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On feedback equivalence to port controlled Hamiltonian systems

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Abstract

In the last few years port controlled Hamiltonian (PCH) systems have emerged as an interesting class of nonlinear models suitable for a large number of physical applications. In this paper we study the question of feedback equivalence of nonlinear systems to PCH systems. More precisely, we give conditions under which a general nonlinear system can be transformed into a PCH system via static state feedback. We consider the two extreme cases where the target PCH system is completely a priori fixed or completely free, as well as the case where it is only partially predetermined. When the energy function is free a set of partial differential equations needs to be solved, on the other hand, if it is fixed we have to deal with a set of algebraic equations. In the former case, we give some verifiable necessary and sufficient conditions for solvability. As a by-product of our analysis we obtain some stabilization results for nonlinear systems.

Keywords: Hamiltonian systems; Linear-gradient systems; Passivity-based control; Feedback equivalence

Notation. All vectors are column vectors, including the gradient which is denoted $\nabla_x = \partial/\partial x$. When clear from the context the subindex will be omitted. For all vectors and matrices which are functions of some variables we will write explicitly this dependence only the first time they are defined. Throughout the paper we will assume that all functions are sufficiently smooth and, with some abuse of notation, treat the vector functions as *elements* of a linear space, instead of sets, e.g., their span as the argument ranges on some set. Finally, no particular attention is given to the characterization of the domain of validity of our statements, to which the local qualifier should be attached. The statements become global if some rank conditions of state-dependent matrices that are assumed to hold only locally, are actually true uniformly in the state.

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1. Introduction

The importance of the notion of passivity for analysis and control design can, nowadays, hardly be over-estimated, see e.g. [17,15,12]. The central question of transforming a non-passive system into a passive system via state-feedback was elegantly settled in [4] where succinct, necessary and sufficient, geometric conditions are given. In spite of the unquestionable beauty of this result feedback equivalence to a (general) passive system has been used more as a conceptual framework to understand stabilization mechanisms than as an actual controller design procedure.

On the other hand, feedback equivalence to port controlled Hamiltonian (PCH) models, which are a class of passive systems, has attracted the attention of many researchers lately, in particular for stabilization objectives. A PCH system (with dissipation) is defined as [17]

$$\Sigma_{\mathrm{PCH}}\begin{cases}\dot{x}=[J(x)-R(x)]\nabla H(x)+G(x)u,\\ \quad x\in\mathbb{R}^n,\ u\in\mathbb{R}^m,\ m<n,\\ y=G^\top(x)\nabla H(x),\quad y\in\mathbb{R}^m,\end{cases}\tag{1}$$

where $H:\mathbb{R}^n\to\mathbb{R}$ is the total stored energy, $J=-J^\top$ is known as the interconnection matrix, $R=R^\top\geqslant 0$ represents the dissipation and G is the input matrix that is assumed full rank. The vector signals u and y are the conjugated port variables and their product $u^\top y$ has units of power. It is easy to see that, if the total energy function is non-negative, then PCH systems are passive. Furthermore, if H has an *isolated* minimum at a point $x_\star\in\mathbb{R}^n$, that is

$$x_\star=\arg\min H(x)\tag{2}$$

then $x_\star$ is a stable equilibrium of Σ_{PCH} with $u\equiv 0$.

As explained in [17] PCH systems constitute an extension of classical Hamiltonian and Euler–Lagrange models that naturally incorporate interaction with the environment (through power port variables) and capture the essential physical property of power conservation that is elegantly articulated using the concept of Dirac structure. Given these nice features of PCH models it is then natural to ask when an affine nonlinear system like

$$\Sigma_{f,G}:\ \dot{x}=f(x)+G(x)u\tag{3}$$

with G full rank is transformable, via feedback, into a PCH system? The investigation of this question is the topic of interest of the present work.

Our work has been largely inspired by the recent interesting paper [16] where some of the questions addressed here are studied for the particular case of symplectic Hamiltonian systems with fixed symplectic structure. (See also [10,13].) As thoroughly explained in the concluding remarks, the present work contains some extensions of their results.

2. Problem formulation

As will become clear in the sequel it is convenient to first consider feedback equivalence to pseudo-gradient systems of the form

$$\Sigma_{PG}:\ \dot{x}=F(x)\nabla H(x),$$

where $F:\mathbb{R}^n\to\mathbb{R}^{n\times n}$ is arbitrary. Then, imposing the constraint that

$$F+F^\top\leqslant 0,\tag{4}$$

address feedback equivalence to PCH systems.[1] Finally, with the stabilization objective in mind, we will additionally be interested in the case when (2) holds, where $x_\star\in\mathbb{R}^n$ is an *admissible equilibrium* of $\Sigma_{f,G}$, that is, such that $f(x_\star)\in\operatorname{Im}G(x_\star)$.

The definitions below are instrumental to provide concise statements of our results.

Definition 1. The affine system $\Sigma_{f,G}$ in Eq. (3) is feedback equivalent to a *pseudo-gradient system* and, for short, denote it as $\Sigma_{f,G}\in\mathscr{F}_{PG}$, if there exists a state feedback $\beta:\mathbb{R}^n\to\mathbb{R}^m$ such that the *matching equations*

$$f(x)+G(x)\beta(x)=F(x)\nabla H(x)\tag{5}$$

hold.

Definition 2. $\Sigma_{f,G}$ is feedback equivalent to a *PCH system*—denote it as $\Sigma_{f,G}\in\mathscr{F}_{\mathrm{PCH}}$—if $\Sigma_{f,G}\in\mathscr{F}_{PG}$ with F satisfying (4).

[1] It is obvious that (4) is equivalent to the existence of matrices $J=-J^\top$ and $R=R^\top\geqslant 0$ such that $F=J-R$.

Definition 3. $\Sigma_{f,G}$ is feedback equivalent to a *stable* PCH system, denoted $\Sigma_{f,G} \in \mathscr{F}^S_{\mathrm{PCH}}$, if $\Sigma_{f,G} \in \mathscr{F}_{\mathrm{PCH}}$ with H satisfying (2)—for an admissible equilibrium $x_\star$ of $\Sigma_{f,G}$.

Depending on the prior assumptions made on the target dynamics, Σ_{PG}, the feedback equivalence question generates different mathematical problems. In the remaining of the paper we give conditions for feedback equivalence in the following four cases:

(i) F and H a priori fixed,
(ii) F and H free,
(iii) F free and H a priori fixed,
(iv) F a priori fixed and H free.

Clearly, in case (iii) the matching (5) defines a set of algebraic equations in the unknowns F and β. On the other hand, case (iv) leads to a set of PDEs for H—parameterized in F and β.

Remark 1. Transforming a system to be controlled into a PCH system is the central idea of the interconnection and damping assignment passivity-based control method first introduced in [13], where the perspective of case (iv) above is adopted. A summary of some recent developments may be found in [11], see also [6,18] for its applications to power systems. In [8], case (iii) above is considered. See also [3,1] for the case of feedback equivalence to Lagrangian systems.

Remark 2. As indicated in [8], the PCH form is invariant to change of coordinates. This justifies the use of the term "feedback equivalence" in the definitions. Also, with some abuse of notation we have used the words "equivalence to PCH systems" in Definitions 2 and 3 without defining the port variables. However, notice that if $\Sigma_{f,G} \in \mathscr{F}_{\mathrm{PCH}}$ then with the new control input $u = \beta(x) + v$ it is possible to define a bona fide PCH system with port variables $(v, \tilde{y})$ of the form

$$\dot{x} = F(x)\nabla H(x) + G(x)v,$$
$$\tilde{y} = G^\top(x)\nabla H(x).$$

3. Conditions for feedback equivalence: full matching equations

In this section we give conditions for feedback equivalence using the full matching equations (5). Then, in the next section concentrate on "the equations in Im G".

To formulate our results we find convenient to define the parameterized closed-loop vector field

$$\tilde{f}_\beta(x) := f(x) + G(x)\beta(x),$$

and define the set of controls that assign the admissible equilibrium $x_\star$ to $\Sigma_{f,G}$ as[2]

$$\mathscr{G}_{x_\star} := \{\beta : \mathbb{R}^n \to \mathbb{R}^m \mid \tilde{f}_\beta(x_\star) = 0\}.$$

Invoking Lemma 9.2.1, pp. 433, of [9] we can prove that for each $\beta \in \mathscr{G}_{x_\star}$, we can define a matrix $A_\beta : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ such that

$$\tilde{f}_\beta(x) = A_\beta(x)(x - x_\star). \tag{6}$$

Similarly, for all functions H satisfying (2) we introduce the factorization

$$\nabla H(x) = \Psi(x)(x - x_\star), \tag{7}$$

where $\Psi : \mathbb{R}^n \to \mathbb{R}^{n\times n}$.

The proposition below gives conditions for feedback equivalence for all four cases, (i)–(iv), of the previous section.

Proposition 1. *Consider the affine system $\Sigma_{f,G}$ of Eq.* (3) *with G full rank.*

(i) *If F and H are* a priori *fixed, with F, H satisfying* (4) *and* (2), *respectively, then*

$$\Sigma_{f,G} \in \mathscr{F}^S_{\mathrm{PCH}} \Leftrightarrow (F\nabla H - f) \in \mathrm{Im}\, G.$$

(ii) *If F and H are free then $\Sigma_{f,G} \in \mathscr{F}_{PG}$. Furthermore, if there exists $\beta \in \mathscr{G}$ such that*

$$PA_\beta + A_\beta^\top P \leqslant 0 \tag{8}$$

for some constant $P = P^\top > 0$, where A_β is given in (6), *then $\Sigma_{f,G} \in \mathscr{F}^S_{\mathrm{PCH}}$.*

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[2] This set $\mathscr{G}_{x_\star}$ is clearly non-empty as it contains the element $\beta = -(G^\top G)^{-1}G^\top f$—invertibility of $G^\top G$ stemming from the fact that it is the Gram matrix of a set of linearly independent vectors.

(iii) *If F is free and H is* a priori *fixed and satisfying* (2) *with* Ψ, *as defined in* (7), full rank *then* $\Sigma_{f,G} \in \mathcal{F}_{PG}$. *Furthermore, if there exists* $\beta \in \mathcal{G}$ *such that*

$$\Psi^\top A_\beta + A_\beta^\top \Psi \leqslant 0, \tag{9}$$

where A_β *is given in* (6), *then* $\Sigma_{f,G} \in \mathcal{F}^S_{\text{PCH}}$.

(iv) *If F is* a priori *fixed and H is free with F* full rank *and satisfying* (4) *then* $\Sigma_{f,G} \in \mathcal{F}_{\text{PCH}}$ *if and only if the* $n/2(n-1)$ *PDEs*

$$\nabla[F^{-1}(f+G\beta)] = (\nabla[F^{-1}(f+G\beta)])^\top \tag{10}$$

admit a solution for β.

Proof. (i) This result follows trivially from the matching equation (5).

(ii) Select $\beta \in \mathcal{G}_{x_\star}$ and define the matrix A_β as in (6). Fix $H = \frac{1}{2}(x-x_\star)^\top P(x-x_\star)$, for some constant full rank $P \in \mathbb{R}^{n\times n}$. The proof that $\Sigma_{f,G} \in \mathcal{F}_{PG}$ is completed defining $F = A_\beta P^{-1}$. To prove the second statement select the matrix $P = P^\top > 0$ solution of (8) and define

$$J := \tfrac{1}{2}(F - F^\top) = -J^\top,$$
$$R := -\tfrac{1}{2}(F + F^\top) \geqslant 0$$

where the inequality follows from (8).

(iii) Select $\beta \in \mathcal{G}_{x_\star}$ and define the matrix A_β as in (6). Since H satisfies (2) we can define Ψ as in (7). The proof that $\Sigma_{f,G} \in \mathcal{F}_{PG}$ is completed defining $F = A_\beta \Psi^{-1}$. To prove the second statement define J and R as above, where the inequality follows now from (9).

(iv) If the matrix F is full rank, Poincare's Lemma gives us directly a necessary and sufficient condition for feedback equivalence. Indeed, the vector field $F^{-1}(f+G\beta)$ is a gradient vector field, that is, (5) is satisfied for some scalar function H, if and only if (10) holds. □

4. Conditions for feedback equivalence: projected matching equations

In this section we show that it is possible to characterize feedback equivalence using a projection of the matching equations. (Although this fact is very easy to establish, the lack of such a formal statement was a source of some confusion in the literature, see e.g., [11].) This result is also of interest because, concentrating on the projected equations, allows to give verifiable necessary and sufficient conditions for the existence of solutions of the PDEs that arise in case (iv) of the previous section.

Towards this end, we introduce the following:

Definition 4. We say that a matrix $G^\perp : \mathbb{R}^n \to \mathbb{R}^{(n-m)\times n}$ is a full-rank left annihilator of G if $G^\perp G = 0$ and rank $G^\perp = n - m$.

We recall a basic linear algebra lemma.

Lemma 1. *Consider two linear subspaces* $\mathcal{S}_1, \mathcal{S}_2 \subset \mathbb{R}^n$. *If,* $\dim \mathcal{S}_1 = \dim \mathcal{S}_2$ *and* $\mathcal{S}_1 \subset \mathcal{S}_2$ (*or* $\mathcal{S}_2 \subset \mathcal{S}_1$), *then* $\mathcal{S}_1 = \mathcal{S}_2$.

Proposition 2. *Consider the affine system* $\Sigma_{f,G}$ *in Eq.* (3), *with F satisfying* (4). *Then* $\Sigma_{f,G} \in \mathcal{F}_{\text{PCH}}$ *if and only if the projected matching equations* (*PMEs*)

$$G^\perp(f - F\nabla H) = 0 \tag{11}$$

hold for an arbitrary full-rank left annihilator of G.

Proof. Clearly, $\Sigma_{f,G} \in \mathcal{F}_{\text{PCH}} \Leftrightarrow (f - F\nabla H) \in \operatorname{Im} G$. For the sake of completeness we prove now the well-known identity

$$\operatorname{Ker} G^\perp = \operatorname{Im} G. \tag{12}$$

First, note that both spaces have the same dimension, m. Consider then the chain of implications:

$$\begin{aligned} a \in \operatorname{Im} G &\Leftrightarrow \exists b \in \mathbb{R}^n : a = Gb \\ &\Rightarrow G^\perp a = G^\perp G b = 0 \\ &\Rightarrow a \in \operatorname{Ker} G^\perp \\ &\Rightarrow \operatorname{Im} G \subset \operatorname{Ker} G^\perp. \end{aligned}$$

Finally, we can invoke Lemma 1 to conclude (12). □

If F is *a priori* fixed and H is free equations (11) define a set of $n - m$ PDEs. To derive sufficient conditions for their solvability we need two preliminary lemmata, which are largely inspired by [16]—more precisely, by their coordinate-free Theorem 4.1. The proofs of the lemmata are given in the Appendix.

Lemma 2. *Given $W:\mathbb{R}^n\to\mathbb{R}^{q\times n}$ and $s:\mathbb{R}^n\to\mathbb{R}^q$, with $q<n$. The following statements are equivalent.*

(A) $\exists H:\mathbb{R}^n\to\mathbb{R}$ *such that*

$$W(x)\nabla H(x)=s(x). \tag{13}$$

(B) $\exists \tilde{H}:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}$ *such that*

$$W(x)\nabla_x\tilde{H}(x,z)-s(x)\nabla_z\tilde{H}(x,z)=0, \tag{14}$$

$$\nabla_z\tilde{H}(x,z)\neq 0. \tag{15}$$

Lemma 3. *Let W and s be as in Lemma 2 and define the distributions*

$$\Delta := \text{involutive closure span}\,\{W^\top(x)\}$$
$$\tilde{\Delta} := \text{involutive closure span}\left\{\begin{bmatrix} W^\top(x)\\ s^\top(x)\end{bmatrix}\right\}.$$

Assume Δ and $\tilde{\Delta}$ are regular. Then (B) of Lemma 2 holds if and only if

$$\dim\Delta=\dim\tilde{\Delta}. \tag{16}$$

We are in a position to present the main proposition of this section.

Proposition 3. *Consider the affine system $\Sigma_{f,G}$ of Eq. (3). Let F be* a priori *fixed and H free, with F satisfying (4). Let*

$$W:=G^\perp F,\quad s:=G^\perp f, \tag{17}$$

compute the distributions Δ and $\tilde{\Delta}$ as in Lemma 3 and assume that they are regular. Then $\Sigma_{f,G}\in\mathscr{F}_{\mathrm{PCH}}$ if and only if condition (16) holds.

Proof. With W and s as in (17) the PDEs (11) take the form (13) with $q=n-m$. The proof follows immediately from Lemmata 2 and 3, and Proposition 2. □

Remark 3. A full-rank left annihilator for G can be easily constructed as follows. Define the partition

$$G(x)=\begin{bmatrix} G_1(x)\\ G_2(x)\end{bmatrix},\quad G_1:\mathbb{R}^n\to\mathbb{R}^{(n-m)\times m},$$
$$G_2:\mathbb{R}^n\to\mathbb{R}^{m\times m},$$

where rank $G_2=m$ in some neighborhood of interest.[3] Then define

$$G^\perp=[\,I_{n-m}\quad -G_1G_2^{-1}\,].$$

See also [5] for some alternative constructions and further discussion on the set of solutions of the equations of interest.

5. Concluding remarks

In this paper we have investigated the problem of feedback equivalence—via state-feedback—of affine systems to PCH systems. Such a transformation lies at the core of some stabilization techniques recently reported in the literature. We also recall that it has been considered as one of the open problems in mathematical systems theory [2]. Necessary and sufficient conditions, expressed in terms of solvability of sets of PDEs or algebraic equations, are given.

A comparison of our work with the interesting results of [16] is in order. In the spirit of [16] verifiable necessary and sufficient conditions for feedback equivalence are given in Proposition 1 (case (i)), with fixed interconnection and damping structure and fixed Hamiltonian. This is the extension of Proposition 3.1 of [16] to our case. Also, similarly to Theorem 4.1 of [16], necessary and sufficient conditions are given in Proposition 3 for the solvability of the PDEs.

Several variations to the feedback equivalence problem studied here are of practical interest and have been studied in the literature. For instance, in some cases it is reasonable to make the interconnection matrix J dependent on the control—e.g., in switched systems like power converters this is the natural way of modelling [7]. In this case, the matching equation (5) becomes

$$f(x)+G(x)\beta(x)=[J(x,\beta(x))-R(x)]\nabla H(x),$$

leading to a completely different characterization of feedback equivalence. This approach has been adopted in [14] yielding some practically interesting stabilizing control laws.

[3] This partition can always be locally achieved simply swapping and relabelling the state equations. However, the size of the neighborhood where rank $G_2=m$ might be smaller than the region where rank $G=m$.

As is well-known, solving PDEs and nonlinear algebraic equations is not an easy task. In [5] the possibility of reducing their number, which turns out to be determined by a simple rank condition, is investigated.

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Appendix A.

In this appendix we give the proofs of Lemmata 1 and 2. As indicated in Section 4, they heavily borrow from [16].

A.1. Proof of Lemma 2

(A) $\Rightarrow$ (B) With H solution of (13) define $\tilde{H}(x,z) := H(x) + z$, which clearly solves (14) and satisfies (15).

(B) $\Rightarrow$ (A) Given $\tilde{H}$ solution of (14) we know from the implicit function theorem and (15) that the equation $\tilde{H}(x,z) = 0$ admits a (local) solution in z. Let us call this solution $z = H(x)$, that is[4]

$$\tilde{H}(x,z)|_{z=H(x)} = \tilde{H}(x, H(x)) = 0,$$

Taking the gradient of $\tilde{H}(x,z)$ we get

$$\nabla_x \tilde{H}(x,z)|_{z=H(x)} + \nabla_z \tilde{H}(x,z)|_{z=H(x)} \nabla H(x) = 0.$$

Using again (15) we obtain

$$\nabla H(x) = \frac{-1}{\nabla_z \tilde{H}(x,z)|_{z=H(x)}} \nabla_x \tilde{H}(x,z)|_{z=H(x)}.$$

[4] For clarity, we write explicitly the arguments of the functions.

Replacing this expression in (13) we get

$$\begin{aligned} W(x)\nabla H(x) - s(x) &= W(x)\left(\frac{-1}{\nabla_z \tilde{H}(x,z)} \nabla_x \tilde{H}(x,z)\right)\Bigg|_{z=H(x)} - s(x) \\ &= \frac{-1}{\nabla_z \tilde{H}(x,z)|_{z=H(x)}} \\ &\quad \times \underbrace{(W(x)\nabla_x \tilde{H}(x,z)|_{z=H(x)})}_{-s(x)\nabla_z \tilde{H}(x,z)|_{z=H(x)}} - s(x), \end{aligned}$$

where we have used (14) to get the underbrace. Since the right-hand side is equal to zero $H(x)$ solves (13) completing the proof. □

A.2. Proof of Lemma 3

To prove sufficiency, note that $\dim \tilde{\Delta} \leqslant n$. Hence by involutivity and regularity, there exists a function $\tilde{H}(x,z)$ such that $\mathrm{d}\tilde{H} \in \tilde{\Delta}^{\perp}$. Hence, the first claim in (B) of Lemma 2 holds.

To prove the second claim we proceed by contradiction. Suppose that at some point $(\bar{x}, \bar{z})$ one has

$$\left.\frac{\partial \tilde{H}}{\partial z}\right|_{(\bar{x},\bar{z})} = 0.$$

Then, at this point,

$$\begin{bmatrix} 0_n \\ 1 \end{bmatrix} \in \ker\{\mathrm{d}\tilde{H}\}. \tag{A.1}$$

Note now that, by regularity of Δ, there exist an integer $r \leqslant n$ and (n-dimensional) vectors $\alpha_1(x)$ to $\alpha_r(x)$ such that

$$\Delta = \mathrm{span}\{\alpha_1, \alpha_2, \ldots, \alpha_r\}. \tag{A.2}$$

This implies that, for some functions $\beta_1(x)$ to $\beta_r(x)$,

$$\tilde{\Delta} = \mathrm{span}\left\{\begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}, \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}, \ldots, \begin{bmatrix} \alpha_r \\ \beta_r \end{bmatrix}\right\}. \tag{A.3}$$

However, Eq. (A.1) implies that

$$\dim \tilde{\Delta} > \dim \Delta,$$

hence a contradiction, which completes the proof. □

To prove necessity, suppose that (B) of Lemma 2 holds. Then, by Frobenius theorem, and regularity of $\tilde{\Delta}$ we have that

$$\dim \tilde{\Delta} = n_1 \leqslant n$$

for some constant n_1. Note now that, by Eqs. (A.2) and (A.3), and regularity of Δ and $\tilde{\Delta}$, one has

$$\dim \Delta = \dim \tilde{\Delta} = n_1,$$

which completes the proof.

References

[1] G. Blankenstein, R. Ortega, A. van der Schaft, The matching conditions of controlled lagrangians and interconnection and damping passivity-based control, Int. J. Control 75 (9) (2002) 645–665.
[2] A. Bloch, N. Leonhard, J. Marsden, Mechanical feedback control systems, in: V. Blondel, E. Sontag, M. Vidyasagar, J. Willems (Eds.), Open Problems in Mathematical Systems Theory, Springer, New York, 1999, pp. 39–44.
[3] A. Bloch, N. Leonhard, J. Marsden, Controlled lagrangians and the stabilization of mechanical systems, IEEE Trans. Automat. Control 45 (12) (2000) 2253–2270.
[4] C. Byrnes, A. Isidori, J.C. Willems, Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems, IEEE Trans. Automat. Control 36 (11) (1991) 1228–1240.
[5] D. Cheng, R. Ortega, A. Astolfi, E. Panteley, Reducing the number of PDEs in interconnection and damping assignment passivity-based control, IFAC World Congress 2005, Prague, July 4–8, 2005.
[6] D. Cheng, T. Shen, T.J. Tarn, Pseudo-Hamiltonian realization and its application, Comm. Inform. Syst. 2 (2) (2002) 91–120.
[7] G. Escobar, A. van der Schaft, R. Ortega, A Hamiltonian viewpoint in the modeling of switching power converters, Automatica, 1999.
[8] K. Fujimoto, T. Sugie, Canonical transformations and stabilization of generalized Hamiltonian systems, Syst. Control Lett. 42 (3) (2001) 217–227.
[9] A. Isidori, Nonlinear Control Systems, 3rd ed., Springer, New York, 1995.
[10] R. McLachlan, G. Quispel, N. Robidoux, Geometric integration using discrete gradients, Phil. Trans. Roy. Soc. Lond. 357 (1999) 1021–1045.
[11] R. Ortega, Some applications and extensions of interconnection and damping assignment passivity-based control, Proceedings of IFAC Wokshop on Lagrangian and Hamiltonian Methods in Nonlinear Systems, Seville, Spain, April 3–5, 2003.
[12] R. Ortega, A. Loria, P.J. Nicklasson, H. Sira-Ramirez, Passivity-Based Control of Euler–Lagrange Systems, Communications and Control Engineering, Springer, Berlin, September, 1998.
[13] R. Ortega, A. van der Schaft, B. Maschke, G. Escobar, Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems, Automatica 38 (4) (2002).
[14] H. Rodriguez, R. Ortega, G. Escobar, N. Barabanov, A robustly stable output feedback saturated controller for the boost DC-to-DC converter, Syst. Control Lett. 40 (1) (2000) 1–8.
[15] R. Sepulchre, M. Jankovic, P.V. Kokotović, Constructive Nonlinear Control, Springer, Berlin, 1996.
[16] P. Tabuada, G. Pappas, From nonlinear to Hamiltonian via feedback, IEEE Trans. Automat. Control 48 (8) (2003) 1439–1442.
[17] A.J. van der Schaft, L_2-Gain and Passivity Techniques in Nonlinear Control, Springer, Berlin, 1999.
[18] Y. Wang, D. Cheng, C. Li, Y. Ge, Dissipative Hamiltonian realization and energy-based L_2 disturbance attenuation control of multimachine power systems, IEEE Trans. Automat. Control 48 (8) (2003) 1428–1433.