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Interconnection and Damping Assignment Passivity-Based Control: A Survey

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Interconnection and damping assignment passivity-based control is a technique that regulates the behavior of non-linear systems assigning a desired (port-controlled Hamiltonian) structure to the closed-loop. Since the introduction of this controller design methodology five years ago, many theoretical extensions and practical applications have been reported in the literature. The theoretical developments include some variations and shortcuts that are useful when dealing with particular classes of systems, and the incorporation of additional features to handle control scenarios other than just stabilization. On the application side the method has provided solutions to a wide variety of physical problems. The purpose of this paper is to review the fundamental theory, main new results and practical applications of this control system design approach as well as to discuss the current open problems and future directions.

Keywords: Hamiltonian Systems; Interconnection; Non-linear Systems; Passivity; Passivity-Based Control; Stabilization

1. Introduction

In the last few years we have witnessed in the control literature, both theoretical and applied, an ever increasing predominance of control techniques that respect, and effectively exploit, the structure of the system over the more classical techniques that try to impose some predetermined dynamic behavior – usually through non-linearity cancellation and high gain (sometimes euphemistically called "nonlinearity domination"). The enormous interest that this research line has raised leads us to believe that we are finally seeing the twilight of the "high-gain era" in nonlinear control, to enter a period where we will "respect and learn to live" with the nonlinearities.

The property of passivity plays a central role in most of these developments. Passivity-based control (PBC) is a generic name, introduced in [42], to define a controller design methodology which achieves stabilization by rendering the system passive with respect to a desired storage function and injecting damping. Although there are many variations of this basic idea, PBCs may be broadly classified into two large groups, "classical" PBC where we *a priori select* the storage function to be assigned (typically quadratic in the increments) and then design the controller that renders the storage function non-increasing. This approach, clearly reminiscent of standard Lyapunov methods, has been very successful to control physical systems described by Euler–Lagrange equations of motion, which as thoroughly detailed in [41], includes mechanical, electrical and electromechanical applications. Unfortunately, it has been shown in [41] (see also [46]) that to assign the given storage function – which does

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not necessarily have the interpretation of total energy – these designs carry out an inversion of the system along the reference trajectories, which destroys the Lagrangian structure and imposes an unnatural stable invertibility requirement to the system.

In the second class of PBCs we do not fix the closed-loop storage function, but instead select the desired structure of the closed-loop system, for example, Lagrangian or port-controlled Hamiltonian (PCH), and then characterize all assignable energy functions compatible with this structure. This characterization is given in terms of the solution of a partial differential equation (PDE). The most notable examples of this approach are the controlled Lagrangian and the interconnection and damping assignment (IDA) methods of [8,14] and [45,46], respectively. In IDA-PBC the particular PDE that we have to solve is parameterized by three (designer chosen) matrices that are related with the interconnection between the subsystems, the damping and the kernel of the systems input matrix, respectively. Several interpretations can be given to the role played by these matrices. At the most basic – computational – level they can be simply viewed as degrees-of-freedom to simplify the solution of the PDE. From a systems theoretic viewpoint they may be seen, either as multipliers (from classical Input–Output Theory [16]) that help us enforce the required passivity property, or as dynamic couplings that permit the propagation of dissipation. In the case of physical systems the interconnection and the damping matrices determine the energy exchange and the dissipation of the system, respectively, and consequently they can often be judiciously chosen invoking this kind of physical considerations. All of these interpretations are illustrated in the paper with physical examples.

Since the introduction of IDA-PBC many theoretical extensions and practical applications of this controller design technique have been reported in the literature. Among the practical applications one has mass-balance systems [36], electrical motors [9,47], magnetic levitation systems [19,44], power systems [22,34], power converters [52,53], underwater vehicles [3], spacecrafts [5] and mechanical systems [2,20,43]. The purpose of this paper is to collect and present some of the new theoretical results in a unified way and to discuss the current research and future directions.

2. The IDA-PBC Methodology

IDA-PBC was introduced in [45,46] as a procedure to control physical systems described by PCH models of the form[1]

$$\begin{aligned} \dot{\boldsymbol{x}} &= [\mathcal{J}(\boldsymbol{x}) - \mathcal{R}(\boldsymbol{x})]\nabla H + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}, \\ \boldsymbol{y} &= \boldsymbol{g}^{\top}(\boldsymbol{x})\nabla H, \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\boldsymbol{u} \in \mathbb{R}^m$, $m < n$ is the control action, $H : \mathbb{R}^n \to \mathbb{R}$ is the total stored energy, $\mathcal{J}(\boldsymbol{x}) = -\mathcal{J}^{\top}(\boldsymbol{x}), \mathcal{R}(\boldsymbol{x}) = \mathcal{R}^{\top}(\boldsymbol{x}) \geq 0$ are the natural interconnection and damping matrices, respectively, and $\boldsymbol{u}, \boldsymbol{y} \in \mathbb{R}^m$ are conjugated variables whose product has units of power. The choice of PCH models was motivated by the fact that they are natural candidates to describe many physical systems – as thoroughly discussed in [57] and references therein.

2.1. General Nonlinear Systems

Unfortunately, in some engineering applications physical PCH models are too complex for control design and a reduction stage is usually needed. These reductions are usually *ad-hoc* and destroy the PCH structure. On the other hand, they yield well-established models that are widely accepted in practice – hence the interest of extending IDA-PBC to a more general class of systems as described in the following proposition.

Proposition 1. Consider the system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}. \tag{2}$$

Assume there are matrices $\boldsymbol{g}^{\perp}(\boldsymbol{x}), \mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) = -\mathcal{J}_{\mathrm{d}}^{\top}(\boldsymbol{x})$, $\mathcal{R}_{\mathrm{d}}(\boldsymbol{x}) = \mathcal{R}_{\mathrm{d}}^{\top}(\boldsymbol{x}) \geq 0$ and a function $H_{\mathrm{d}} : \mathbb{R}^n \to \mathbb{R}$ that verifies the PDE

$$\boldsymbol{g}^{\perp}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{g}^{\perp}(\boldsymbol{x})[\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x})]\nabla H_{\mathrm{d}}, \tag{3}$$

where $\boldsymbol{g}^{\perp}(\boldsymbol{x})$ is a full-rank left annihilator of $\boldsymbol{g}(\boldsymbol{x})$, that is, $\boldsymbol{g}^{\perp}(\boldsymbol{x})\boldsymbol{g}(\boldsymbol{x}) = 0$, and $H_{\mathrm{d}}(\boldsymbol{x})$ is such that

$$\boldsymbol{x}_{\star} = \arg\min H_{\mathrm{d}}(\boldsymbol{x}), \tag{4}$$

with $\boldsymbol{x}_{\star} \in \mathbb{R}^n$ the equilibrium to be stabilized. Then, the closed-loop system (2) with $\boldsymbol{u} = \boldsymbol{\beta}(\boldsymbol{x})$, where

$$\begin{aligned} \boldsymbol{\beta}(\boldsymbol{x}) = {} & [\boldsymbol{g}^{\top}(\boldsymbol{x})\boldsymbol{g}(\boldsymbol{x})]^{-1}\boldsymbol{g}^{\top}(\boldsymbol{x}) \\ & \times \{[\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x})]\nabla H_{\mathrm{d}} - \boldsymbol{f}(\boldsymbol{x})\}, \end{aligned} \tag{5}$$

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[1] All vectors defined in the paper are *column* vectors, even the gradient of a scalar function that we denote with the operator $\nabla_x = \partial/\partial \boldsymbol{x}$. When clear from the context the subindex will be omitted. Also, we use $(\cdot)'$ to denote differentiation for functions of scalar arguments.

takes the PCH form

$$\dot{\boldsymbol{x}} = [\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x})]\nabla H_{\mathrm{d}}, \tag{6}$$

with $\boldsymbol{x}_\star$ a (locally) stable equilibrium. It will be asymptotically stable if, in addition, $\boldsymbol{x}_\star$ is an isolated minimum of $H_{\mathrm{d}}(\boldsymbol{x})$ and the largest invariant set under the closed-loop dynamics (6) contained in

$$\left\{\boldsymbol{x} \in \mathbb{R}^n \mid [\nabla H_{\mathrm{d}}]^\top \mathcal{R}_{\mathrm{d}}(\boldsymbol{x}) \nabla H_{\mathrm{d}} = 0\right\}, \tag{7}$$

equals $\{\boldsymbol{x}_\star\}$. An estimate of its domain of attraction is given by the largest bounded level set $\{\boldsymbol{x} \in \mathbb{R}^n \mid H_{\mathrm{d}}(\boldsymbol{x}) \le c\}$.

Proof. Setting up the right-hand side of (2), with $\boldsymbol{u} = \beta(\boldsymbol{x})$, equal to the right-hand side of (6) we get the matching equation

$$\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\beta(\boldsymbol{x}) = [\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x})]\nabla H_{\mathrm{d}}. \tag{8}$$

Multiplying on the left by $\boldsymbol{g}^\perp(\boldsymbol{x})$ we obtain the PDE (3). The expression of the control is obtained by multiplying on the left the pseudo-inverse of $\boldsymbol{g}(\boldsymbol{x})$. Stability of $\boldsymbol{x}_\star$ is established noting that, along the trajectories of (6), we have

$$\dot{H}_{\mathrm{d}} = -[\nabla H_{\mathrm{d}}]^\top \mathcal{R}_{\mathrm{d}}(\boldsymbol{x}) \nabla H_{\mathrm{d}} \le 0.$$

Hence, $H_{\mathrm{d}}(\boldsymbol{x})$ qualifies as a Lyapunov function. Asymptotic stability follows immediately invoking La Salle's invariance principle and the condition (7). Finally, to ensure that the solutions remain bounded, we give the estimate of the domain of attraction as the largest bounded level set of $H_{\mathrm{d}}(\boldsymbol{x})$. □

2.2. Solving the Matching Equation

It is clear from the proposition that the key step in the design procedure is the solution of (3). We underscore the fact that in this equation:

(i) $\mathcal{J}_{\mathrm{d}}(\boldsymbol{x})$ and $\mathcal{R}_{\mathrm{d}}(\boldsymbol{x})$ are free – up to the constraint of skew-symmetry and positive semidefiniteness, respectively;
(ii) $H_{\mathrm{d}}(\boldsymbol{x})$may be totally, or partially, fixed provided we can ensure (4) – and probably a properness condition;
(iii) there is an additional degree-of-freedom in $\boldsymbol{g}^\perp(\boldsymbol{x})$ which is not uniquely defined by $\boldsymbol{g}(\boldsymbol{x})$.[2]

Therefore, to solve this equation there are, at least, three ways to proceed:

Non-Parameterized IDA In one extreme case, which was the original one adopted in [46], we *fix* the desired interconnection ($\mathcal{J}_{\mathrm{d}}(\boldsymbol{x})$) and dissipation ($\mathcal{R}_{\mathrm{d}}(\boldsymbol{x})$) matrices – hence the name IDA – as well as $\boldsymbol{g}^\perp(\boldsymbol{x})$. This yields a PDE whose solutions define the admissible energy functions ($H_{\mathrm{d}}(\boldsymbol{x})$) for the given interconnection and damping matrices. Among the family of solutions we select one that satisfies (4).

Algebraic IDA At the other extreme, originally proposed in [21], one *fixes* the desired energy function, then (3) becomes an algebraic equation in $\mathcal{J}_{\mathrm{d}}(\boldsymbol{x})$, $\mathcal{R}_{\mathrm{d}}(\boldsymbol{x})$ and $\boldsymbol{g}^\perp(\boldsymbol{x})$.

Parameterized IDA For some physical systems it is desirable to restrict the desired energy function to a certain class, for instance, for mechanical systems the sum of a potential energy term, that depends only on the generalized positions, and the kinetic energy that is quadratic in the generalized momenta [43]. Fixing the *structure* of the energy function yields a new PDE for its unknown terms and, at the same time, imposes some constraints on the interconnection and damping matrices.

Example. To illustrate the three approaches consider the problem of position regulation of the microelectromechanical system depicted in Fig. 1. The dynamical equations of motion are given by [30], see also [57]

$$\begin{aligned}
\dot{q} &= \frac{p}{m}, \\
\dot{p} &= -k(q - q_\star) - \frac{Q^2}{2A\epsilon} - \frac{b}{m}p, \\
\dot{Q} &= -\frac{qQ}{RA\epsilon} + \frac{1}{R}u,
\end{aligned} \tag{9}$$

where the state of the system is the air gap q (with $q_\star$ the equilibrium to be stabilized), the momentum p and

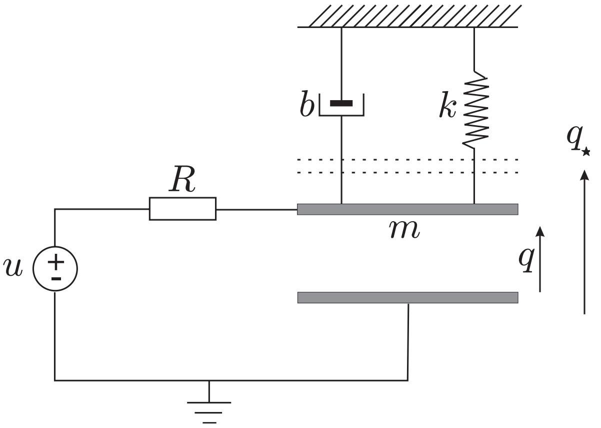

Fig. 1. Model of an electrostatic microactuator.

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[2] As reported in [1], this degree of freedom can be used to linearize a nonlinear PDE that appears in mechanical systems.

the charge of the device Q. The plate area, the mass of the plate and the permittivity in the gap are represented by A, m and ϵ, respectively. The spring and friction coefficients are given respectively by the positive constants k and b. The input resistance is R and u represents the input voltage which is the control action. As pointed out in [30], p is usually not available for measurement. Equations (9) can be represented as a PCH system of the form (1) where

$$\mathcal{J}=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad \mathcal{R}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \dfrac{1}{R} \end{bmatrix},\quad g=\begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{R} \end{bmatrix},$$

with the energy function

$$H(q,p,Q)=\frac{1}{2}k(q-q_\star)^2+\frac{1}{2m}p^2+\frac{q}{2A\epsilon}Q^2.$$

Let us start with the Algebraic IDA and assume we want to assign a quadratic energy function, that is,

$$H_{\mathrm{d}}(q,p,Q)=\frac{\gamma_1}{2}(q-q_\star)^2+\frac{1}{2m}p^2+\frac{\gamma_2}{2}Q^2,$$

where γ_1,γ_2 are positive constants. Denote the desired interconnection and damping matrices, which are to be determined, as

$$\mathcal{J}_{\mathrm{d}}=\begin{bmatrix} 0 & J_{12} & J_{13} \\ -J_{12} & 0 & J_{23} \\ -J_{13} & -J_{23} & 0 \end{bmatrix},\quad \mathcal{R}_{\mathrm{d}}=\begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}. \tag{10}$$

After some simple calculations we get that Eq. (3) becomes an algebraic equation, with one possible solution, the following matrices:

$$\mathcal{J}_{\mathrm{d}}(Q)=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -\dfrac{Q}{2A\epsilon\gamma_2} \\ 0 & \dfrac{Q}{2A\epsilon\gamma_2} & 0 \end{bmatrix},\quad \mathcal{R}_{\mathrm{d}}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & r_3 \end{bmatrix}$$

with $\gamma_1=k$ and $\gamma_2,r_3>0$ and free parameters. The control law is obtained as

$$\beta(q,p,Q)=\left(\frac{1}{2A\epsilon\gamma_2}\frac{p}{m}-r_3\gamma_2\right)RQ+\frac{1}{A\epsilon}qQ.$$

Note that the control depends on the unmeasurable state p. To apply the non-parameterized IDA, we use the same structure of $(\mathcal{J}_{\mathrm{d}}-\mathcal{R}_{\mathrm{d}})$ in (10), this yields the PDEs

$$\begin{aligned} &r_1\nabla_q H_{\mathrm{d}}+J_{12}\nabla_p H_{\mathrm{d}}+J_{13}\nabla_Q H_{\mathrm{d}}=\frac{p}{m},\\ &-J_{12}\nabla_q H_{\mathrm{d}}-r_2\nabla_p H_{\mathrm{d}}+J_{23}\nabla_Q H_{\mathrm{d}}\\ &\quad=-k(q-q_\star)-\frac{Q^2}{2A\epsilon}-\frac{b}{m}p. \end{aligned}$$

As there are no clear indications on how to choose the elements of $\mathcal{J}_{\mathrm{d}}-\mathcal{R}_{\mathrm{d}}$, we fix them as constants. Based on physical considerations we can select the matrix $\mathcal{J}_{\mathrm{d}}-\mathcal{R}_{\mathrm{d}}$ as

$$\mathcal{J}_{\mathrm{d}}-\mathcal{R}_{\mathrm{d}}=\begin{bmatrix} 0 & J_{12} & 0 \\ -J_{12} & -r_2 & J_{23} \\ 0 & -J_{23} & -r_3 \end{bmatrix}.$$

The solution of the PDEs yields a cumbersome expression and an even more complicated controller that still depends on p. That is, setting $r_2=J_{12}b$ we get for the desired energy function

$$\begin{aligned} &H_{\mathrm{d}}(q,p,Q)\\ &\quad=\frac{1}{2J_{12}}\left(\frac{1}{m}p^2+\frac{k}{2A\epsilon}(q-q_\star)^2+\frac{J_{23}^2}{3J_{12}^2A\epsilon}q^3\right.\\ &\qquad\left.+\frac{J_{23}}{J_{12}A\epsilon}q^2Q+\frac{1}{A\epsilon}qQ^2\right)+\psi\left(\frac{J_{23}}{J_{12}}q+Q\right), \end{aligned}$$

where $\psi(\cdot)$ is a free function to be selected to guarantee the equilibrium assignment condition (4). The expression for the controller is given by

$$\begin{aligned} &\beta(q,p,Q)\\ &\quad=-\frac{J_{23}R}{J_{12}m}p-r_3R\left(\frac{J_{23}}{2J_{12}^2A\epsilon}q^2+\frac{1}{4J_{12}A\epsilon}q+\psi'_Q\right)\\ &\qquad+\frac{1}{A\epsilon}qQ. \end{aligned}$$

Finally, we observe that the mechanical part of the system suggests to consider an energy function consisting of the sum of the open-loop kinetic energy and a function to be defined, that is,

$$H_{\mathrm{d}}(q,p,Q)=\frac{1}{2m}p^2+\varphi(q,Q).$$

This parameterization fixes $(\mathcal{J}_{\mathrm{d}}-\mathcal{R}_{\mathrm{d}})$ as

$$\mathcal{J}_{\mathrm{d}}-\mathcal{R}_{\mathrm{d}}=\begin{bmatrix} 0 & 1 & 0 \\ -1 & -b & 0 \\ 0 & 0 & -r_3 \end{bmatrix}.$$

The solution of the PDEs (3) yields

$$\varphi(q,Q)=\frac{1}{2}k(q-q_\star)^2+\frac{1}{2A\epsilon}qQ^2+\psi(Q),$$

with $\psi(Q)$ again free for the equilibrium assignment. After some simple calculations we obtain the nice output-feedback control

$$\beta(q,Q)=-(r_3R-1)\frac{1}{A\epsilon}qQ-r_3R\psi',$$

which does not depend on the unmeasurable state p. This controller contains, as a particular case with

$r_3 = 1/R$ and $\psi(Q)$ quadratic, the linear charge feedback controller studied in [30], but it also allows to use nonlinear functions to guarantee, for instance, saturation levels. We should also mention that the control scheme of [30] has been obtained using a damping control plus a control by interconnection approach.

In the following sections we will come back to a discussion of the three methods, as well as variations of them, and discuss their relative merits and shortcomings. We note that with the Algebraic IDA-PBC viewpoint, we can consider the use of time-varying (and possibly non-smooth) energy functions. Exploiting these new feature of IDA-PBC it has been possible to solve many interesting open problems, including tracking [19] and stabilization of nonholonomic systems [20]. (See also [50] for an application of this idea to electromechanical systems and [37] for output-feedback control of induction motors.)

2.3. Three Variations

A first, simple, variation of the IDA-PBC method given above, which was observed in [21], is obtained adding the possibility of *coordinate changes*. Indeed, if we write the system dynamics (2) in the new coordinate z, with $\boldsymbol{x} = \phi(\boldsymbol{z})$, where ϕ is a diffeomorphism, and apply the construction of Proposition 1 we obtain a modified PDE

$$\tilde{\boldsymbol{g}}^{\perp}(z)(\nabla\phi)^{-1}f(\phi(z)) = \tilde{\boldsymbol{g}}^{\perp}(z)\left[\tilde{\mathcal{J}}_{\mathrm{d}}(z) - \tilde{\mathcal{R}}_{\mathrm{d}}(z)\right]\nabla H_{\mathrm{d}},$$

where

$$\begin{aligned}\tilde{\mathcal{J}}_{\mathrm{d}}(z) &= (\nabla\phi)^{-1}\mathcal{J}_{\mathrm{d}}(\phi(z))(\nabla\phi)^{-\top} = -\tilde{\mathcal{J}}_{\mathrm{d}}^{\top}(z),\\ \tilde{\mathcal{R}}_{\mathrm{d}}(z) &= (\nabla\phi)^{-1}\mathcal{R}_{\mathrm{d}}(\phi(z))(\nabla\phi)^{-\top} = \tilde{\mathcal{R}}_{\mathrm{d}}^{\top}(z) \geq 0,\\ \tilde{\boldsymbol{g}}(z) &= (\nabla\phi)^{-1}\boldsymbol{g}(\phi(z)),\end{aligned}$$

and we view the choice of ϕ as a new degree-of-freedom for our design.

The formulation of IDA-PBC presented above aims at *exact* model matching. In some applications this objective may be too stringent – for instance, we might not be able to solve the PDE (3) – hence, we allow in the target dynamics a *perturbation* that does not destroy the desired stability properties [10]. This relaxed objective can be formulated with a target dynamics

$$\dot{\boldsymbol{x}} = [\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x})]\nabla H_{\mathrm{d}} + \boldsymbol{\xi}(\boldsymbol{x}),$$

where $\boldsymbol{\xi}(\boldsymbol{x})$ is some state-dependent disturbance that should satisfy

$$\dot{H}_{\mathrm{d}} = -[\nabla H_{\mathrm{d}}]^{\top}\mathcal{R}_{\mathrm{d}}(\boldsymbol{x})\nabla H_{\mathrm{d}} + [\nabla H_{\mathrm{d}}]^{\top}\boldsymbol{\xi}(\boldsymbol{x}) \leq 0,$$

to preserve stability. The disturbance $\boldsymbol{\xi}(\boldsymbol{x})$ may be seen as an additional degree-of-freedom for the design as it transforms the PDE (3) into

$$\boldsymbol{g}^{\perp}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{g}^{\perp}(\boldsymbol{x})\{[\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x})]\nabla H_{\mathrm{d}} + \boldsymbol{\xi}(\boldsymbol{x})\}.$$

For systems which are not *affine in the control*, say $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \beta(\boldsymbol{x}))$, the procedure described in Proposition 1 is not applicable. In this case the matching objective of IDA-PBC can be achieved by proceeding in the following alternative way. From (8), and assuming the matrix $\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x})$ is non-singular,[3] we obtain $\boldsymbol{K}(\boldsymbol{x}) = \nabla H_{\mathrm{d}}$, where we have defined the vector function

$$\boldsymbol{K}(\boldsymbol{x}) := [\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x})]^{-1}[\boldsymbol{f}(\boldsymbol{x}, \beta(\boldsymbol{x}))].$$

We recall now the following lemma.

Lemma 1. (*Poincare's Lemma*). *Given* $\boldsymbol{K} : \mathbb{R}^n \to \mathbb{R}^n$, $K \in \mathcal{C}^1$. There exists $H_{\mathrm{d}} : \mathbb{R}^n \to \mathbb{R}$ such that $\nabla H_{\mathrm{d}} = \boldsymbol{K}$ if and only if

$$\nabla\boldsymbol{K} = (\nabla\boldsymbol{K})^{\top},$$

where

$$\nabla\boldsymbol{K} = \begin{bmatrix} \dfrac{\partial k_1}{\partial x_1} & \cdots & \dfrac{\partial k_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial k_n}{\partial x_1} & \cdots & \dfrac{\partial k_n}{\partial x_n} \end{bmatrix}.$$

Fixing $\mathcal{J}_{\mathrm{d}}(\boldsymbol{x})$ and $\mathcal{R}_{\mathrm{d}}(\boldsymbol{x})$ and applying the lemma yields a set of PDEs, directly in terms of the control signal, that solve the IDA-PBC problem. The approach is also adequate when the target dynamics is of the form

$$\dot{\boldsymbol{x}} = [\mathcal{J}_{\mathrm{d}}(\boldsymbol{x}, \beta(\boldsymbol{x})) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x}, \beta(\boldsymbol{x}))]\nabla H_{\mathrm{d}},$$

where we underscore the dependence of the interconnection and damping matrices on the control. This choice is desirable for a class of physical systems, for instance power converters [18], that operate commuting between different dynamical systems and the control action is precisely the switching policy. In this case, to preserve the physical nature of the system, we postulate a target dynamics of the form given above.

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[3] This assumption is done with little loss of generality, because otherwise there would exist equilibria of the closed-loop system which are not extrema of the desired energy function.

Example. In [53] we used this last variation of the method to design an output-feedback IDA-PBC for the classical DC-to-DC Boost Converter, whose model is given by

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & -u \\ u & -\dfrac{1}{R} \end{bmatrix} \nabla H + \begin{bmatrix} E \\ 0 \end{bmatrix},$$

where $H(\boldsymbol{x}) = (1/2L)x_1^2 + (1/2C)x_2^2$. The control objective is to stabilize an equilibrium $\boldsymbol{x}_\star = \left(\frac{L}{RE}V_\star^2, CV_\star\right)$ measuring only x_2 and without knowledge of R.[4] Note that, as discussed above, the control action enters into the interconnection matrix. We propose to preserve this interconnection, but modify the damping as $\mathcal{R}_\mathrm{d} = \mathrm{diag}\{R_\mathrm{a}, 0\}$ where we underscore that we have *removed* the admittance $\left(\frac{1}{R}\right)$ from the second line and added a damping ($R_\mathrm{a} > 0$) to the first one. Using the definitions above we obtain

$$\boldsymbol{K}(\boldsymbol{x}) = \nabla H_\mathrm{a} = \frac{1}{\beta(x_2)} \begin{bmatrix} -\dfrac{1}{RC}x_2 \\ -\dfrac{1}{L}R_\mathrm{a}x_1 - E + \dfrac{R_\mathrm{a}}{RC}\dfrac{x_2}{\beta(x_2)} \end{bmatrix},$$

where we have made explicit our desire to make the control dependent only on x_2. The PDE to be solved, $\frac{\partial K_2}{\partial x_1}(\boldsymbol{x}) = \frac{\partial K_1}{\partial x_2}(\boldsymbol{x})$, is actually an ODE

$$\beta' = \frac{\alpha}{x_2}\beta(x_2),$$

where $\alpha \overset{\Delta}{=} 1 - (R_\mathrm{a}RC/L)$. Solving this ODE and doing some simple calculations to fix the minimum at the desired equilibrium yields the desired control action

$$\beta(x_2) = \frac{E}{V_\star}\left(\frac{x_2}{x_{2\star}}\right)^\alpha,$$

where $0 < \alpha < 1$ is a tuning parameter. The energy function is

$$H_\mathrm{d}(\boldsymbol{x}) = \frac{1}{2L}x_1^2 + \frac{1}{2C}x_2^2 + \kappa_1 x_2^{2(1-\alpha)} - (\kappa_2 + \kappa_3 x_1)x_2^{1-\alpha},$$

with $\kappa_i > 0$ some constants.

2.4. Linear and Hybrid Systems

In [48] IDA-PBC applied to linear time-invariant systems is studied. The authors first prove that all stable (resp., asymptotically stable) linear systems can be written in PCH form with constant matrices $\mathcal{J}$ and $\boldsymbol{g}$, a quadratic positive definite energy function and a positive semidefinite (resp., definite) constant damping matrix $\mathcal{R}$. The problem of stabilization of general linear PCH systems is then studied. It is shown that existence of a linear static state feedback that preserves the PCH form and assigns a positive definite $H_\mathrm{d}(\boldsymbol{x})$ is equivalent to a set of linear matrix inequalities – whose feasibility can be easily tested with standard convex programming tools. Two different equivalences are given, one where $\mathcal{J}_\mathrm{d}$ and $\mathcal{R}_\mathrm{d}$ do not appear explicitly, styming the possibility of fixing some of its elements. Under the weak assumption of no uncontrollable poles at the origin, equivalence with other linear matrix inequalities that now contain these matrices is established.

The IDA-PBC method has been adapted in [26] to handle PCH systems with impulses. Constructive sufficient conditions to design hybrid feedback stabilizers for these class of systems, preserving a hybrid Hamiltonian structure, are given. Furthermore, an inverse optimal hybrid feedback control framework is developed that characterizes a class of IDA-PBCs that guarantee hybrid sector and gain margins to multiplicative input uncertainty of hybrid Hamiltonian systems.

3. Some Properties of IDA-PBC

In this section, we review some interesting system-theoretic properties of IDA-PBC that have been reported in the literature.

3.1. Energy-Balancing

The stabilization mechanism of IDA-PBC is particularly clear when applied to PCH systems (1) with some "suitable" damping properties. Indeed, it has been shown in [46] that if the natural damping of the PCH system satisfies[5]

$$\mathcal{R}(\boldsymbol{x})(\nabla H_\mathrm{d} - \nabla H) = 0, \tag{11}$$

and no additional damping is injected, that is, $\mathcal{R}_\mathrm{d}(\boldsymbol{x}) = \mathcal{R}(\boldsymbol{x})$ then (along the trajectories of the closed-loop system) the desired energy function may be expressed as

$$H_\mathrm{d}(\boldsymbol{x}(t)) = H(\boldsymbol{x}(t)) - \int_0^t \boldsymbol{u}^\top(s)\boldsymbol{y}(s)\,\mathrm{d}s. \tag{12}$$

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[4] The zero dynamics with respect to x_2 is unstable [41], hence this robust output-feedback problem is far from trivial and remained open for many years.

[5] Since $\mathcal{R}(x)$ is usually diagonal, this condition requires that no damping is present in the coordinates that need to be shaped, that is, the coordinates where the function $H(\boldsymbol{x})$ has to be modified.

As this expression reveals, in this case, the IDA-PBC assigns as energy function the difference between the energy stored in the system and the energy supplied to it from the environment, hence we say that the controller is *energy-balancing*. In Section 4, where we discuss mechanical systems, it is shown that IDA-PBC is energy-balancing if we modify only the potential energy of the system.

Even when (11) is not satisfied the control action of IDA-PBC admits an energy-balancing-like interpretation. In [28] it is shown that if the interconnection and the damping are not modified, that is, $\mathcal{J}_{\rm d}(\boldsymbol{x}) = \mathcal{J}(\boldsymbol{x})$, $\mathcal{R}_{\rm d}(\boldsymbol{x}) = \mathcal{R}(\boldsymbol{x})$ and the matrix $\mathcal{J}(\boldsymbol{x}) - \mathcal{R}(\boldsymbol{x})$ is full rank, then $H_{\rm d}(\boldsymbol{x}(t))$ still satisfies (12) with $\boldsymbol{y}$ replaced by the "new output"

$$\begin{aligned}\tilde{\boldsymbol{y}} = &-\boldsymbol{g}^\top(\boldsymbol{x})[\mathcal{J}(\boldsymbol{x}) - \mathcal{R}(\boldsymbol{x})]^{-\top}\\ &\times \{[\mathcal{J}(\boldsymbol{x}) - \mathcal{R}(\boldsymbol{x})]\nabla H + g(x)u\}.\end{aligned}$$

As a partial converse of this result we also have that for single input systems that verify condition (11) the new output $\tilde{y}$ exactly coincides with the natural one $\boldsymbol{y}$. Although $\boldsymbol{u}^\top\tilde{y}$ does not have (in general) units of power anymore, it is also shown in [28] that for electromechanical systems the new port variables are obtained from a classical Thevenin–Norton transformation of the voltage–current port variables $(\boldsymbol{u}, \boldsymbol{y})$.

3.2. Universal Stabilization Property

It may be argued that, aiming at a PCH closed-loop system, we restrict the class of systems that can be stabilized via IDA-PBC. The lemma below, established in [46], proves that this presumption is incorrect since the IDA-PBC methodology is "universally stabilizing", in the sense that it generates *all* asymptotically stabilizing controllers for systems of the form (2).

Lemma 2. [46] If the system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{f}(\boldsymbol{x}) \in \mathcal{C}^1$ has an asymptotically stable equilibrium point $\boldsymbol{x}_\star$, then there exist a $\mathcal{C}^1$ positive definite (w.r.t. $\boldsymbol{x}_\star$) function $H_{\rm d}(\boldsymbol{x})$ and $\mathcal{C}^0$ matrix functions $\mathcal{J}_{\rm d}(\boldsymbol{x}) = -\mathcal{J}_{\rm d}^\top(\boldsymbol{x})$, $\mathcal{R}_{\rm d}(\boldsymbol{x}) = \mathcal{R}_{\rm d}^\top(\boldsymbol{x}) \geq 0$ such that

$$\boldsymbol{f}(\boldsymbol{x}) = [\mathcal{J}_{\rm d}(\boldsymbol{x}) - \mathcal{R}_{\rm d}(\boldsymbol{x})]\nabla H_{\rm d}. \tag{13}$$

We should point out that the proof of the lemma is constructive, provided we know the Lyapunov function $H_{\rm d}(\boldsymbol{x})$.

3.3. Selection of $\mathcal{J}_{\rm d}(\boldsymbol{x}), \mathcal{R}_{\rm d}(\boldsymbol{x})$ and $\boldsymbol{g}^\perp(\boldsymbol{x})$

It is clear from Proposition 1 that a suitable selection of $\mathcal{J}_{\rm d}(\boldsymbol{x}), \mathcal{R}_{\rm d}(\boldsymbol{x})$ and $\boldsymbol{g}^\perp(\boldsymbol{x})$ is essential for the success of the parameterized and the non-parameterized versions of IDA-PBC. Adopting a purely computational viewpoint they can be assimilated to "additional control actions" that are used to simplify the solution of the PDE (3). For instance, they can be selected to verify the integrability conditions of Frobenius' Theorem for

$$\boldsymbol{g}^\perp(\boldsymbol{x})[\mathcal{J}_{\rm d}(\boldsymbol{x}) - \mathcal{R}_{\rm d}(\boldsymbol{x})]\nabla H_{\rm d} = 0,$$

whose solution is (generically) necessary for the solvability of (3). This purely analytical viewpoint is, however, of little practical use and it is often preferable to try to incorporate the available prior knowledge about the application at hand, particularly when dealing with physical systems. In this section we discuss some considerations/interpretations pertaining to these matrices that are useful for this task.

Sometimes it is natural to split the control action into energy shaping and damping injection terms as $\boldsymbol{u} = \boldsymbol{u}_{\rm es} + \boldsymbol{u}_{\rm di}$ – see Section 4. This fixes the matrix $\mathcal{R}_{\rm d}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{x})K_v\boldsymbol{g}^\top(\boldsymbol{x})$ with $K_v > 0$. Then, the PDE becomes

$$\boldsymbol{g}^\perp(\boldsymbol{x})\mathcal{J}_{\rm d}(\boldsymbol{x})\nabla H_{\rm d} = \boldsymbol{g}^\perp(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}),$$

and

$$\begin{aligned}\boldsymbol{u}_{\rm es}(\boldsymbol{x}) &= [\boldsymbol{g}^\top(\boldsymbol{x})\boldsymbol{g}(\boldsymbol{x})]^{-1}\boldsymbol{g}^\top(\boldsymbol{x})\{\mathcal{J}_{\rm d}(\boldsymbol{x})\nabla H_d - \boldsymbol{f}(\boldsymbol{x})\},\\ \boldsymbol{u}_{\rm di}(\boldsymbol{x}) &= -K_v\boldsymbol{g}^\top(\boldsymbol{x})\nabla H_{\rm d}.\end{aligned}$$

This partition proceeds from the premise that damping is introduced to enforce asymptotic stability to an otherwise stable system. This viewpoint may be restrictive and damping may also be used to achieve other control objectives. In Section 2.3, we presented an example where the resistive terms on a DC-to-DC power converter are swapped to obtain an output-feedback solution to the stabilization problem. Damping swapping between the electrical and the mechanical coordinates of a magnetic levitation system is also used in [44] to eliminate from the control a high order term that saturated the input. Also, in [37] we showed that output feedback stabilization of an induction motor is not possible with a two-step design.

In some applications it is possible to choose $\mathcal{J}_{\rm d}(\boldsymbol{x})$ from *physical considerations*. In [22,44] it is introduced to enforce a *coupling* between electrical and mechanical coordinates in a magnetic levitation system and a synchronous generator, respectively. More precisely, the open-loop interconnection

$$\mathcal{J} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix},$$

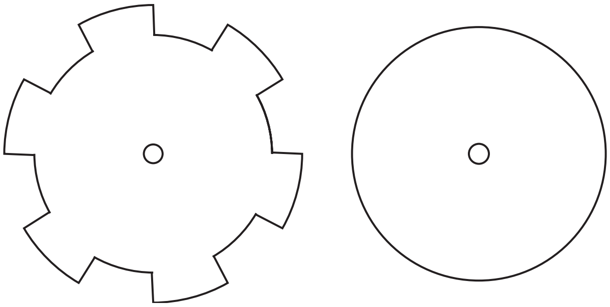

Fig. 2. Indented rotor in open-loop and its desired virtual behavior in closed-loop.

that has no (explicit) coupling between the first coordinate (corresponding to flux) and the last two (position and momenta), is replaced by a desired interconnection

$$\mathcal{J}_\mathrm{d}(\boldsymbol{x}) = \begin{bmatrix} 0 & 0 & \alpha(\boldsymbol{x}) \\ 0 & 0 & 1 \\ -\alpha(\boldsymbol{x}) & -1 & 0 \end{bmatrix},$$

with $\alpha(\boldsymbol{x})$ a free function to be designed. It is shown in those papers that the introduction of the additional coupling is actually necessary to achieve stabilization via IDA-PBC.

Another example where physics can be used to select $\mathcal{J}_\mathrm{d}(\boldsymbol{x})$ is reported in [47] where permanent magnet synchronous motors are considered. The argument there is that, for energy consumption considerations, we would like to make an indented rotor motor behave like a smooth rotor (also called isotropic) motor – see Fig. 2.

The corresponding interconnection matrices are

$$\mathcal{J}_\text{indented}(\boldsymbol{x}) = \begin{bmatrix} 0 & 0 & x_2 \\ 0 & 0 & -(x_1+\Phi) \\ -x_2 & x_1+\Phi & 0 \end{bmatrix},$$

$$\mathcal{J}_\text{smooth}(\boldsymbol{x}) = \begin{bmatrix} 0 & L_0 x_3 & 0 \\ -L_0 x_3 & 0 & -\Phi \\ 0 & \Phi & 0 \end{bmatrix},$$

where x_1, x_2 are currents, x_3 is the mechanical momentum, Φ is the back emf constant and L_0 is the stator inductance, which are equal in an isotropic motor. To achieve the desired *virtual* behavior in closed-loop we select $\mathcal{J}_\mathrm{d}(\boldsymbol{x}) = \mathcal{J}_\text{smooth}(\boldsymbol{x})$. We refer the reader to [47], and references therein, for further details.

In other cases, the interconnection is introduced to *propagate passivity*. To illustrate this point consider the system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & \alpha(\boldsymbol{x}) \\ -\alpha(\boldsymbol{x}) & -r \end{bmatrix} \nabla H_\mathrm{d} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

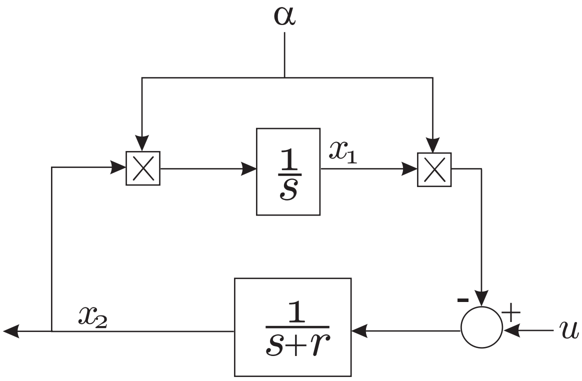

Fig. 3. Feedback interconnection of a passive and a strictly passive system.

with $H_\mathrm{d}(\boldsymbol{x}) = \frac{1}{2}(x_1^2 + x_2^2)$ and $r > 0$. As seen in Fig. 3 the introduction of the function $\alpha(\boldsymbol{x})$ permits the interconnection of an output strictly passive system, $1/(s+r)$, and a passive system, $1/s$, that still defines an output strictly passive operator $u \to x_2$.[6] This passivity propagation property, which is clearly preserved for "concatenated structures", for instance

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & \beta(\boldsymbol{x}) & 0 \\ -\beta(\boldsymbol{x}) & 0 & \alpha(\boldsymbol{x}) \\ 0 & -\alpha(\boldsymbol{x}) & -r \end{bmatrix} \nabla H_\mathrm{d} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u,$$

is effectively exploited by the backstepping procedure [29] in the stabilization of triangular systems. From $\boldsymbol{g}^\perp(\boldsymbol{x})\boldsymbol{g}(\boldsymbol{x}) = 0$ and the full-rank condition on $\boldsymbol{g}^\perp(\boldsymbol{x})$ we conclude that the rows of $\boldsymbol{g}^\perp(\boldsymbol{x})$ span the kernel of $\boldsymbol{g}^\top(\boldsymbol{x})$, but the choice of this basis is not unique. The non-uniqueness of the left-annihilator $\boldsymbol{g}(\boldsymbol{x})$ provides an additional degree-of-freedom to IDA-PBC. For illustration let us assume $m = n - 1$, in mechanical systems with n denoting the generalized coordinates, this situation is called "underactuation degree one" and is further elaborated in Section 4. In this case the left annihilators of $\boldsymbol{g}(\boldsymbol{x})$ constitute a basis of the one-dimensional linear space $\ker\{\boldsymbol{g}^\top(\boldsymbol{x})\}$, which leads to the following simple fact.

Fact 1. If $\tilde{\boldsymbol{g}}^\perp(\boldsymbol{x})\boldsymbol{g}(\boldsymbol{x}) = 0$ then $\boldsymbol{g}^\perp(\boldsymbol{x}) \triangleq \eta(\boldsymbol{x})\tilde{\boldsymbol{g}}^\perp(\boldsymbol{x})$, with $\eta(\boldsymbol{x})$ an arbitrary scalar function, is also a left annihilator of $\boldsymbol{g}(\boldsymbol{x})$.

Although the new free function $\eta(\boldsymbol{x})$ does not affect the PDE (3), we will see in Section 4 that it plays a key role in the new PDEs that appear in parameterized IDA-PBC of mechanical systems.

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[6] This follows from the well-known fact (see Chapter 6.4, Exercise 5 of [16]), that pre- and post-multiplying a passive operator by arbitrary (possibly unbounded) non-singular matrices does not destroy its passivity property.

4. Mechanical Systems

We have argued in Section 2 that it is sometimes desirable to restrict the desired energy function to a certain class. In this section we discuss how this variation of the method simplifies its application to mechanical systems.

4.1. A Parameterized Energy Function

In [43] we have applied parameterized IDA-PBC for mechanical systems described by the Hamiltonian equations

$$\begin{bmatrix} \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \nabla \boldsymbol{q} H \\ \nabla \boldsymbol{p} H \end{bmatrix} + \begin{bmatrix} 0 \\ G(\boldsymbol{q}) \end{bmatrix} u, \qquad (14)$$

with total energy $H(\boldsymbol{q},\boldsymbol{p}) = \frac{1}{2}\boldsymbol{p}^\top M^{-1}(\boldsymbol{q})\boldsymbol{p} + V(\boldsymbol{q})$, where $\boldsymbol{q} \in \mathbb{R}^n, \boldsymbol{p} \in \mathbb{R}^n$ are the generalized position and momenta, respectively, $M(\boldsymbol{q}) = M^\top(\boldsymbol{q}) > 0$ is the inertia matrix, $V(\boldsymbol{q})$ is the potential energy and $\mathrm{rank}(G) = m < n$. It has been proposed to fix the desired energy function

$$H_\mathrm{d}(\boldsymbol{q},\boldsymbol{p}) = \tfrac{1}{2}\boldsymbol{p}^\top M_\mathrm{d}^{-1}(\boldsymbol{q})\boldsymbol{p} + V_\mathrm{d}(\boldsymbol{q}),$$

where $M_\mathrm{d}(\boldsymbol{q})$ and $V_\mathrm{d}(\boldsymbol{q})$ represent the (to be defined) closed-loop inertia matrix and potential energy function, respectively, and we require that $M_\mathrm{d}(\boldsymbol{q}) = M_\mathrm{d}^\top(\boldsymbol{q}) > 0$ and $\boldsymbol{q}_\star = \arg\min V_\mathrm{d}(\boldsymbol{q})$.

Fixing the desired energy function also fixes the desired interconnection matrix as

$$\begin{aligned} \mathcal{J}_\mathrm{d}(\boldsymbol{q},\boldsymbol{p}) &= \begin{bmatrix} 0 & M^{-1}(\boldsymbol{q})M_\mathrm{d}(\boldsymbol{q}) \\ -M_\mathrm{d}(\boldsymbol{q})M^{-1}(\boldsymbol{q}) & J_2(\boldsymbol{q},\boldsymbol{p}) \end{bmatrix} \\ &= -\mathcal{J}_\mathrm{d}^\top(\boldsymbol{q},\boldsymbol{p}), \end{aligned}$$

where the skew-symmetric matrix $J_2(\boldsymbol{q}, \boldsymbol{p})$ is a *free parameter*. On the other hand, it is also proposed in [43] to split the control into $u = u_\mathrm{es}(\boldsymbol{q}, \boldsymbol{p}) + u_\mathrm{di}(\boldsymbol{q}, \boldsymbol{p})$, as suggested in Section 3.3. This restricts the desired damping matrix to be of the form

$$\mathcal{R}_\mathrm{d}(\boldsymbol{q}) = \begin{bmatrix} 0 & 0 \\ 0 & G(\boldsymbol{q})K_v G^\top(\boldsymbol{q}) \end{bmatrix} \geq 0, \quad \text{where } K_v > 0.$$

As shown in [43] the PDEs of the IDA-PBC can be naturally separated into the terms that depend on $\boldsymbol{p}$ and terms which are independent of $\boldsymbol{p}$, that is, those corresponding to the kinetic and the potential energies, respectively. This leads to

$$\begin{aligned} &G^\perp(\boldsymbol{q})\{\nabla \boldsymbol{q}(\boldsymbol{p})^\top M^{-1}(\boldsymbol{q})\boldsymbol{p}) \\ &\quad - M_\mathrm{d}(\boldsymbol{q})M^{-1}(\boldsymbol{q})\nabla \boldsymbol{q}(\boldsymbol{p}^\top M_\mathrm{d}^{-1}(\boldsymbol{q})\boldsymbol{p}) \\ &\quad + 2J_2(\boldsymbol{q},\boldsymbol{p})M_\mathrm{d}^{-1}(\boldsymbol{q})\boldsymbol{p}\} = 0, \qquad (15) \\ &G^\perp(\boldsymbol{q})\{\nabla \boldsymbol{q} V - M_\mathrm{d}(\boldsymbol{q})M^{-1}(\boldsymbol{q})\nabla \boldsymbol{q} V_\mathrm{d} = 0, \qquad (16) \end{aligned}$$

where $G^\perp(\boldsymbol{q})G(\boldsymbol{q}) = 0$. The first equation is a nonlinear PDE that using $J_2(\boldsymbol{q}, \boldsymbol{p})$ (which has to be linear in $\boldsymbol{p}$) has to be solved for the unknown elements of the closed-loop inertia matrix $M_\mathrm{d}(\boldsymbol{q})$. Given $M_\mathrm{d}(\boldsymbol{q})$, equation (16) is a simple linear PDE, hence the main difficulty is in the solution of (15).

There are two "extreme" particular cases of our procedure. First, if we do not modify the interconnection matrix then we recover the well-known potential energy shaping procedure of PBC that has its roots in [56]. Indeed, if $M_\mathrm{d}(\boldsymbol{q}) = M(\boldsymbol{q})$ and $J_2(\boldsymbol{q}, \boldsymbol{p}) = 0$, then (15) is obviated and the energy-shaping term of the controller reduces to

$$u_\mathrm{es}(\boldsymbol{q}) = (G^\top(\boldsymbol{q})G(\boldsymbol{q}))^{-1}G^\top(\boldsymbol{q})(\nabla \boldsymbol{q} V - \nabla \boldsymbol{q} V_\mathrm{d}),$$

which is the familiar potential energy shaping control. On the other extreme, if we do not change the potential energy, but only modify the kinetic energy, then it is shown in [13,43], that for a particular choice of $J_2(\boldsymbol{q}, \boldsymbol{p})$ we recover the controlled-Lagrangian method of [14]. Now, if we shape both kinetic and potential energies, but still fix $J_2(\boldsymbol{q}, \boldsymbol{q})$, then IDA-PBC coincides with the method proposed in [8,27]. Actually, the approach of [14] is more modest than IDA-PBC and [18,27] in that a physically inspired parameterization for the modified kinetic energy is assumed *a priori*.

As indicated in [58] the Lagrangian and Hamiltonian settings can be made equivalent adding to the former gyroscopic forces. This fact was exploited in [15] to prove that, considering these enlarged set of desired Lagrangians, both methods are equivalent. The benefits of adding gyroscopic forces (or equivalently, of adopting the PCH formalism of IDA-PBC) are clearly shown in [35] where, thanks to the addition of these terms, the calculations for a classical example can be worked out.

4.2. Underactuation Degree One

In [1] it has been shown that, for a class of mechanical systems with an underactuation degree one, it is possible to completely obviate the PDEs. The class consists of systems of the form

$$\begin{aligned} \dot{\boldsymbol{q}} &= M^{-1}(q_r)\boldsymbol{p}, \\ \dot{\boldsymbol{p}} &= \boldsymbol{s}(q_r) + G(q_r)\boldsymbol{u}, \end{aligned} \qquad (17)$$

where q_r, with r an integer taking values in the set $\{1, \ldots, n\}$, is a distinguished element of $\boldsymbol{q} \in \mathbb{R}^n, \boldsymbol{p} \in \mathbb{R}^n$ and $\boldsymbol{u} \in \mathbb{R}^{n-1}$. Structures of the form (17) result from the reduction, via singular perturbations or a preliminary feedback action, of certain classes of mechanical systems. A complete characterization of the class of mechanical systems feedback equivalent to (17) is also given in the paper.

Two key properties of (17) are exploited in [1]:

(i) since $M(q_r)$ and $M_{\mathrm{d}}(q_r)$ depend only on the coordinate q_r the kinetic energy PDE (15) becomes an ODE;
(ii) restricting $M_{\mathrm{d}}(q_r)$ to a particular structure, the latter can be explicitly solved with a suitable choice of $J_2(q_r)$ and $G^{\perp}(q_r)$ – the latter being possible to define because of the underactuation degree one (see Fact 1).

The main result of the paper is the following.

Proposition 2. [1] Consider the system (17). Fix

$$M_{\mathrm{d}}(q_r) = \int_{q_{r\star}}^{q_r} G(\mu)\Psi(\mu)G^{\top}(\mu)\mathrm{d}\mu + M_{\mathrm{d}}^0,$$

and assume there are $\Psi(q_r) = \Psi^{\top}(q_r) \in \mathbb{R}^{(n-1)\times(n-1)}$ and $M_{\mathrm{d}}^0 = (M_{\mathrm{d}}^0)^{\top} \in \mathbb{R}^{n\times n}$, $M_{\mathrm{d}}^0 > 0$ such that for some left annihilator $G^{\perp}(q_r)$ of $G(q_r)$[7]

$$|\tilde{G}^{\perp}(q_{r\star})M_{\mathrm{d}}(q_{r\star})M^{-1}(q_{r\star})\boldsymbol{e}_r| \geq \epsilon > 0.$$

Then, there exists a matrix $J_2(q_r)$ and a function $\eta(q_r)$ such that the kinetic energy PDE (15) with $G^{\perp}(q_r) = \eta(q_r)G^{\perp}(q_{r_\star})$ is solved. Furthermore, the solution of the potential energy PDE (16) is given by

$$V_{\mathrm{d}}(\boldsymbol{q}) = -\frac{1}{\rho}\int_0^{q_r} G^{\perp}(\mu)\boldsymbol{s}(\mu)\,\mathrm{d}\mu + \Phi(\boldsymbol{z}(\boldsymbol{q})), \quad (18)$$

where $\boldsymbol{z}(\boldsymbol{q})$ is an $n-1$ dimensional vector whose elements are of the form

$$z_i(\boldsymbol{q}) := q_i - \frac{1}{\rho}\int_0^{q_r} G^{\perp}(\mu)M_{\mathrm{d}}(\mu)M^{-1}(\mu)e_i\,\mathrm{d}\mu,$$
$$i = 1, \ldots, n,\ i \neq r$$

$\Phi(\boldsymbol{z})$ is an arbitrary differentiable function, and ρ is a constant.

Using this construction we automatically verify that $M_{\mathrm{d}}(q_r) > 0$, hence the only condition needed to ensure stability is $\boldsymbol{q}_\star = \arg\min V_{\mathrm{d}}(\boldsymbol{q})$. From (18), and taking into account that $\Phi(\boldsymbol{z})$ is free, we see that this translates into the "Hessian" assumption for the q_r coordinate

$$-\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}q_r}\{G^{\perp}\boldsymbol{s}\}(q_{r\star}) > 0.$$

It is interesting to compare these results with those of [7], particularly Theorem 3.1 of that paper. Auckly and Kapitanski do not consider structure modification (equivalently, gyroscopic forces), but they find similar simplifications in the matching conditions and they derive an interesting integrability result for systems which are underactuated by one degree.

4.3. Friction Effects

Let us now study the effect of (linear) *friction* on the system, which adds a matrix $R_2 = \mathrm{diag}\{r_i\} \geq 0$ to the (2, 2)-block of the interconnection matrix of (14) as

$$\mathcal{J} - \mathcal{R} = \begin{bmatrix} 0 & I_n \\ -I_n & R_2 \end{bmatrix}.$$

On the one hand, we see that for potential energy shaping controllers the damping condition (11) for energy-balancing stabilization becomes

$$\begin{bmatrix} 0 & 0 \\ 0 & R_2 \end{bmatrix}\begin{bmatrix} \nabla\boldsymbol{q}(V_{\mathrm{d}} - V) \\ 0 \end{bmatrix} = 0,$$

which is clearly satisfied for all $V(\boldsymbol{q})$, $V_{\mathrm{d}}(\boldsymbol{q})$, R_2. Hence, if kinetic energy is not modified, IDA-PBC is energy-balancing. Unfortunately, as we will see in the example below, in some underactuated system applications we have to modify the kinetic energy to shape the potential energy function.

On the other hand, in a recent interesting paper [49] it has been shown that when the kinetic energy is also modified, and friction is not taken into account in the design, then it may have a destabilizing effect. Indeed, if friction is neglected the closed-loop system contains a perturbation term

$$\begin{bmatrix} \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{p}} \end{bmatrix} = [\mathcal{J}_{\mathrm{d}}(\boldsymbol{q},\boldsymbol{p}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{q},\boldsymbol{p})]\nabla H_{\mathrm{d}} + \begin{bmatrix} 0 \\ R_2 M^{-1}(\boldsymbol{q})\boldsymbol{p} \end{bmatrix},$$

and the derivative of the energy function becomes

$$\dot{H}_{\mathrm{d}} = -\boldsymbol{p}^{\top}M_{\mathrm{d}}^{-1}(\boldsymbol{q})[G(\boldsymbol{q})K_vG^{\top}(\boldsymbol{q}) + R_2M^{-1}(\boldsymbol{q})M_{\mathrm{d}}(\boldsymbol{q})]M_{\mathrm{d}}^{-1}(\boldsymbol{q})\boldsymbol{p}. \quad (20)$$

The matrix in brackets may be sign indefinite disqualifying $H_{\mathrm{d}}(\boldsymbol{q}, \boldsymbol{p})$ as a Lyapunov function. Note, however, that this *does not* mean that the IDA-PBC design is unstable. Actually, some subsequent analysis has revealed that, for the examples reported in [43], it is always possible to increase the damping injection to ensure a margin of stability *vis-à-vis* this unmodeled effect.

A detailed analysis of this phenomenon has been reported in [25] where it has been established that a necessary and sufficient condition to ensure passivity of the closed-loop, is that the restriction of

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[7] $\boldsymbol{e}_i \in \mathbb{R}^n$ are the vectors of the standard Euclidian basis.

$R_2M^{-1}(\boldsymbol{q})M_d(\boldsymbol{q})$ to the kernel of $G(\boldsymbol{q})$ is positive semidefinite. That is, the effect of the damping on the non-actuated coordinates is not detrimental.

4.4. Example

Let us illustrate with a simple example some of the issues raised above. Consider the inertia wheel pendulum shown in Fig. 4, which consists of a physical pendulum with a balanced rotor at the end. The motor torque produces an angular acceleration of the end-mass which generates a coupling torque at the pendulum axis. The dynamic equations, after a simple change of coordinates and scaling, can be written as [43]

$$\dot{\boldsymbol{q}} = \boldsymbol{p},$$
$$\dot{\boldsymbol{p}} = \begin{bmatrix} m_3 \sin(q_1) \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u,$$

with the open-loop energy function $H(\boldsymbol{q}, \boldsymbol{p}) = 1/2\boldsymbol{p}^\top \boldsymbol{p} + m_3(\cos(q_1) - 1)$. The equilibrium to be stabilized is the upward position with the inertia disk aligned, which corresponds to $q_{1_\star} = q_{2_\star} = 0$.

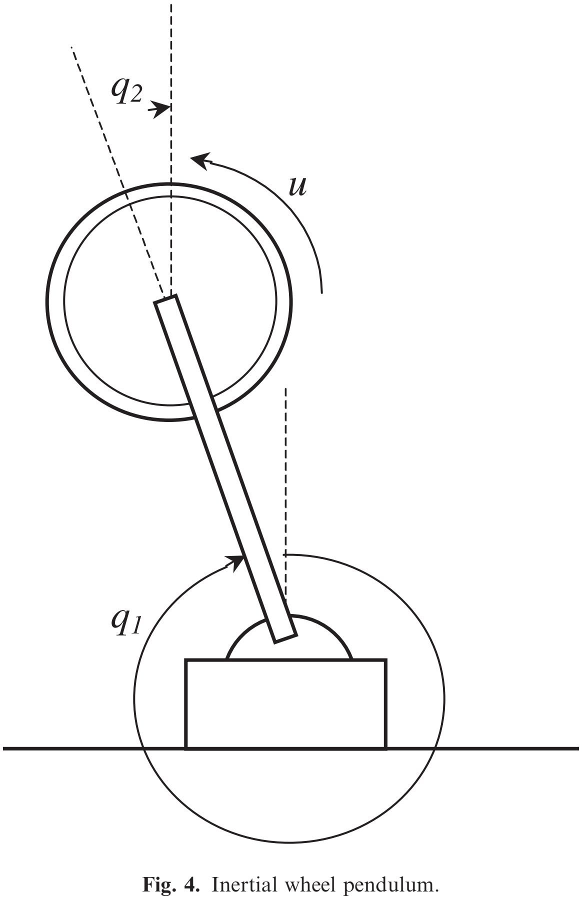

Fig. 4. Inertial wheel pendulum.

From (16) and $G^\perp = [1, 1]$, it can easily be checked that the potential energy of this system cannot be modified without changing the inertia matrix. Hence, we propose

$$M_d = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix},$$

that we take constant with $m_{11} > 0$, $m_{11}m_{22} > m_{12}^2$, to ensure $M_d > 0$. As M and M_d are constants and $J_2 = 0$ the PDE (15) is obviated. The PDE (16) takes the form

$$(m_{11} + m_{12})\nabla_1 V_d + (m_{22} + m_{12})\nabla_2 V_d = m_3 \sin(q_1).$$

Hence, we impose the constraint $m_{11} + m_{12} \neq 0$. The general solution of this PDE is

$$V_d(\boldsymbol{q}) = \frac{m_3}{m_{11} + m_{12}} \cos q_1 + \Phi(z(\boldsymbol{q})),$$

where $z(\boldsymbol{q}) = (q_2 - ((m_{12} + m_{22})/(m_{11} + m_{12}))\,(q_1))$ is the characteristic of the PDE and $\Phi(z(\boldsymbol{q}))$ is an arbitrary differentiable function that we select as $\Phi(z(\boldsymbol{q})) = (\gamma/2)z^2(\boldsymbol{q})$, $\gamma > 0$. From the expression of $V_d(\boldsymbol{q})$ above it is clear that to ensure it has a minimum at zero we need an additional constraint

$$m_{11} + m_{12} < 0. \tag{21}$$

Finally, we obtain an almost globally stabilizing controller given by

$$u = \frac{m_{12}}{m_{11} + m_{12}} m_3 \sin(q_1) - \tilde{\gamma}\left(q_2 - \frac{m_{12} + m_{22}}{m_{11} + m_{12}} q_1\right),$$

where $\tilde{\gamma}$ is free. Now, if damping of the form $R_2 = \operatorname{diag}\{0, r_2\}$ is present in the system, some simple calculations show that the matrix $GK_vG^\top + R_2M^{-1}M_d$ of (20) is sign indefinite for all $K_v > 0$, and all $M_d > 0$ verifying (21). This analysis, carried out in [25], reveals that the proposed IDA-PBC cannot assign $H_d(\boldsymbol{q}, \boldsymbol{p})$ as a Lyapunov function. However, on the lighter side, the linearization of the closed-loop system with $R_2 = 0$ has – for all admissible M_d and K_v – all its eigenvalues on the open left half plane, proving a robustness margin with respect to (small) unmodeled friction.

5. Electromechanical Systems

IDA-PBC has been successful in various types of electromechanical system applications including magnetic levitation [21,50], motor control [9,10,37,47] and transient stabilization of synchronous generators [22,34,38]. We review briefly some of these results providing, in some cases, new insights and alternative solutions.

5.1. Position Control

IDA-PBC was used in [50] to solve a, quite general, *position* control problem. Similar to the mechanical systems of Section 4.2, the structure of the energy function was fixed, and the PDE for the new parameters was explicitly solved using the elements of $\mathcal{J}_{\mathrm{d}}$.

We consider an electromechanical system consisting of n_{e} windings, with possible permanent magnets or a salient rotor where the relationship between the flux linkage vector $\boldsymbol{\lambda} \in \mathbb{R}^{n_{\mathrm{e}}}$ and the current vector $\boldsymbol{i} \in \mathbb{R}^{n_{\mathrm{e}}}$ is given by $\boldsymbol{\lambda} = L(\theta)\boldsymbol{i} + \boldsymbol{\mu}(\theta)$, with $\theta \in \mathbb{R}$ the mechanical angular position, and $L(\theta) = L^{\top}(\theta) > 0$ the $n_{\mathrm{e}} \times n_{\mathrm{e}}$ multiport inductance matrix of the windings. The vector $\boldsymbol{\mu}(\theta)$ represents the flux linkages due to the possible existence of permanent magnets. Assuming all the electrical coordinates are actuated,[8] the voltage balance equation yields

$$\dot{\boldsymbol{\lambda}} + R\boldsymbol{i} = \boldsymbol{u}, \tag{22}$$

where $\boldsymbol{u} \in \mathbb{R}^{n_{\mathrm{e}}}$ is the vector of voltages applied to the windings, $R = R^{\top} > 0$ is the matrix of electrical resistance of the windings. The coupling between the electrical and the mechanical subsystems is established through the torque of electrical origin

$$\boldsymbol{\tau}(\boldsymbol{i}, \theta) = \tfrac{1}{2}\boldsymbol{i}^{\top}L'\boldsymbol{i} + \boldsymbol{i}^{\top}\boldsymbol{\mu}'. \tag{23}$$

The model is completed by replacing the latter in the mechanical dynamics

$$J\ddot{\theta} = -r_m\dot{\theta} + \boldsymbol{\tau}(\boldsymbol{i}, \theta) - V', \tag{24}$$

where $J > 0$ is the rotational (translational) inertia of the mechanical subsystem, $r_m \geq 0$ is the viscous friction coefficient and the scalar function $V(\theta)$ is the potential energy. As shown in [41] the model contains, as particular cases, magnetic levitated systems, as well as permanent magnet synchronous and stepping motors.

The system may be written in PCH form (1)

$$\begin{bmatrix} \dot{\boldsymbol{\lambda}} \\ \dot{\theta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -R & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -r_m \end{bmatrix} \begin{bmatrix} \nabla_{\boldsymbol{\lambda}} H \\ \nabla_{\theta} H \\ \nabla_{p} H \end{bmatrix} + \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \boldsymbol{u}, \tag{25}$$

where

$$H(\boldsymbol{\lambda}, \theta, p) = \frac{1}{2}[\boldsymbol{\lambda} - \boldsymbol{\mu}(\theta)]^{\top} L(\theta)^{-1} [\boldsymbol{\lambda} - \boldsymbol{\mu}(\theta)] + V(\theta) + \frac{1p^2}{2J}, \tag{26}$$

$p \triangleq J\dot{\theta}$ is the mechanical momentum.

The control objective is asymptotic *regulation* of θ to a constant position $\theta_{\star}$, hence the desired equilibria are of the form $[\boldsymbol{\lambda}_{\star}, \theta_{\star}, 0]^{\top}$, where $\boldsymbol{\lambda}_{\star} = L(\theta_{\star})\boldsymbol{i}_{\star} + \boldsymbol{\mu}(\theta_{\star})$ and $(\boldsymbol{i}_{\star})$ is the solution of (23), (24) for $\theta = \theta_{\star}$, that is,

$$\tfrac{1}{2}\boldsymbol{i}_{\star}^{\top} L'(\theta_{\star})\boldsymbol{i}_{\star} + \boldsymbol{i}_{\star}^{\top}\boldsymbol{\mu}'(\theta_{\star}) - V'(\theta_{\star}) = 0. \tag{27}$$

It is interesting at this point to compare the problem described above with the one of mechanical systems discussed in Section 4. The equilibria (of the open-loop system) are not only determined by the mechanical potential energy, but they also (generally) correspond to non-zero electric energy. To be able to assign the equilibria of the closed-loop system via a selection of the potential energy only, we will choose a desired energy function of the form

$$H_{\mathrm{d}}(\boldsymbol{\lambda}, \theta, p) = \frac{1}{2}[\boldsymbol{\lambda} - \boldsymbol{\mu}_{\mathrm{d}}(\theta, p)]^{\top} L(\theta)^{-1} [\boldsymbol{\lambda} - \boldsymbol{\mu}_{\mathrm{d}}(\theta, p)] + V_{\mathrm{d}}(\theta) + \frac{1}{2J}p^2, \tag{28}$$

where $\boldsymbol{\mu}_{\mathrm{d}}(\theta, p)$ is a function to be defined, on which we impose the constraint

$$\boldsymbol{\lambda}_{\star} = \boldsymbol{\mu}_{\mathrm{d}}(\theta_{\star}, 0). \tag{29}$$

In this way, the equilibria will coincide with the extrema of $V_{\mathrm{d}}(\theta)$, and we simply have to select a function with a unique isolated minimum at $\theta_{\star}$.

Now, to assign the proposed energy function preserving the PCH structure we propose to modify the original interconnection and damping structures to take the form

$$\begin{bmatrix} \dot{\boldsymbol{\lambda}} \\ \dot{\theta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -R & \boldsymbol{\alpha} & \gamma \\ -\boldsymbol{\alpha}^{\top} & 0 & 1 \\ \gamma^{\top} & -1 & -r_a(p) \end{bmatrix} \begin{bmatrix} \nabla_{\boldsymbol{\lambda}} H_{\mathrm{d}} \\ \nabla_{\theta} H_{\mathrm{d}} \\ \nabla_{p} H_{\mathrm{d}} \end{bmatrix}, \tag{30}$$

where $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$, $r_a(p)$ are the *free* parameters (possibly functions of the state space variables) that we will use to assign the desired energy function, and we select $r_a(p) > 0$.

Proposition 3. [50] For any scalar function $r_a(p) > 0$ and any function $V_{\mathrm{d}}(\theta)$, there exists a function $\boldsymbol{\mu}_{\mathrm{d}}(\theta, p)$ satisfying (29), vector functions $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$ and a static state-feedback control $\boldsymbol{\beta}(\boldsymbol{\lambda}, \theta, p)$ such that the original dynamics (25) in closed-loop with $\boldsymbol{u} = \boldsymbol{\beta}(\boldsymbol{\lambda}, \theta, p)$ matches the desired dynamics (30) in some neighborhood of the equilibrium.[9]

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[8] This assumption is essential to avoid fixing some elements of the matrix $\mathcal{J}_{\mathrm{d}}$, as in the case of mechanical systems.

[9] In [50] it is shown that for typical applications, including rotating machines, matching (and subsequent stabilization) is global.

In the proposition above, no assumptions are made about the form of the parameters that define the system dynamics. In particular, no assumption of Blondel–Parks transformability – essential for the developments in [41] – is required. The price paid for achieving this level of generality, is *complexity*, requiring full state measurement and knowledge of system parameters. Some results on partial state feedback control are reported in [51]. On the other hand, it is our contention that the solid theoretical foundation of the proposed controller structure provides a suitable starting point for the derivation of (more practical) control schemes under some simplifying assumptions on the model.

5.2. An Interlaced Algebraic-Parameterized IDA-PBC

To solve a power systems problem, yet another variation of IDA-PBC was introduced in [38]. The main idea is to interlace the steps of solution of the PDE and determination of $\mathcal{J}_d$ as follows:

(i) first "evaluate" the PDE in *some subspace* of the state-space where the "solution" can be easily computed, and then
(ii) find the matrices $\mathcal{J}_d$, $\mathcal{R}_d$, $\boldsymbol{g}$ that will ensure the solution is valid in the whole state-space.

To explain the procedure we use here a permanent magnet synchronous motor (PMSM) example. Application of IDA-PBC to speed control of PMSMs has been reported in [47], see also Section 3.3 The controllers were tested experimentally and shown to be downward compatible with existing engineering practice, in the sense that the proposed controllers reduce to the standard ones for some particular tuning parameters and under some simplifying assumptions on the models.

The model in the so-called dq coordinates takes the form

$$\begin{aligned} L_d \frac{di_d}{dt} &= -R_s i_d + \omega L_q i_q + v_d, \\ L_q \frac{di_q}{dt} &= -R_s i_q - \omega L_d i_d - \omega \Phi + v_q, \\ J \frac{d\omega}{dt} &= n_p((L_d - L_q) i_d i_q + \Phi i_q) - \tau_l, \end{aligned}$$

where ω is the angular velocity, v_d, v_q, i_d, i_q are voltages and currents, n_p is the number of pole pairs, L_d and L_q are stator inductances, R_s is stator winding resistance, τ_l is a constant unknown load torque, Φ and J are the dq back emf constant and the moment of inertia respectively, and all the parameters are positive.

The desired equilibrium – applying a "maximum torque per ampere" principle – is $\left(0,\ L_q\tau_l/n_p\Phi,\ J/n_p\omega_\star\right)$. We then fix the *structure* of the desired energy function

$$H_d(i_d, i_q, \omega) = \tfrac{1}{2}\gamma_1 i_d^2 + \psi(i_q) + \tfrac{1}{2}\gamma_2(\omega - \omega_\star)^2,$$

where $\psi(i_q)$ is a function to be defined, $\gamma_i > 0$ are free parameters, and select

$$\begin{aligned} &\mathcal{J}_d(i_d, i_q) - \mathcal{R}_d \\ &\quad = \begin{bmatrix} -r_1 & J_{12}(i_d, i_q) & J_{13}(i_d, i_q) \\ -J_{12}(i_d, i_q) & -r_2 & J_{23}(i_d, i_q) \\ -J_{13}(i_d, i_q) & -J_{23}(i_d, i_q) & 0 \end{bmatrix}, \end{aligned}$$

with $r_i > 0$, and the functions $J_{ij}(i_d, i_q)$ to be defined. The ODE to be solved, corresponding to the unactuated coordinate, is

$$\begin{aligned} &-J_{13}(i_d, i_q)\gamma_1 i_d - J_{23}(i_d, i_q)\psi' \\ &\quad = \frac{1}{J}\left(n_p(L_d - L_q) i_d i_q + n_p \Phi i_q - \tau_l\right). \end{aligned} \tag{31}$$

The key step here is to "evaluate" the ODE on the plane $\{i_d = 0\}$. This yields

$$\psi' = \frac{1}{J_{23}(0, i_q)J}(\tau_l - n_p \Phi i_q),$$

that can be easily integrated if we take J_{23} to be constant. We note that the (positive Hessian) minimum condition, $\psi'' > 0$ imposes $J_{23} < 0$.

Now, we plug back the proposed ψ into the ODE (35) and compute the function $J_{13}(i_q)$ as

$$J_{13}(i_q) = \frac{n_p}{\gamma_1 J}(L_q - L_d) i_q.$$

The design is completed by computing the control law

$$\begin{aligned} v_d &= \frac{R_s}{L_d} i_d - \frac{L_q}{L_d} i_q \omega - r_1 \gamma_1 i_d \\ &\quad + \frac{1}{J_{23}J} J_{12}(i_d, i_q)(\tau_l - n_p \Phi i_q) \\ &\quad + \frac{n_p \gamma_2}{\gamma_1 J}(L_q - L_d) i_q (\omega - \omega_\star), \end{aligned} \tag{32}$$

$$\begin{aligned} v_q &= \frac{R_s}{L_q} i_q + \frac{L_d}{L_q} i_d \omega + \frac{\Phi}{L_q}\omega - J_{12}(i_d, i_q)\gamma_1 i_d \\ &\quad - \frac{r_2}{J_{23}J}(\tau_l - n_p \Phi i_q) + J_{23}\gamma_2(\omega - \omega_\star). \end{aligned} \tag{33}$$

These expressions can be considerably simplified selecting the free parameters $r_1, r_2, \gamma_1, \gamma_2 > 0, J_{12} \in \mathbb{R}$, $J_{23} < 0$. Also, as in [47] an observer for τ_l may be added.

The procedure explained above to solve (31) admits the following alternate interpretation. Fixing J_{23} to be constant and J_{13} independent of i_{d}, we can split (31) into a part that is linear on i_{d} and another one, containing ψ', that does not depend on i_{d}. In this respect, the procedure is similar to the one needed in Section 4.1 to obtain (15) and (16).

6. Extensions

In this section, we present some extensions to the basic method presented in Proposition 1 that incorporate additional features and allows to handle control scenarios other than just stabilization.

6.1. Exploiting the Passivity of PCH Systems

The next two propositions are immediate corollaries of the passivity property of PCH systems. The first one states that we can add an integrator around the passive output preserving stability – this feature is essential in all applications where the presence of noise and modeling errors induce steady-state errors. The second one shows that to inject additional damping, instead of the passive output, we can feed back the filtered signal preserving stability. This property, already reported in a Lagrangian framework in [40], is of particular interest in mechanical systems where the passive output is velocity, hence its measurement may be obviated.

Proposition 4 (*Integral action*). Consider the system of Proposition 1 in closed-loop with $\boldsymbol{u}=\boldsymbol{\beta}(\boldsymbol{x})+\boldsymbol{v}$, where

$$\dot{\boldsymbol{v}} = -K_I\boldsymbol{g}^\top(\boldsymbol{x})\nabla H_{\mathrm{d}},$$

with $K_I = K_I^\top > 0$. Then, all stability properties of $\boldsymbol{x}_\star$ stated in Proposition 1 are preserved.

Proof. The closed-loop clearly takes the PCH form

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{v}} \end{bmatrix} = \begin{bmatrix} \mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x}) & \boldsymbol{g}(\boldsymbol{x})K_I \\ -K_I\boldsymbol{g}^\top(\boldsymbol{x}) & 0 \end{bmatrix} \begin{bmatrix} \nabla_{\boldsymbol{x}} W \\ \nabla_{\boldsymbol{v}} W \end{bmatrix},$$

where

$$W(\boldsymbol{x},\, \boldsymbol{v}) \overset{\Delta}{=} H_{\mathrm{d}}(\boldsymbol{x}) + \tfrac{1}{2}\boldsymbol{v}^\top K_I^{-1}\boldsymbol{v},$$

qualifies now as Lyapunov function. □

Proposition 5 (*Adding damping with a dynamic extension*). For the system of Proposition 1 assume the control $\boldsymbol{u} = \boldsymbol{\beta}(\boldsymbol{x}) - K_{\mathrm{di}}\boldsymbol{g}^\top(\boldsymbol{x})\nabla H_{\mathrm{d}}$, with $K_{\mathrm{di}} = K_{\mathrm{di}}^\top > 0$, achieves asymptotic stability of $\boldsymbol{x}_\star$. Then, the dynamic feedback $\boldsymbol{u} = \boldsymbol{\beta}(\boldsymbol{x}) + \boldsymbol{\chi}$, where

$$\tau\dot{\boldsymbol{\chi}} = -\boldsymbol{\chi} - K_{\mathrm{di}}\boldsymbol{h}(\boldsymbol{x}), \tag{34}$$

with $\tau > 0$, also ensures $\boldsymbol{x}_\star$ is asymptotically stable.

Proof. Defining $W(\boldsymbol{x},\boldsymbol{\chi}) \overset{\Delta}{=} H_{\mathrm{d}}(\boldsymbol{x}) + \frac{\tau}{2}\boldsymbol{\chi}^\top K_{\mathrm{di}}^{-1}\boldsymbol{\chi}$, we get the new PCH system for the closed-loop

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{\chi}} \end{bmatrix} = \begin{bmatrix} \mathcal{J}_{\mathrm{d}}(\boldsymbol{x}) - \mathcal{R}_{\mathrm{d}}(\boldsymbol{x}) & \frac{1}{\tau}\boldsymbol{g}(\boldsymbol{x})K_{\mathrm{di}} \\ -\frac{1}{\tau}K_{\mathrm{di}}\boldsymbol{g}^\top(\boldsymbol{x}) & -\frac{1}{\tau^2}K_{\mathrm{di}} \end{bmatrix} \begin{bmatrix} \nabla_{\boldsymbol{x}} W \\ \nabla_{\boldsymbol{\chi}} W \end{bmatrix},$$

for which

$$\dot{W} = -\dot{H}_{\mathrm{d}} - \boldsymbol{\chi}^\top K_{\mathrm{di}}^{1}\boldsymbol{\chi}.$$

The proof is completed with a classical La Salle argument noting from (34) that

$$\boldsymbol{\chi}(t) = 0 \;\Rightarrow\; \boldsymbol{g}^\top(\boldsymbol{x}(t))\frac{\partial H_{\mathrm{d}}}{\partial \boldsymbol{x}}(\boldsymbol{x}(t)) = 0. \qquad \square$$

6.2. Damping Injection

The next proposition shows that the effect on the passive output of disturbances entering in the input channel can be attenuated (in the $\mathcal{L}_2$ sense) increasing the damping injection.

Proposition 6 ($\mathcal{L}_2$-gain assignment). Consider the system of Proposition 1 in closed-loop with $\boldsymbol{u} = \boldsymbol{\beta}(\boldsymbol{x}) - K_{\mathrm{di}}\boldsymbol{g}^\top(\boldsymbol{x})\nabla H_{\mathrm{d}} + \boldsymbol{v}$, where $K_{\mathrm{di}} = K_{\mathrm{di}}^\top \geq \delta I > 0$. Then, the map $\boldsymbol{v} \mapsto z$ with $z = \boldsymbol{g}^\top(\boldsymbol{x})\nabla H_{\mathrm{d}}$ has $\mathcal{L}_2$ gain smaller than $1/\delta$.

Proof. The proof follows from direct application of Lemma 2.2.14 in [57], which establishes that output strictly operators are finite-gain stable, and retracing the proof for the system above. □

The proposition below, which was reported in [59], shows that it is possible to adaptively inject damping to a PCH system even in the case when the energy function is only partially known – provided it depends linearly on the unknown parameters.

Proposition 7 (*Adaptive damping injection*). Consider the PCH system (1) and assume the energy function is radially unbounded, satisfies $\boldsymbol{x}_\star = \arg\min H(\boldsymbol{x})$, and may be linearly parameterized as

$$H(\boldsymbol{x}) = \boldsymbol{\theta}^\top\phi(\boldsymbol{x}),$$

where $\boldsymbol{\theta} \in \mathbb{R}^q$ is a vector of unknown parameters and $\phi(\boldsymbol{x})$ is a known vector function. Then, (1) in closed-loop with the adaptive damping injection

$$\boldsymbol{u} = -K_{\mathrm{di}}\boldsymbol{g}^\top(\boldsymbol{x})\left(\frac{\partial\phi}{\partial\boldsymbol{x}}\right)^\top\hat{\boldsymbol{\theta}},$$

where $K_{\mathrm{di}} = K_{\mathrm{di}}^\top > 0$ and $\hat{\theta}$ is adjusted with

$$\dot{\hat{\boldsymbol{\theta}}} = -\frac{\partial\phi}{\partial\boldsymbol{x}}\left[2\mathcal{R}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})K_{\mathrm{di}}\boldsymbol{g}^\top(\boldsymbol{x})\right]\left(\frac{\partial\phi}{\partial\boldsymbol{x}}\right)^\top\hat{\theta} \tag{35}$$

ensures all signals are bounded and $\boldsymbol{x}_\star$ is a stable equilibrium.

Proof. Consider the Lyapunov function candidate

$$W(\boldsymbol{x},\hat{\theta}) = H(\boldsymbol{x}) + \tfrac{1}{2}\tilde{\boldsymbol{\theta}}^\top\tilde{\boldsymbol{\theta}},$$

where $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$. After some simple calculations it can be shown that, along the closed-loop trajectories, we have

$$\dot{W} = -\,\hat{\theta}^\top \frac{\partial\phi}{\partial \boldsymbol{x}}\left[\mathcal{R}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})K_{\mathrm{di}}\boldsymbol{g}^\top(\boldsymbol{x})\right]\left(\frac{\partial\phi}{\partial \boldsymbol{x}}\right)^\top\hat{\boldsymbol{\theta}} \\ -\tilde{\boldsymbol{\theta}}^\top\frac{\partial\phi}{\partial \boldsymbol{x}}\mathcal{R}(\boldsymbol{x})\left(\frac{\partial\phi}{\partial \boldsymbol{x}}\right)^\top\tilde{\boldsymbol{\theta}} \le 0,$$

which completes the proof. □

Stability in the proposition above is ensured via a suitable selection of a non-increasing adjustable gain in the damping injection term. The (rather unpleasant) "non-increasing" qualifier stems from the fact that $|\hat{\boldsymbol{\theta}}(t)|^2 \le |\hat{\boldsymbol{\theta}}(0)|^2$ for all $t \ge 0$, which follows from direct inspection of the parameter update law (35).

6.3. Regulation and Disturbance Suppression with Internal Model

The problem of rejection of (matched) disturbances generated from an (Poisson stable) exosystem for PCH systems has been studied in [4]. In particular, the following scenario has been considered, and solved. Given a PCH system with matched additive disturbance

$$\dot{\boldsymbol{x}} = [\mathcal{J}(\boldsymbol{x}) - \mathcal{R}(\boldsymbol{x})]\nabla H + \boldsymbol{g}(\boldsymbol{x})(\boldsymbol{u} - \boldsymbol{d}), \tag{36}$$

where

$$\dot{\boldsymbol{w}} = \mathcal{J}_{\mathrm{ex}}\frac{\partial H_{\mathrm{ex}}}{\partial \boldsymbol{w}}, \quad \boldsymbol{d} = \boldsymbol{g}_{\mathrm{ex}}^\top\frac{\partial H_{\mathrm{ex}}}{\partial \boldsymbol{w}}, \quad \boldsymbol{w} \in \mathbb{R}^s$$

with $\mathcal{J}_{\mathrm{ex}} = -\mathcal{J}_{\mathrm{ex}}^\top$ and

$$H_{\mathrm{ex}}(\boldsymbol{w}) = \tfrac{1}{2}\boldsymbol{w}^\top E\boldsymbol{w}$$

for some positive definite matrix E, find a dynamic controller that, measuring only the plant state $\boldsymbol{x}$, ensures asymptotic stability of some desired equilibrium $\boldsymbol{x}_\star$. Note that the class of considered disturbances contains, as a particular case, the classical sinusoidal disturbance. Since $\boldsymbol{w}$ is not known, the solution of the problem requires the incorporation of the internal model of the disturbance $\boldsymbol{d}$ whose goal is to generate the steady-state control input needed to compensate for it. A key step in the construction proposed in [4] is the realization of this internal model in such a way that, in some suitable coordinates, we can inject damping to the coordinates associated to the dynamic extension without the need to measure $\boldsymbol{w}$.

It is rather remarkable that with this ingenious construction it is possible to "separate" the tasks of IDA-PBC stabilization of the unforced system, using for instance Proposition 1, and the design of the internal model, the former being completely independent of the parameters $\mathcal{J}_{\mathrm{ex}}$, $\boldsymbol{g}_{\mathrm{ex}}$ and H_{ex}.

An alternate, and in some sense, more general scenario is considered in [24], where only the output of the system is assumed to be available for measurement, and a general regulation problem is addressed. A first result in the paper proves that, if the control that solves the Francis–Byrnes–Isidori equations (for general nonlinear systems) is polynomial, then the internal model admits a PCH realization. As a consequence of this result we have that, if the system is PCH, then the output regulation problem reduces to a problem of stabilization of an extended PCH system. It is then proven that, under the standard assumptions for output regulation and the hypothesis that the unforced system is stable, the zero-error manifold can be rendered globally attractive via a damping injection from the output of an extended system. For the problem of regulation, this output coincides with the plant output and injecting the damping is immediate. For the tracking case, however, this output is the tracking error which is not a passive output anymore and the damping injection stage remains to be solved.

6.4. Constrained PCH Systems and Sliding Mode Control

IDA-PBC has been used in [32] as a first step in the design of controllers that drive the systems trajectories towards a given n–m-dimensional sub-manifold of the state space, say

$$\mathcal{S} \overset{\Delta}{=} \{\boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{s}(\boldsymbol{x}) = 0\},$$

where $\boldsymbol{s}$: $\mathbb{R}^n \to \mathbb{R}^m$ is a continuous function. An important application of this idea, which is explored in the paper, is in the design of sliding mode controllers for PCH systems which have some nice energy interpretations.

Following the procedure of the control by interconnection of [57], see also [44], a dynamic PCH controller (with state $\boldsymbol{\xi} \in \mathbb{R}^m$) is coupled with the plant via a power-preserving interconnection. To effectively shape the energy of the interconnected system m dynamical invariants, defined by the Casimir functions $\boldsymbol{\xi} - \boldsymbol{s}(\boldsymbol{x})$, have to be created. IDA-PBC is used at this stage to modify the plant interconnection and damping to verify the necessary (and quite restrictive) conditions for existence of these Casimir functions, yielding a reduced dynamics of the form

$$\dot{\boldsymbol{x}} = [\mathcal{J}_{\rm d}(\boldsymbol{x}) - \mathcal{R}_{\rm d}(\boldsymbol{x})]\nabla H_{\rm d} + \mathcal{J}_{\rm d}(\boldsymbol{x})\frac{\partial \boldsymbol{s}}{\partial \boldsymbol{x}}\frac{\partial H_{\rm c}}{\partial \boldsymbol{s}},$$

where $H_{\rm c}$: $\mathbb{R}^m \to \mathbb{R}^m$ is the energy of the controller to be designed.

Under the standard assumption of unitary relative degree – with respect to the output $\boldsymbol{s}(\boldsymbol{x})$ – of the original PCH system, and an additional hypothesis of integrability of the function

$$\left(\boldsymbol{g}^\top(\boldsymbol{x})\frac{\partial \boldsymbol{s}}{\partial \boldsymbol{x}}\right)^{-1}\left[\boldsymbol{g}^\top(\boldsymbol{x})\nabla H_{\rm d} + \boldsymbol{s}(\boldsymbol{x})\right],$$

the authors prove the existence of an energy function for the control that ensures attractivity of $\mathcal{S}$. The (rather technical) integrability requirement is needed because, as shown above, the controller energy function enters into the (reduced) PCH system dynamics in the form of a gradient. A sliding mode action may be enforced commuting the controller energy function according to the position of the state relative to $\mathcal{S}$. A sliding motion will then be created if a set of $2m$ inequalities that restrict the slope of the controller energy function are satisfied. As a by product of this research the classical result that states that the zero-dynamics of Hamiltonian systems without dissipation and Hamiltonian vector field inputs are Hamiltonian is extended to the more general PCH models with damping. IDA-PBC for constrained PCH systems (for instance, mechanical systems with kinematic constraints) was also treated in [11].

7. Concluding Remarks and Future Research

We have reviewed in this paper a series of applications and extensions of the basic IDA-PBC design methodology laid out in [46]. The wealth of material contained here reveals the wide diversity of this research line, which is being enthusiastically pursued by many different control groups.

Among the manifold of research directions that are currently being investigated or remain open includes the following:

Tracking. The basic IDA-PBC is restricted to stabilization of fixed points and, with the notable exceptions of [19,24], tracking remains an essentially open issue. As pointed out in [24] the question of tracking exosystem-generated references may be cleanly recasted as a damping injection problem, but unfortunately with "unmatched" signals. In some cases it is possible to adapt the procedure to treat the stabilization of periodic orbits, see [2] for an example, where a "Mexican sombrero" $H_{\rm d}(\boldsymbol{x})$ of the form depicted in Fig. 5 is assigned to the closed-loop. See also [17] where similar objectives are pursued.

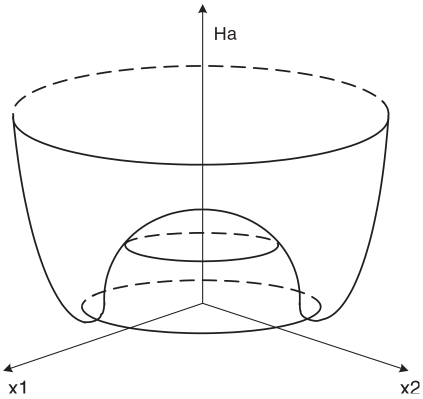

Fig. 5. Desired energy function for generation of periodic orbits.

Solving the matching equation. The need to have an explicit solution of the matching equation (3) remains an important stumbling block to make IDA-PBC a systematic design procedure. For mechanical systems, it has been shown in [13] that the λ-method of [8], developed for the Controlled Lagrangian method, can be adapted for IDA-PBC, yielding a bilinear PDE. See also the ν-method described in [7]. Using all the available degrees-of-freedom (i.e. $\mathcal{J}_{\rm d}(\boldsymbol{x})$, $\mathcal{R}_{\rm d}(\boldsymbol{x})$, $\boldsymbol{g}^\perp(\boldsymbol{x})$, change of coordinates, perturbed target dynamics) and combining the two extreme approaches described in Section 2.2, this obstacle has been overcome for a large collection of practical applications. A particularly promising approach, that allowed the solution of the longstanding problem of transient stabilization of multimachine power systems with transfer conductances [38], is the interlaced technique described in Section 5.2.

Robustness and adaptation. The current framework to assess robustness of controller designs is based on contrived (but mathematically convenient) uncertainty structures that are difficult to justify from physical considerations.[10] Developing a robustness and adaptation theory that would accommodate interconnection of (partially uncertain) parameterized PCH systems seems to be a reasonable alternative to reverse this trend.

Asymptotic matching. The final aim of IDA-PBC is, in essence, model matching which in many applications might be too strong a requirement. The possibility of accommodating a disturbance – linearly

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[10] The prevalent configuration assumes some kind of matching condition that can be "dominated" with high-gain.

bounded by ∇H_d – as discussed in Section 2.3 provides some additional flexibility. Another possibility, that would clearly relax the stringent conditions imposed by the method, is to aim at *asymptotic* model matching. This perspective was adopted in [5] to develop a working design in a satellite application. This research led to the development of a new, immersion and invariance, technique for stabilization of general nonlinear systems [6], that could be instrumental to derive systematic tools to attain our objective of asymptotically matching a PCH system.

Power shaping. A practical drawback of energy-shaping control is the limited ability to "speed up" the transient response leading to somehow sluggish transients and below par overall performance levels. In a recent paper [39] we prove that for a class of RLC circuits with convex energy function and weak electromagnetic coupling it is possible to "add a differentiation" to the port terminals preserving passivity – with a new storage function that is directly related to the circuit power. A complete characterization of the linear RLC circuits that enjoy these new property is given in [23]. A corollary of this result is that it is possible to formulate the stabilization problem in terms of power (as opposed to energy) shaping, adding "derivative" actions in the control that naturally yield faster responses. Current research is underway in two directions: extending the technique to other physical systems and assessing the effective advantages of the alternate framework with respect to energy-shaping based schemes like IDA-PBC. (See [12] for the geometric formalization of the ideas in [28,29].)

Infinite dimensional systems. The PCH modeling framework for these systems has already been laid out in [33] and some preliminary results are available on control by interconnection [31,54]. See also [55].

From a control engineering viewpoint the introduction of a "new" controller design technique is justified only if it can outperform the existing schemes for a practical problem or provide solutions to a challenging task that remained open. As witnessed by the list of references IDA-PBC has amply and satisfactorily fulfilled these expectations. The wide attention that PBC in general, and IDA-PBC in particular, has attracted makes us confident that the technique will prove instrumental to solve other practical problems in the future.

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