_1$  are constants to be defined. Notice that the  $L_gV$  control design proposed in other papers represents a particular case of this design method. We have the following result.

**Proposition 2** Consider the SMIB system (1) in closed-loop with the static state feedback controller

$$u = -k_v b_1 (\cos x_{1*} - \cos x_1) - \alpha_1 \alpha_2 \left( \frac{b_3}{b_1} + k_v \right) \tilde{x}_1 - \alpha_1 x_2 - \left( \frac{b_3}{b_1} \alpha_2 - b_4 + k_v \alpha_2 \right) \tilde{x}_3 \quad (7)$$

with the tuning parameters  $k_v \geq 0$ ,  $\alpha_2 > 0$  and  $\alpha_1 < 0$  verifying<sup>2</sup>

$$\alpha_2 \geq \frac{b_1 b_4}{b_3}, \quad \alpha_1 < -\frac{b_1}{\alpha_2} \quad (8)$$

Then, the closed-loop system takes the form (5), (6) and

(i)  $x_*$  is the *unique equilibrium* in  $\mathcal{D}_\delta$ .

(ii)  $x_*$  is asymptotically stable with Lyapunov function  $H_d(x) = H(x) + H_a(x)$ , where

$$H_a = b_1 \alpha_1 [\tilde{x}_1 \cos x_{1*} - \sin x_1 + \frac{b_4 \alpha_1}{2 b_3} (\tilde{x}_1^2 + \frac{2}{\alpha_1} \tilde{x}_1 \tilde{x}_3 + x_{1*}^2)] + \frac{1}{2} (\alpha_2 - \frac{b_1 b_4}{b_3}) (\alpha_1 \tilde{x}_1 + \tilde{x}_3)^2 \quad (9)$$

and  $H$  is given by (3).

(iii) Estimates of its domain of attraction are *all* the sublevel sets  $\Omega_c^d \triangleq \{x \mid H_d(x) \leq c\}$  that are contained in  $\mathcal{D}_\delta$ .

**Proof (sketch)** With the choices of  $\mathbf{J}_d, \mathbf{R}_d$  in (6) we see that the model (5) will be matched, with an energy function of the form  $H_d(x) = H(x) + H_a(x)$ , for all  $H_a(x)$  satisfying

$$\frac{\partial H_a}{\partial x_2} = 0, \quad -\frac{\partial H_a}{\partial x_1} + \alpha_1 \frac{\partial H_a}{\partial x_3} = -\alpha_1 \frac{\partial H}{\partial x_3}$$

if we apply the control  $u = -\frac{b_3}{b_1} \frac{\partial H_a}{\partial x_3} - \alpha_1 x_2 - k_v \frac{\partial H_d}{\partial x_3}$ . The first equation indicates that  $H_a$  cannot depend on  $x_2$ , a restriction that does not hamper the energy shaping, as in this coordinate the function  $H$  already has a minimum at the desired (zero) value. The second equation is a linear PDE that can be easily solved using symbolic programming (e.g., Maple) and the solution is

$$H_a = \alpha_1 b_1 [x_1 \cos x_{1*} - \sin x_1 + \frac{b_4}{b_3} x_1 (\tilde{x}_3 + \frac{\alpha_1}{2} x_1)] + \Phi(\alpha_1 x_1 + x_3) \quad (10)$$

<sup>2</sup> In particular, we can take  $\alpha_2 = \frac{b_1 b_4}{b_3}$ , which yields the partial state feedback control  $u = -\alpha_1 (b_4 \tilde{x}_1 + x_2)$ , with  $\alpha_1 < -\frac{b_3}{b_4}$ . Also, notice that with  $k_v = 0$  the controller is *linear*.

where  $\Phi$  is an (arbitrary) differentiable function. This function must be chosen such that  $H_d$  has an isolated minimum at  $x_*$ . It is easy to verify that with the choice

$$\Phi = -\frac{b_1 b_4 \alpha_1}{b_3} x_{1*} (\alpha_1 \tilde{x}_1 + \tilde{x}_3) + \frac{\gamma}{2\alpha_1^2} (\alpha_1 \tilde{x}_1 + \tilde{x}_3)^2$$

we satisfy  $\frac{\partial H_d}{\partial x}(x_*) = 0$  for any  $\gamma \in \mathfrak{R}$ . Replacing  $\Phi$  in (10), and defining  $\alpha_2 \triangleq \frac{\gamma}{\alpha_1^2} + \frac{b_1 b_4}{b_3}$  we obtain (up to an additive constant) the Lyapunov function  $H_d$ .

Now we will choose  $\alpha_2$  to ensure that  $H_d(x)$  has a minimum at  $x_*$  and, at the same time, check its strong convexity. With some lengthy, but straightforward calculations, we can prove that: given a small  $\delta > 0$ , and  $\alpha_1, \alpha_2$  as defined by the proposition,  $\frac{\partial^2 H_d}{\partial x^2}(x) - \epsilon I > 0, \forall x \in \mathcal{D}_\delta$  and

$$\forall \epsilon \in \left( 0, \min \left\{ b_1 \delta \cos\left(\frac{\pi}{2} - \delta\right), 1, \alpha_2, \frac{-b_1 \sin \delta}{\alpha_1} \right\} \right)$$

This property has the following implications: (1)  $x_*$  is a strict local minimum of  $H_d$ , hence,  $x_*$  is a stable equilibrium; (2) all sublevel sets  $\Omega_c^d$  contained in  $\mathcal{D}_\delta$  are closed; (3)  $x_*$  is the unique extremum of  $H_d$  in  $\mathcal{D}_\delta$ .

The proof that  $x_*$  is asymptotically stable follows from an invariant set analysis noting that  $\dot{H}_d = -\left(\frac{\partial H_d}{\partial x}\right)^\top \mathbf{R}_d \frac{\partial H_d}{\partial x}$  for the closed-loop dynamics (5).

**Remark 3** It is not possible to conclude positively from Propositions 1 and 2 that  $\Omega_c \subset \Omega_c^d$ . However, since the region of strong convexity of  $H_d$  is at least the whole set  $\mathcal{D}_\delta$ , a considerably larger estimate for the domain of attraction is expected in closed-loop, particularly when the system is highly stressed.

**Remark 4** We should underscore that there is a large degree of freedom on the choice of the feedback gains. This feature is important when control saturation considerations should be taken into account. This situation should be contrasted with other nonlinear controller design techniques which feed-back higher order terms that, obviously, inject high gains into the loop and saturate the actuator. One further degree of freedom is provided by our choice of the function  $\Phi$  in (10), that may be taken such that its contribution in the control satisfies some specified bounds—without affecting the convexity properties of  $H_d$ . See Ortega et al. (2002) for further discussion.

## 5 Stabilization Mechanism

In this section we elaborate on the stabilization mechanism of the proposed controller. In particular, we show how the boundedness and damping properties of the

open-loop SMIB system (1) are improved by the proposed controller, which also gives insight into the choice of its tuning parameters. We then relate these boundedness properties to transient stability.

Some simple calculations with (1) show that, with  $u \equiv 0$ , we have that  $x_3$  is bounded and that the set  $\{x \mid |\tilde{x}_3| \leq \frac{b_3}{b_4}\}$  is *globally attractive*. Further, using the identity  $\sin x_1 = \sin x_{1*} + (\cos x_{10})\tilde{x}_1$ , where  $x_{10}$  is a value between  $x_1$  and  $x_{1*}$ , we obtain the linear differential operator representation

$$(p^2 + b_2 p + b_1 x_{3*} \cos x_{10})\tilde{x}_1 = -(b_1 \sin x_1)\tilde{x}_3 \quad (11)$$

where  $p \triangleq \frac{d}{dt}$ , and we treat  $\cos x_{10}, \sin x_1$  as arbitrary, bounded time-varying functions. If  $x_1(t)$  remains in  $\mathcal{D}_\delta$  the third term in the parenthesis is positive, and the “frozen” operator is stable. Furthermore, if  $|x_2(t)|$  is sufficiently small, the operator is slowly time-varying and  $\mathcal{L}_\infty$  stable. This property, combined with the fact that  $\tilde{x}_3 \in \mathcal{L}_\infty$ , establishes boundedness of the overall system.

The interest of studying the asymptotic sets stems from the fact that a simple invariant set analysis with  $\dot{H} = 0$  shows that, if the trajectories remain within  $\mathcal{D}_\delta$ , then they will asymptotically converge to the desired equilibrium  $x_*$ . This behavior is a folk theorem among power engineers that roughly says that “trajectories that skip only finitely many cycles converge to an equilibrium” (pp. 200, Sastry, 1999). However, there is no *a priori* guarantee that  $x(t)$  will remain in  $\mathcal{D}_\delta$ , and there may exist initial conditions for which it will actually escape from this region.

Let us see now how the proposed feedback affects these properties. To this end, define  $w = \alpha_1 \tilde{x}_1 + \tilde{x}_3$ . Then, along the closed loop dynamics with the control (7), we have

$$\dot{w} = -\frac{\alpha_2}{b_1} (b_3 + k_v b_1) w - (b_3 + k_v b_1) (\cos x_1 - \cos x_{1*}) \quad (12)$$

hence,  $w \in \mathcal{L}_\infty$ , and the *globally attractive* set now becomes  $\{x \mid |\alpha_1 \tilde{x}_1 + \tilde{x}_3| \leq \frac{b_3}{\alpha_2}\}$ . See Fig. 2. Furthermore, the linear differential operator relating  $w$  and  $\tilde{x}_1$  is obtained as

$$[p^2 + b_2 p + b_1 (x_{3*} \cos x_{10} - \alpha_1 \sin x_1)]\tilde{x}_1 = -(b_1 \sin x_1)w \quad (13)$$

The following observations are in order:

- While in the open loop system the asymptotic behavior was restricted to a band  $|\tilde{x}_3| \leq \frac{b_3}{b_4}$ , in closed loop the asymptotic operating region is  $|\alpha_1 \tilde{x}_1 + \tilde{x}_3| \leq \frac{b_3}{\alpha_2}$ . As depicted in Fig. 2, by decreasing  $\alpha_1$  we can “rotate” the admissible region, to move it inside  $\mathcal{D}_\delta$ . Furthermore, by increasing  $\alpha_2$ , we can make the band arbitrarily small in

an arbitrarily small time. Increasing the damping injection gain  $k_v$  has an effect similar to increasing  $\alpha_2$ . These are the stabilizing effects of the tuning parameters.

– Comparing (11) with (13)—and recalling that  $\alpha_1 < 0$ —we see that we have added a stabilizing term which is effective even outside  $\mathcal{D}_\delta$ .

– The  $L_g V$  controller of Bazanella et al. (1999) and Machowski et al. (1997) yields the voltage dynamics

$$\dot{\tilde{x}}_3 = \left(k_v + \frac{b_3}{b_1}\right)[-b_4\tilde{x}_3 + b_3(\cos x_1 - \cos x_{1*})]$$

Hence, the stabilization mechanism explained above is conspicuous by its absence in this controller.

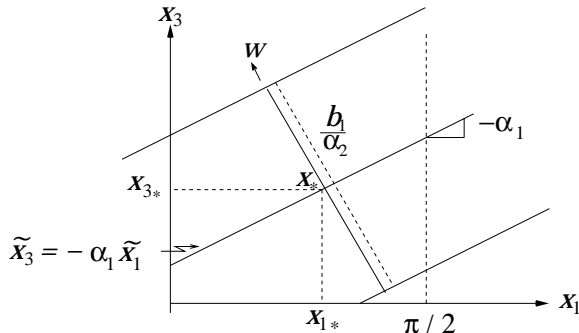


Fig. 2. Closed-loop globally attractive set  $\{x \mid \alpha_1 \tilde{x}_1 + \tilde{x}_3 \leq \frac{b_1}{\alpha_2}\}$

## 6 Simulation Results

To illustrate the behavior of the proposed controller we present in this section some simulation results.

### 6.1 A Simple Example

The parameters of the SMIB model (1),  $b_1 = 34.29, b_2 = 0, b_3 = 0.149, b_4 = 0.3341, P = 28.22$  are in pu and are taken from Anderson & Fouad (1977) and the operating conditions are as in Bazanella et al. (1999).

We analyze the response of the system to a short circuit which consists of the connection of a small impedance between the machine's terminal and ground. The impedance is disconnected after a certain time (called the clearing time) and the system is back to its pre-disturbance topology. During the fault the trajectories diverge, the largest time interval before instability, called the *critical clearing time*, is determined via simulation

We carried out simulations in two operating conditions, one stressed condition with  $E = 0.2405$  and the other in a more realistic operating condition with  $E = 0.2593$ . Table 1 shows the critical clearing time of the systems

for various controls, as we see from the table the *energy-shaping* term indeed increases the clearing time. On the other hand, the *damping-injection* term, which significantly improves the transient performance as shown in Fig. 3 and Fig. 4, may reduce the critical clearing time. The trajectories shown in these figures were started in  $x(0) = x_*$ , and the fault introduced at  $t = 1$  sec and removed after a clearing time  $t_{cl}$ .

$u$	$u = 0$	$u = (7), k_v = 0$	$k_v = 0.01$	$k_v = 0.07$
$E = 0.2405$	40	200	200	180
$E = 0.2593$	120	240	240	220

Table 1: Critical clearing time (m sec)

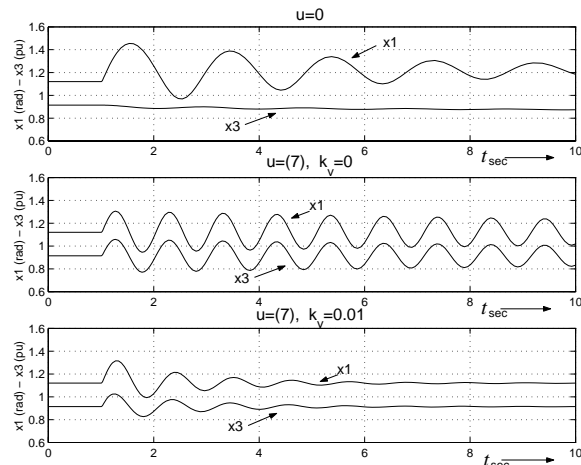


Fig. 3. Load angle and internal voltage for  $E=0.2405$ ;  $k_v = 0.07$ ;  $t_{cl} = 40$  m sec

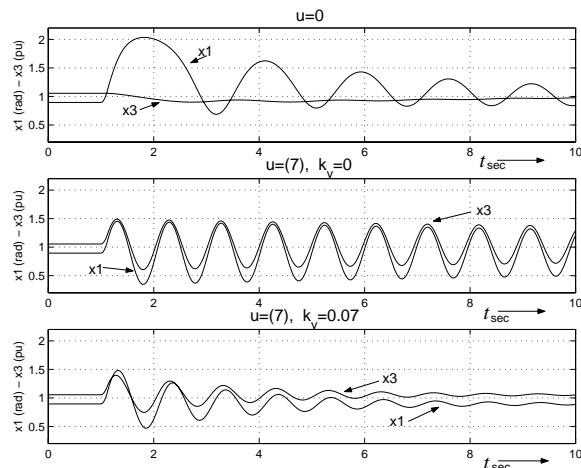


Fig. 4. Load angle and internal voltage for  $E=0.2593$ ;  $k_v = 0.07$ ;  $t_{cl} = 120$  m sec

To compare the estimates of the domain of attraction provided by the Lyapunov functions  $H$  and  $H_d$  we present in Figure 5 the projection on the plane  $x_1 - x_3$

of their sublevel sets. Notice that the sublevel sets of  $H$  (dashed line) open up very close to the equilibrium  $x_*$  while the ones corresponding to  $H_d$  (solid line) cover up a very large neighborhood of  $x_*$ . To highlight the dramatic increase in domain of attraction we present also a zoom of this figure, where only the sublevel sets of  $H$  are depicted.

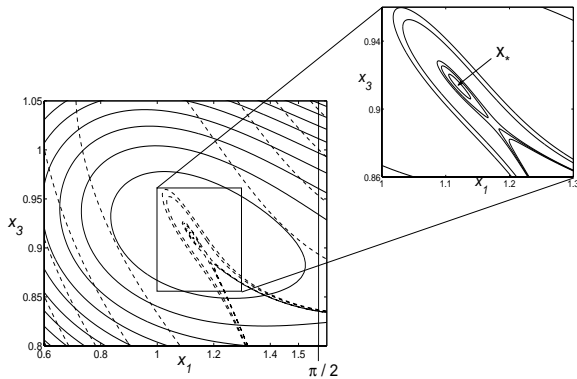


Fig. 5. Level sets of  $H$  (dashed line) and  $H_d$  (solid line)

## 6.2 A Benchmark Case Study

The second case study is taken from Kundur (1994), Example 13.2, and includes the effects of generator field and rotor circuits dynamics, resulting in a sixth order model (eqs. 12.83, 12.84 and 12.158 in Kundur (1994)). In this case, the disturbance is a three-phase fault in one of the two transmissions lines circuits, cleared by isolating the faulted circuit simultaneously at both ends. The results obtained with a conventional AVR+PSS excitation control reported in Kundur (1994) are also given for comparison. The parameters of the controller are set as  $\alpha_1 = -\frac{b_1}{\alpha_2} = -0.6$ ,  $\alpha_2 = \frac{b_1 b_4}{b_3} = 120$ ,  $k_v = 1$ .

Figure 6 presents the system's response to a short-circuit with 70 m sec clearing time in three situations: no control, conventional AVR+PSS control with the parameters given in Kundur (1994), and the proposed controller. Table 2 presents the critical clearing times obtained in each one of these three control scenarios. It is seen that the critical clearing time obtained with the proposed controller is the same as with the AVR+PSS control. Figure 7 presents the systems response to a short-circuit with clearing time equal to critical.

Method	No Control	IDA	AVR and PSS
$t_{cr}$	70	100	100

TABLE 2. Critical clearing time (m sec)

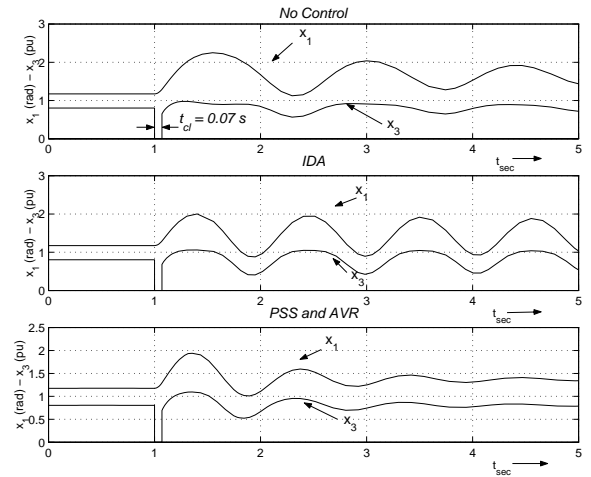


Fig. 6. Load angle and internal voltage  $t_{cl} = 70$  m sec

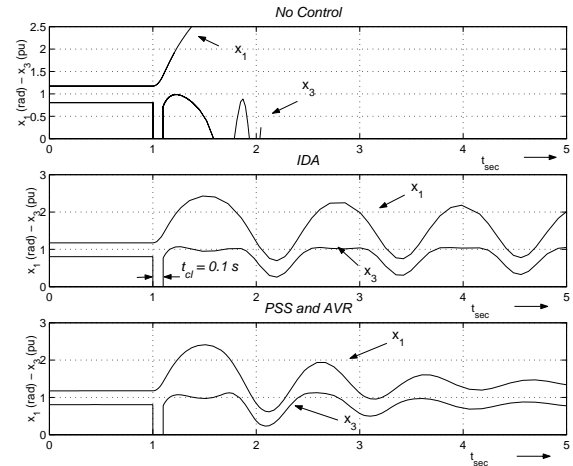


Fig. 7. Load angle and internal voltage  $t_{cl} = 100$  m sec

## 7 Conclusions

We have shown that a simple, suitably tuned, linear state-feedback excitation controller for synchronous machines enlarges the domain of attraction of the open-loop equilibrium and increases the damping. The controller is derived using the recently developed IDA methodology of Ortega et al. (2002). A key step in the procedure is the inclusion of an interconnection between the electrical and the mechanical dynamics. A by-product of our developments is the definition of a new Lyapunov function for this system that may be of interest for other stability studies. The methodology strictly contains the earlier proposed  $L_gV$  design.

As a consequence of the enlargement of the domain of attraction of the post-fault equilibrium, critical-clearing times for faults are increased. In a simple case study a dramatic increase of the critical clearing time for solid short-circuits has been observed. A benchmark has also been presented, in which the critical clearing time ob-

tained with the proposed controller has been similar to the one obtained with the conventional AVR+PSS control structure. Further increase can be obtained by combining the proposed controller with some action aiming at reducing the state trajectory during the fault, just as done by fast AVR's with high ceiling voltage in the conventional AVR+PSS control structure. Additional studies are necessary to provide optimal parameter tuning and better design options in the design methodology ( $\mathbf{J}_d$  and  $\mathbf{R}_d$ ), which will result in improved performance of the controller and possibly an output feedback control law.

Current research is under way mainly in four directions: (i) incorporation of adaptation features to estimate the parameters and the possibly unknown equilibrium, as done for instance in Shen et al. (2000), Bazanella et al. (1999) and Galaz, Ortega & Bazanella (2001); (ii) implementation of designs that feed-back the shaft speed only; (iii) utilization of FACTS devices, which act at the level of the interconnection matrix,<sup>3</sup> and (iv) extension of the results to the multi-machine case.

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## References

- [1] Anderson P.M., & Fouad, A.A. (1977). *Power Systems Control and Stability*, The Iowa State University Press.
- [2] Bazanella, A., Kototovic, P. & e Silva, A.S. (1999). A dynamic extension for  $L_gV$  controllers, *IEEE Transactions on Automatic Control*, *44*, 588–592.
- [3] Boyd, S. & Vandenberghe, L. (2000). Introduction to convex optimization with engineering applications, Lecture notes, Stanford University.
- [4] Ghandhari, M., Andersson, G., Pavella, M. & Ernst, D. (2001). A control strategy for controllable series capacitor in electric power systems, *Automatica*, *37*, 1575–1583.
- [5] Chiang, H-D., Hirsch M. & Wu F. (1988). Stability regions of nonlinear autonomous dynamical systems, *IEEE Transactions on Automatic Control*, *33*, 16–26.
- [6] King, C.A., Chapman, J.W. & Ilic, M.D. (1994). Feedback linearizing excitation control on a full-scale power system model, *IEEE Transactions on Power Systems*, *9*, 1102–1109.
- [7] Kundur, P. (1994). *Power System Stability and Control*, McGraw-Hill.
- [8] De Leon-Morales, J., Busawon, K. & Acha-Daza, S. (2001). A robust observed-based controller for synchronous generators, *International Journal of Electrical Power & Energy Systems*, *23*, 195–211.
- [9] Galaz, M., Ortega, R. & Bazanella, A. (2001). A consistent parameter estimator for excitation control of synchronous generators, *Technical Report LSS-SUPELEC*.
- [10] Liu, Q.J., Sun, Y.Z., Song, Y.H. & Tielong Shen. (2001). Nonlinear coordinated excitation and SMES controller based on Hamiltonian structure for multimachine power system. Transient stability improvement, *submitted to IEEE*.
- [11] Lu, Q. & Sun, Y.Z. (1989). Nonlinear stabilizing control of multimachine power systems, *IEEE Transactions on Power Systems*, *4*, 236–241.
- [12] Machowski, J., Bialek, J.W. & Bumby, J.R. (1997). *Power System Dynamics and Stability*, John Wiley & Sons.
- [13] Mielczarsky, W. & Zajaczkowski, A.M. (1994). Nonlinear field voltage control of a synchronous generator using feedback linearization, *Automatica*, *30*, 1625–1630.
- [14] Moon, Y-H., Choi, B-K. & Roh, T-H. (2000). Estimating the domain of attraction for power systems via a group of damping-reflected energy functions, *Automatica*, *36*, 419–425.
- [15] Ortega, R., Stankovic A. & Stefanov, P. (1998). A passivation approach to power systems stabilization, *IFAC Symp. Nonlinear Control Systems Design*, Enschede, NL, July 1–3.
- [16] Ortega, R., van der Schaft, A.J., Maschke, B.M. & Escobar, G. (2002). Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems, *Automatica*, *38*, 585–896.
- [17] Paganini, F. & Lesieutre B. (1999). Generic properties, one-parameter deformations and the BCU method, *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, *46*(6), 760–763.
- [18] Pai, M.A. (1989). *Energy Function Analysis for Power System Stability*, Kluwer Academic Publishers, Boston, MA.
- [19] Sastry, S. (1999). *Nonlinear Systems: Analysis, Stability and Control*, Springer-Verlag.
- [20] Shen, T., Ortega, R., Lu, Q., Mei, S. & Tamura, K. (2000). Adaptive  $L_2$  control of hamiltonian systems with parametric perturbation and application to power systems, *IEEE Conf. on Dec. and Control*, Sydney, Australia, December 2000. (To appear in *Asian J of Control*, 2002.)
- [21] Special Issue on the Technology of Power System Competition, (2000). *Proceedings of the IEEE*, *88*(2), eds. D.S. Kirschen, R. Bacher & G.T. Heydt. February 2000.
- [22] Sun, Y.Z., Song, Y.H. & Li, X. (2000). Novel energy-based Lyapunov function for controlled power systems, *IEEE Power Engineering Review*, 55–57, May 2000.
- [23] Tsolas, N.A., Arapostathis, A. & Varaiya, P.P. (1985). A structure preserving energy function for power system transient stability analysis, *IEEE Transactions on Circuits and Systems*, *32*, 1041–1049.
- [24] Wang, Y., Hill, D.J., Middleton, R.H. & Gao, L. (1993). Transient stability enhancement and voltage regulation of power system, *IEEE Transactions on Power Systems*, *8*, 620–627.
- [25] Zaborsky, J., Huang, G. & Zheng B. (1988). A Counterexample on a theorem by Tsolas *et al.* and an Independent Result by Zaborsky *et al.*, *IEEE Transactions on Automatic Control*, *33*, 316–317.

<sup>3</sup> In Liu, Sun, Song & Shen (2001) an  $L_gV$  controller is designed for a FACTS-based system.