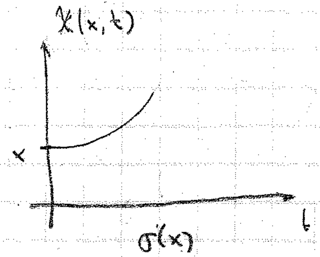
12/03

<u>Fundam. problem</u> $f: \mathbb{R}^n \to \mathbb{R}^n$, $h: \mathbb{R}^n \to \mathbb{R}^p$



Given $\dot{x} = f(x)$, $y = h(x)$

with $\chi(x,t)$ sol. with $IC = x$ at time t

From $\{h(\chi(x,s)) : 0 \le s \le t\}$ find $\hat{\chi}(t)$ s.t.

$$\lim_{t \to \sigma(x)} |\hat{\chi}(t) - \chi(x,t)| = 0$$

Take $\sigma = \infty$

<u>Observer</u> (Dyn syst) $\Psi: \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}^m$, $\phi: \mathbb{R}^m \to \mathbb{R}^n$ $(m \ge n)$

s.t. $\forall\, x_0, z_0$

$$\left\{\begin{array}{l} \dot{x} = f(x) \\ \dot{z} = \Psi(z, h(x)) \\ \hat{x} = \phi(z) \end{array}\right. \Rightarrow \lim_{t \to \infty} |\hat{x}(t) - x(t)| = 0$$

<u>Luenberger '64</u> $\boxed{\dot{x} = Fx, \ y = Hx}$ Reconstruct Tx, $T \in \mathbb{R}^{m \times n}$, $m \ge n$ $[\quad]$

<u>Fact 1</u> Given F, H & T. If we can find $A \in \mathbb{R}^{m \times m}$, $A \in \mathcal{H}$, $B \in \mathbb{R}^{m \times p}$ s.t.

$$\boxed{TF - AT = BH} \quad (\#)$$ Resonant eqs

Then, $\boxed{\dot{z} = Az + By}$ ensures $z - Tx \to 0$

<u>Idea</u> A is a contraction

$\dot{z}_1 = Az_1 + By$, $z_1(0)$
$\dot{z}_2 = Az_2 + By$, $z_2(0)$

$\Rightarrow \dot{\overline{z_1 - z_2}} = A(z_1 - z_2)$

$\Rightarrow z_1 - z_2 \to 0$

<u>Proof</u>

$$\dot{\overline{z - Tx}} = Az + \underbrace{BHx - TFx}_{-AT} = A(z - Tx)$$

<u>Remarks</u> (i) $\exists\, T^I \in \mathbb{R}^{n \times m}$ s.t. $T^I T = I_n$ then $\boxed{\hat{x} = T^I z}$

(ii) $\lambda_i(F) \ne \lambda_i(A) \Rightarrow \exists\, T$

(iii) (F, H) obs & (A, B) controll $\Rightarrow \exists\, T^I$

<u>Fact 2</u> $m = n$ & $|T| \ne 0$. Let $\dot{\hat{x}} = F\hat{x} + \underbrace{T^{-1}B}_{K}(y - H\hat{x})$ then $\hat{x} - x \to 0$

Proof $\hat{x} = T^{-1} z$, $\dot z = Az + By \Rightarrow \dot{\hat x} = T^{-1}AT\hat x + T^{-1}BHx \pm T^{-1}BH\hat x$

Now $(\ddagger) \Leftrightarrow F = T^{-1}AT + T^{-1}BH \Rightarrow \dot{\hat x} = F\hat x + T^{-1}BH(x-\hat x)$ $\nabla\nabla$

Remark $A \in \mathcal{H} \Leftrightarrow F - T^{-1}BH \in \mathcal{H}$

Nonlinear extension (KK, SCL'98 No 34, pp 241-7) Fix (A,B) controll., $A \in \mathcal{H}$, (PDE)

Fact 3 $\exists T: \mathbb{R}^n \to \mathbb{R}^m$ s.t. $\boxed{\nabla T f(x) = AT(x) + Bh(x)} \Rightarrow z - T(x) \to 0$

Proof $\dot z - \dot T = Az + Bh - \nabla T f = A(z - T(x))$, $\nabla T = \begin{bmatrix} \nabla_1 T_1 & \cdots & \nabla_m T_1 \\ \vdots & & \\ \nabla_1 T_m & \cdots & \nabla_m T_m \end{bmatrix}$ $\nabla\nabla$

$\nabla_i T_j = \frac{\partial T_j}{\partial x_i}$

Questions (i) $\exists T$ s.t. $\exists T^I$

(ii) Construction: integral form (KE, TAC'03, No 3; KXiao JOTA'02, No 3 pp 932-953)

(iii) NL $B(h(x))$ } Andrieu '05

(iv) Dyn. extension }

NL 9

Example

$\dot x_1 = x_2$, $h = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$, $m = 1$, $A = -1$, $\dot z = -z + B(x_1, x_3)$

$\dot x_2 = \sin x_1$

dyn ext $\longrightarrow$ $\dot x_3 = f_3(x_1)$

PDE: $\nabla_1 T\, x_2 + \nabla_2 T \sin x_1 + \nabla_3 T f_3 = -T + B$

Let $T = \sin x_1 + x_2 x_3$

$x_2 \cancel{\cos x_1} + x_3 \sin x_1 + x_2 f_3 = -\sin x_1 - \cancel{x_2 x_3} + B$

$\downarrow$

$-\cancel{x_3} - \cos \cancel{x_1}$

Fix $f_3 = -x_3 - \cos x_1 \Rightarrow B = (x_3 + 1)\sin x_1$

In summary

$z \to T(x) \Leftarrow$ $\boxed{\begin{aligned} \dot x_3 &= -x_3 - \cos x_1 \\ \dot z &= -z + (x_3+1)\sin x_1 \\ \hat x_2 &= \frac{1}{x_3}(z - \sin x_1) \end{aligned}}$

$\boxed{\text{An I\&I VIEWPOINT}}$ (KCA, ECC'07) $\dot y = f_1(y,x)$, $\dot x = f_2(y,x)$

$\dot \xi = \alpha(y, \xi)$, $\xi \in \mathbb{R}^q$, $q \geq n$

is an obs. $\exists\, \beta: \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^q$, $\phi_y: \mathbb{R}^n \to \mathbb{R}^q$ invertible

($\exists\, \phi_y^I: \mathbb{R}^q \to \mathbb{R}^n$ s.t. $\phi_y^I(\phi_y(x)) = x$) s.t. $\mathcal{M} = \{(y,x,\xi): \beta(y,\xi) = \phi_y(x)\} \subset \mathbb{R}^p \times \mathbb{R}^n \times \mathbb{R}^q$

satisfies (i) $\mathcal{M}$ is invariant; (ii) asymp. attractive.

Remark Tsinias SCL'90 pp 411-418

Idea drive $z = \beta(y,\xi) - \phi_y(x) \to 0 \Rightarrow \hat x = \phi_y^I(\beta(y,\xi)) \to x$

Papers

* KK, SCL'98

* KE, TAC'03 (SCL'06)

– KXiao, JOTA'02 (SCL'06)

– AmP, JOTA'06 (PhD VA, Ch 5-8)
↓ IM Example

– AsP, MCSS'06

* KCA, ECC'07 ⊃ Arcak PK, Automatica'01, pp 1923-1930
↳ mech syst

– Invariant observers, Bonnabel, PM, PR, ACC'06 (PhD Bonnabel)

– Intrinsic observers; Aghannan, PR, TAC'03, pp 936-945 (PhD Aghannan)

– Contraction obs; Slotine (PAVLOV)

– OUTPUT FEEDBACK STABILIZATION ....

Others

– Krener/Respondek, JOTA'85, pp 197-216

– Gauthier/Kupka, JOTA'94, pp 975-994

– Khalil LNCIS'99 (Henk/Fossen)

– SM (Levant)
↳ PMSM Ex
– numerical diff