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Adaptive Synchronization of Bilateral Teleoperators with Time Delay*

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Summary. In this chapter we develop schemes for synchronization of bilateral teleoperators. It is well known that a feedback interconnection of two passive systems is passive. We discuss an extension of this property to the case when there are heterogeneous, constant communication delays in the interconnection. If the interconnected systems are output strictly passive, we show that their feedback interconnection is passive independent of the constant delays. We exploit this property to achieve delay-independent output synchronization. This result is then used in the problem of bilateral teleoperation to synchronize the master/slave velocities in free motion. We also develop an architecture that guarantees state synchronization of the master/slave robots in free motion independent of the constant delay. Experimental results are presented to verify the efficacy of the state-synchronizing architecture.

15.1 Introduction

In this chapter we address the problem of state synchronization in bilateral teleoperation. We refer the reader to [1] for a detailed survey of the various schemes developed for the problem of bilateral teleoperation. Henceforth, we restrict ourselves to the discussion of passivity-based methods in bilateral teleoperation.

The passivity-based-approach developed in [2] and [3] has been the cornerstone of teleoperator control for the last two decades. Subsequent schemes, building upon the above two approaches, have been proposed in [4, 5, 6, 7, 8, 9, 10, 11, 12] among others, to provide performance improvement via position feedback, impedance matching, robustness to time-varying delays and various other objectives.

However, most passivity-based architectures, notable exceptions being [6, 9, 11], dictate transfer of velocity information between the master and the slave. Consequently, mismatch of initial conditions can result in position drift between the master and the slave robots. Therefore, development of a stable high performance bilateral teleoperator necessitates transmission of position information between the master and the slave. This has been attempted in [6, 9], where the coupling gains were delay dependent,

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* This research was partially supported by the Office of Naval Research under Grant N00014-02-1-0011, N00014-05-1-0186, and by the National Science Foundation under Grants ECS-0122412 and INT-0128656.

M. Ferre et al. (Eds.): Advances in Telerobotics, STAR 31, pp. 257–270, 2007.
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and in [11] where position information was encoded by transmitting the integral of the wave-variables [3].

The performance goals in bilateral teleoperation are:

G1: Demonstrate ultimate boundedness of master/slave trajectories, in both free and constrained motion, independent of the time delay.
G2: Synchronize the configuration variables of the master and the slave robot when the slave is allowed to move freely.
G3: Ensure that the force tracking error between the human operator force, and the environmental force experienced by slave on hard contact with the environment, is ultimately bounded.

In this chapter, we concentrate on the first and the second goal in free motion. We study feedback interconnections of passive systems with delays in Sec. 15.2 and develop an output synchronizing control law. The proposed control law is then used for constructing velocity and state-synchronizing algorithms for bilateral teleoperators in Sec. 15.3. Experimental results for the state-synchronizing architecture are presented in Sec. 15.4. Finally in Sec. 15.5, we summarize the results and discuss some future directions of work.

15.2 Passivity

The concept of passivity is one of the most physically appealing concepts of system theory [14] and, as it is based on input-output behavior of an system, is equally applicable to both linear and nonlinear systems. Most of the ideas presented in this section are from [15]. Consider a dynamical system represented by the state space model

$$\dot{x} = f(x,u) \tag{15.1}$$

$$y = h(x,u) \tag{15.2}$$

where f: $R^n \times R^p \to R^n$ is locally Lipschitz, h: $R^n \times R^p \to R^p$ is continuous, $f(0,0) = 0$, $h(0,0) = 0$ and the system has the same number of inputs and outputs.

Definition 1. *The dynamical system (15.1)-(15.2) is said to be passive if there exists a continuously differentiable non-negative definite scalar function $V(x)$: $R^n \to R$ (called the storage function) such that*

$$u^T y \geq \dot{V}, \qquad \forall (x,u) \in R^n \times R^p$$

Moreover, the system is said to be

- *strictly passive if $u^T y \geq \dot{V} + S(x)$ for some positive definite function $S(x)$*
- *lossless if $u^T y = \dot{V}$*
- *input strictly passive if $u^T y \geq \dot{V} + u^T \psi(u)$, where $u^T \psi(u) > 0$ for some function ψ and $\forall u \neq 0$*
- *output strictly passive if $u^T y \geq \dot{V} + y^T \rho(y)$, where $y^T \rho(y) > 0$ for some function ρ and $\forall y \neq 0$*

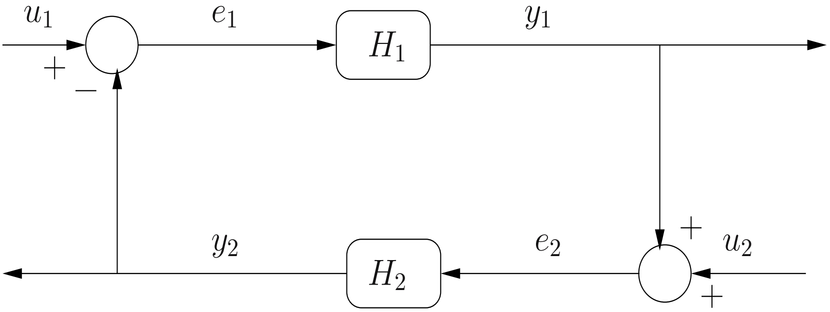

Fig. 15.1. A feedback interconnection

15.2.1 Feedback Interconnection of Passive Systems

At this point we recall a fundamental property of interconnection of passive systems. Assuming that the interconnection is well-posed (see [15]), consider the feedback connection of Fig. 15.1, where each of the feedback components is a time-invariant dynamical system represented by the state model

$$\begin{aligned} \dot{x}_i &= f_i(x_i, e_i) \\ y_i &= h_i(x_i, e_i) \end{aligned} \tag{15.3}$$

The closed-loop (composed of the components H_1 and H_2) then takes the form

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x, u) \end{aligned} \tag{15.4}$$

where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$, $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$. A fundamental result on the feedback interconnection of passive system is the following

Theorem 1. *The feedback connection of two passive systems is passive.*

We refer the reader to [15] for a proof of this result. A similar property follows when the two systems are output strictly passive with

$$e_i^T y_i \geq \dot{V}_i + \delta_i y_i^T y_i \;\; \delta_i > 0 \;\; i = 1, 2 \tag{15.5}$$

In this case it is possible to show that

$$u^T y \geq \dot{V} + \delta y^T y$$

where $V(x) = V_1(x_1) + V_2(x_2)$ and $\delta = \min\{\delta_1, \delta_2\}$.

The effect of delays on linear interconnections of passive systems has been studied in [16] and in the context of large-scale systems in [17]. We next study the extension of the aforementioned interconnection properties to the case when the outputs are delayed. The subsequent result is closely related to the work in [16] where the notion of quasi-dominance was used.

Consider the scenario illustrated in Fig. 15.2 where the delays in the forward and the feedback loop are assumed to be constant and heterogeneous. Our main result in this section follows

Theorem 2. *Consider two output strictly passive systems described by (15.1), (15.2), (15.5) and Fig. 15.2 along with the constant communication delays. Then the feedback interconnection is*

1. *Passive if* $\delta_1 = \delta_2 = 1$.
2. *Strictly output passive if* $\delta_1 = \delta_2 > 1$

Proof. Consider the case when $\delta_1 = \delta_2 = 1$. Then,

$$e_i^T y_i \geq \dot{V}_i + y_i^T y_i \;\; i = 1, 2 \tag{15.6}$$

Using the feedback connection of Fig. 15.2, we have

$$\begin{aligned}
&\dot{V}_1 + \dot{V}_2 = e_1^T y_1 + e_2^T y_2 - y_1^T y_1 - y_2^T y_2 \\
&= (u_1 - y_2(t - T_2))^T y_1 + (u_2 + y_1(t - T_1))^T y_2 - y_1^T y_1 - y_2^T y_2 \\
&= u_1^T y_1 + u_2^T y_2 - y_2(t - T_2)^T y_1 + y_1(t - T_1)^T y_2 - y_1^T y_1 - y_2^T y_2 \\
&\leq u_1^T y_1 + u_2^T y_2 + \frac{1}{2}(y_2(t - T_2)^T y_2(t - T_2) + y_1^T y_1) \\
&+\frac{1}{2}(y_1(t - T_1)^T y_1(t - T_1) + y_2^T y_2) - y_1^T y_1 - y_2^T y_2 \\
&\leq u_1^T y_1 + u_2^T y_2 - \frac{1}{2}(y_2^T y_2 - y_2(t - T_2)^T y_2(t - T_2)) \\
&-\frac{1}{2}(y_1^T y_1 - y_1(t - T_1)^T y_1(t - T_1)) \\
&\leq u_1^T y_1 + u_2^T y_2 - \frac{1}{2}\frac{d}{dt}\Big(\int_{t-T_1}^{t} y_1^T y_1 dk + \int_{t-T_2}^{t} y_2^T y_2 dk\Big) \\
&\Rightarrow \dot{V}_1 + \dot{V}_2 + \dot{V}_{channel} \leq u_1^T y_1 + u_2^T y_2 = u^T y
\end{aligned}$$

where

$$V_{channel} = \frac{1}{2}\frac{d}{dt}\Big(\int_{t-T_1}^{t} y_1^T y_1 dk + \int_{t-T_2}^{t} y_2^T y_2 dk\Big)$$

Therefore, the feedback interconnection is passive with $V(x) = V_1(x_1) + V_2(x_2) + V_{channel}$ as the storage function.

Inorder to prove the second claim, it follows from the above calculations that when $\delta_1 = \delta_2 > 1$,

$$\begin{aligned}
\dot{V}_1 + \dot{V}_2 + \dot{V}_{channel} &\leq u_1^T y_1 + u_2^T y_2 - (\delta_1 - 1)y_1^T y_1 - (\delta_2 - 1)y_2^T y_2 \\
&\leq u^T y - \delta_c y^T y
\end{aligned}$$

where $\delta_c = \min\{(\delta_1 - 1), (\delta_2 - 1)\}$. Hence the feedback interconnection is output strictly passive when $\delta_1 = \delta_2 > 1$.

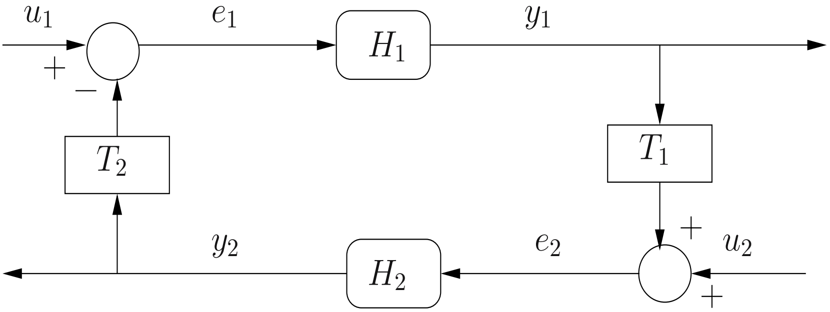

Fig. 15.2. A feedback interconnection with delays

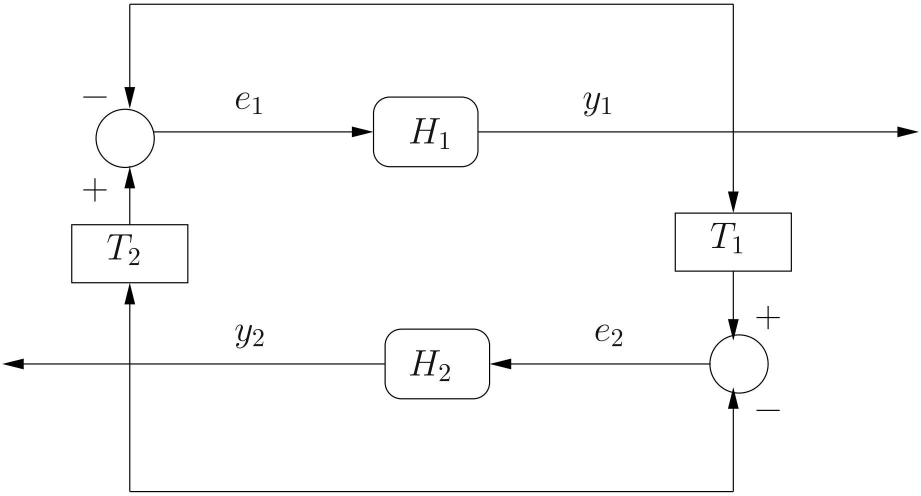

Fig. 15.3. An output synchronizing feedback interconnection

It is well known [14] that output strict passivity can be induced in a passive system with the choice of the control input $u_i = -y_i$. Using this fact and Theorem 2, we propose an output synchronizing control law (see related work in [18, 19]) for the coupled passive systems. The control schematic is illustrated in Fig. 15.3.

Theorem 3. *Consider two passive systems described by (15.1), (15.2) and Fig. 15.3 along with the constant communication delays. Then,*

1. *The signals* $(y_2(t-T_2)-y_1), (y_1(t-T_1)-y_2) \in \mathcal{L}_2[0,\infty)$.
2. *If* $\dot{y}_1, \dot{y}_2 \in \mathcal{L}_\infty[0,\infty)$, *the agents output synchronize in the sense that*

$$\lim_{t\rightarrow\infty} |y_2(t-T_2)-y_1| = \lim_{t\rightarrow\infty} |y_1(t-T_1)-y_2| = 0$$

Proof. Passivity of the individual systems implies that

$$\dot{V}_1 + \dot{V}_2 \leq e_1^T y_1 + e_2^T y_2$$

Observing Fig. 15.3, the above differential inequality can be rewritten as

$$\begin{aligned}
&\dot{V}_1 + \dot{V}_2 \leq (y_2(t-T_2) - y_1)^T y_1 + (y_1(t-T_1) - y_2)^T y_2 \\
&\leq y_2(t-T_2)^T y_1 + y_1(t-T_1)^T y_2 - y_1^T y_1 - y_2^T y_2 \\
&\leq \frac{1}{2}\Big(-(y_2(t-T_2) - y_1)^T (y_2(t-T_2) - y_1) - (y_1(t-T_1) - y_2)^T (y_1(t-T_1) - y_2) \\
&+ y_2(t-T_2)^T y_2(t-T_2) + y_1(t-T_1)^T y_1(t-T_1) - y_1^T y_1 - y_2^T y_2 \Big) \\
&\leq \frac{1}{2}\Big(-(y_2(t-T_2) - y_1)^T (y_2(t-T_2) - y_1) - (y_1(t-T_1) - y_2)^T (y_1(t-T_1) - y_2) \Big) \\
&-\dot{V}_{channel} \\
&\Rightarrow \dot{V}_1 + \dot{V}_2 + \dot{V}_{channel} \leq \frac{1}{2}\Big(-(y_2(t-T_2) - y_1)^T (y_2(t-T_2) - y_1) \\
&\qquad\qquad -(y_1(t-T_1) - y_2)^T (y_1(t-T_1) - y_2) \Big)
\end{aligned}$$

Integrating the above equation, it is easy to see that the signals $(y_2(t-T_2)-y_1)$, $(y_1(t-T_1) - y_2) \in \mathcal{L}_2[0,\infty)$, and hence the first claim holds.

To prove the second claim, it is important to note that a $\mathcal{L}_2[0,\infty)$ signal with a bounded derivative converges asymptotically to the origin (pp. 116, [13]). If $\dot{y}_1, \dot{y}_2 \in \mathcal{L}_\infty[0,\infty)$, the derivatives of the signals $(y_2(t-T_2)-y_1)$, $(y_1(t-T_1)-y_2)$ are bounded. The second claim follows from the above discussion.

15.3 Application to Bilateral Teleoperation

We next apply the above ideas in the context of bilateral teleoperation. Neglecting friction or other disturbances, the Euler-Lagrange equations of motion for n-link master and slave robots are given as [20]

$$\begin{aligned}
M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) &= F_h + \tau_m \\
M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_s - F_e
\end{aligned} \tag{15.7}$$

where q_m, q_s are the $n \times 1$ vectors of joint displacements, $\dot{q}_m$, $\dot{q}_s$ are the $n \times 1$ vectors of joint velocities, τ_m, τ_s are the $n \times 1$ vectors of applied torques, $M(q)$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(q, \dot{q})$ is the $n \times n$ matrix of Centripetal and Coriolis torques and $g(q) = \frac{\partial P}{\partial q}$ is the gradient of the gravitational potential energy $P(q)$. Also, F_h is the human operator force and F_e is the environmental force acting on the slave robot when it contacts the environment. We list here fundamental properties of the master and slave robots that we use in the subsequent analysis

- **Property 1:** The inertia matrix $M(q)$ is symmetric positive definite and there exists a positive constant m such that $mI \leq M(q)$.

- **Property 2:** The Lagrangian dynamics are linearly parameterizable which gives us that
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y_1(q,\dot{q},\ddot{q})\theta$$
where θ is a constant p-dimensional vector of inertia parameters (such as link masses, moments of inertia etc.) and Y_1 is an $n \times p$ matrix of known functions of the generalized coordinates (q_m, q_s) and their higher derivatives.
- **Property 3:** Under an appropriate definition of the matrix C, the matrix $\dot{M}$ - $2C$ is skew symmetric

We study synchronization of bilateral teleoperators in the free motion scenario, and therefore develop synchronization schemes when $F_h, F_e = 0$. These forces can be easily included in the subsequent analysis provided they satisfy the passivity property with respect to the appropriate outputs. At the end of the subsequent synchronization schemes, we will point the reader to appropriate references where the human operator and environment forces have been taken into consideration and briefly discuss the schemes.

15.3.1 Master-Slave Velocity Synchronization

We first develop an algorithm that guarantees velocity synchronization for the bilateral teleoperator. The master/slave robots are said to velocity synchronize if

$$\lim_{t\to\infty} |\dot{q}_m(t-T_1) - \dot{q}_s| = 0 \ ; \ \lim_{t\to\infty} |\dot{q}_s(t-T_2) - \dot{q}_m| = 0 \qquad (15.8)$$

Let the preliminary master/slave torques be given as

$$\tau_m = \bar{\tau}_m + g_m \ ; \ \tau_s = \bar{\tau}_s + g_s \qquad (15.9)$$

where $\bar{\tau}_m, \bar{\tau}_s$ are the output-synchronizing torques that will be designed subsequently. The master/slave dynamics are now given as

$$\begin{aligned} M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m &= \bar{\tau}_m \\ M_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s &= \bar{\tau}_s \end{aligned} \qquad (15.10)$$

The skew-symmetry property (*Property 3*) implies passivity of (15.10) from input $\bar{\tau}_i$ to output $\dot{q}_i$ [20]. This can be demonstrated by choosing

$$V_i(q_i,\dot{q}_i) = \frac{1}{2}\dot{q}_i^T M_i(q_i)\dot{q}_i \quad i = m, s \qquad (15.11)$$

as the positive semi-definite storage function for the system. Differentiating V_i along trajectories of (15.10) we have

$$\begin{aligned} \dot{V}_i &= \dot{q}_i^T(-C_i\dot{q}_i + \bar{\tau}_i) + \frac{1}{2}\dot{q}_i^T\dot{M}_i\dot{q}_i \\ &= \dot{q}_i^T\bar{\tau}_i \qquad \text{(Using Property 3)} \end{aligned}$$

and hence the dynamics are passive with $(\bar{\tau}_i, \dot{q}_i)$ as the input-output pair.

In context of Fig. 15.3, the dynamical system (15.10) can be written as

$$H_1 \left\{ \begin{array}{l} M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m = \bar{\tau}_m \\ y_m = \dot{q}_m \end{array} \right. \tag{15.12}$$

$$H_2 \left\{ \begin{array}{l} M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s = \bar{\tau}_s \\ y_s = \dot{q}_s \end{array} \right. \tag{15.13}$$

Following Theorem 3, the velocity-synchronizing torques are given as

$$\bar{\tau}_m = \dot{q}_s(t - T_2) - \dot{q}_m \ \ ; \ \ \bar{\tau}_s = \dot{q}_m(t - T_1) - \dot{q}_s \tag{15.14}$$

Velocity synchronization for the teleoperator system can be demonstrated (see Theorem 3) by choosing

$$V = \frac{1}{2}\Big(\dot{q}_m^T M_m(q_m)\dot{q}_m + \dot{q}_s^T M_s(q_s)\dot{q}_s + \int_{t-T_1}^{t} \dot{q}_m^T \dot{q}_m dk + \int_{t-T_2}^{t} \dot{q}_s^T \dot{q}_s dk\Big)$$

as the positive semi-definite storage function for the teleoperator system. Using Theorem 3,

$$\dot{V} = -\frac{1}{2}\Big((\dot{q}_s(t-T_2)-\dot{q}_m)^T(\dot{q}_s(t-T_2)-\dot{q}_m)+(\dot{q}_m(t-T_1)-\dot{q}_s)^T(\dot{q}_m(t-T_1)-\dot{q}_s)\Big)$$

Therefore, the signals $(\dot{q}_s(t - T_2) - \dot{q}_m), (\dot{q}_m(t - T_1) - \dot{q}_s) \in \mathcal{L}_2[0, \infty)$. Additionally, as $\dot{V}$ is negative semi-definite, $\dot{q}_m, \dot{q}_s \in \mathcal{L}_\infty[0, \infty)$. Noting that $||C_i(q_i, \dot{q}_i)|| \leq k_c||\dot{q}_i||$, where $k_c > 0$ and $|| \cdot ||$ denotes the induced norm for a matrix and the Euclidean norm for a vector, from the system dynamics (15.12), (15.13), $\ddot{q}_m, \ddot{q}_s \in \mathcal{L}_\infty[0, \infty)$. Invoking the second claim of Theorem 3,

$$\lim_{t\to\infty} |\dot{q}_s(t - T_2) - \dot{q}_m| = \lim_{t\to\infty} |\dot{q}_m(t - T_1) - \dot{q}_s| = 0$$

Remark 1. In addition to velocity synchronization, it is possible to guarantee that the master/slave velocities converge to the origin. This can be achieved by choosing the coupling controls as

$$\begin{array}{l} \bar{\tau}_m = (\dot{q}_s(t - T_2) - \dot{q}_m) - \delta_d\dot{q}_m \\ \bar{\tau}_s = (\dot{q}_m(t - T_1) - \dot{q}_s) - \delta_d\dot{q}_s \quad\quad \delta_d > 0 \end{array} \tag{15.15}$$

Remark 2. The human operator and the environment forces can be included in the above analysis provided the input-output pairs $(-F_h, \dot{q}_m), (F_e, \dot{q}_s)$ are passive [6, 9].

In context of goal G3, using the coupling control (15.15), and on being manipulated by a passive human operator, good force reflection will not be generated on contact with a passive environment. This is due to the asymptotic convergence of the torques $\bar{\tau}_m, \bar{\tau}_s$ to the origin. To overcome this difficulty, and also achieve position synchronization, transmission of position information was advocated in [6, 9]. The proposed schemes resulted in static force reflection on contact with the environment. However, to ensure stability, the position gains were required to be delay-dependent. We next propose a scheme that guarantees delay-independent state synchronization of the master/slave robots in free-motion.

15.3.2 Master-Slave State Synchronization

In free motion, the bilateral teleoperator is said to state synchronize if

$$\begin{aligned} \lim_{t\to\infty} |q_m(t-T_1) - q_s| &= \lim_{t\to\infty} |\dot{q}_m(t-T_1) - \dot{q}_s| = 0 \\ \lim_{t\to\infty} |q_s(t-T_2) - q_m| &= \lim_{t\to\infty} |\dot{q}_s(t-T_2) - \dot{q}_m| = 0 \end{aligned} \tag{15.16}$$

It is well known that passivity can be induced in a Lagrangian system with a different choice of output and a preliminary feedback control. This is know as *Feedback Passivation* [14]. We use this idea to achieve passivity with respect to an output from which both the position and velocity information are available. To this end, choose a preliminary control input for the master and slave robots as

$$\begin{aligned} \tau_m &= \bar{\tau}_m - \hat{M}_m(q_m)\lambda\dot{q}_m - \hat{C}_m(q_m,\dot{q}_m)\lambda q_m + \hat{g}_m(q_m) \\ \tau_s &= \bar{\tau}_s - \hat{M}_s(q_s)\lambda\dot{q}_s - \hat{C}_s(q_s,\dot{q}_s)\lambda q_s + \hat{g}_s(q_s) \end{aligned} \tag{15.17}$$

where $\hat{M}_i, \hat{C}_i, \hat{g}_i \quad i = m, s$ are the estimates of the respective matrices available at that instant, λ is a constant positive definite matrix, and $\bar{\tau}_m$, $\bar{\tau}_s$ are the synchronizing torques that will be described below. As the dynamics are linearly parameterizable ($Property\,2$), the motor torques can also be written as

$$\tau_m = \bar{\tau}_m - Y_m(q_m,\dot{q}_m)\hat{\theta}_m \;\;;\;\; \tau_s = \bar{\tau}_s - Y_s(q_s,\dot{q}_s)\hat{\theta}_s$$

where Y_m, Y_s are known functions of the generalized coordinates, and $\hat{\theta}_m$, $\hat{\theta}_s$ are the time-varying estimates of the manipulators' actual constant p dimensional inertial parameters given by θ_m, θ_s respectively. The master and slave dynamics (15.7) (with F_h, F_e=0) reduce to

$$\begin{aligned} &\dot{q}_m = -\lambda q_m + r_m \\ &M_m\dot{r}_m + C_m r_m = Y_m\tilde{\theta}_m + \bar{\tau}_m \\ &\dot{q}_s = -\lambda q_s + r_s \\ &M_s\dot{r}_s + C_s r_s = Y_s\tilde{\theta}_s + \bar{\tau}_s \end{aligned} \tag{15.18}$$

where r_m, r_s are the new outputs of the master and slave robots respectively, and $\tilde{\theta}_m = \theta_m - \hat{\theta}_m$, $\tilde{\theta}_s = \theta_s - \hat{\theta}_s$ are the estimation errors. Let the time-varying estimates of the uncertain parameters evolve as

$$\dot{\hat{\theta}}_m = \Gamma_m Y_m^T r_m \;\;;\;\; \dot{\hat{\theta}}_s = \Gamma_s Y_s^T r_s \tag{15.19}$$

where Γ_m and Γ_s are constant positive definite matrices. Therefore, in context of Fig. 15.3, the bilateral teleoperator system can be viewed as an interconnection of two systems given by

$$H_1 \left\{ \begin{aligned} &\dot{q}_m = -\lambda q_m + r_m \\ &M_m\dot{r}_m + C_m r_m = Y_m\tilde{\theta}_m + \bar{\tau}_m \\ &\dot{\hat{\theta}}_m = \Gamma_m Y_m^T r_m \\ &y_m = r_m \end{aligned} \right. \tag{15.20}$$

$$H_2 \begin{cases} \dot{q}_s = -\lambda q_s + r_s \\ M_s \dot{r}_s + C_s r_s = Y_s \tilde{\theta}_s + \bar{\tau}_s \\ \dot{\hat{\theta}}_s = \Gamma_s Y_s^T r_s \\ y_s = r_s \end{cases} \tag{15.21}$$

Consider a positive semi-definite storage function for the master/slave robot as

$$V_i(q_i, r_i, \tilde{\theta}_i) = \frac{1}{2}\Big(r_i^T M_i(q_i) r_i + \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i\Big) \ \ i = m, s \tag{15.22}$$

The derivative of this storage function, along trajectories of H_i, is given as

$$\begin{aligned} \dot{V}_i &= r_i^T(-C_i r_i + Y_i \tilde{\theta}_i + \bar{\tau}_i) + \frac{1}{2} r_i^T \dot{M}_i r_i - \tilde{\theta}_i^T Y_i^T r_i \\ &= r_i^T \bar{\tau}_i \end{aligned}$$

Thus, H_1, H_2 are passive with $(\bar{\tau}_m, r_m), (\bar{\tau}_s, r_s)$ as the input-output pairs respectively.

To state synchronize the master/slave robots, the synchronizing torques for the master and slave robots are given as

$$\bar{\tau}_s = (r_m(t - T_1) - r_s) \ \ ; \ \ \bar{\tau}_m = (r_s(t - T_2) - r_m) \tag{15.23}$$

It is to be noted that

$$\begin{aligned} r_m(t - T_1) - r_s &= (\dot{q}_m(t - T_1) + \lambda q_m(t - T_1)) - (\dot{q}_s + \lambda q_s) \\ &= \dot{e}_m + \lambda e_m \end{aligned} \tag{15.24}$$

where $e_m = q_m(t - T_1) - q_s$. Similarly $r_s(t - T_2) - r_m = \dot{e}_s + \lambda e_s$ where $e_s = q_s(t - T_2) - q_m$. (15.24) represents an exponentially stable linear system with input $r_m(t - T_1) - r_s$. It follows that if $(r_m(t - T_1) - r_s), (r_s(t - T_2) - r_m)$ are $\mathcal{L}_2[0, \infty)$ signals that converge asymptotically to zero, then

$$\lim_{t \to \infty} |e_m| = \lim_{t \to \infty} |e_s| = \lim_{t \to \infty} |\dot{e}_m| = \lim_{t \to \infty} |\dot{e}_s| = 0$$

from which state synchronization follows immediately.

Using (15.22), the positive semi-definite storage function for the bilateral teleoperator described by (15.20), (15.21), (15.23) and Fig. 15.3 is given as

$$V = V_m + V_s + \frac{1}{2}\Big(\int_{t-T_1}^{t} r_m^T r_m dk + \int_{t-T_2}^{t} r_s^T r_s dk\Big)$$

From Theorem 3, the derivative of V along trajectories of the system is given as

$$\dot{V} = -\frac{1}{2}\Big((r_s(t-T_2)-r_m)^T(r_s(t-T_2)-r_m)+(r_m(t-T_1)-r_s)^T(r_m(t-T_1)-r_s)\Big)$$

and hence, the signals $(r_m(t - T_1) - r_s), (r_s(t - T_2) - r_m) \in \mathcal{L}_2[0, \infty)$. As $\dot{V}$ is negative semi-definite, $r_m, r_s, \tilde{\theta}_s, \tilde{\theta}_m \in \mathcal{L}_\infty[0, \infty)$. Consequently, from (15.20), (15.21), $\dot{q}_m, q_m, \dot{q}_s, q_s \in \mathcal{L}_\infty[0, \infty)$ and therefore, the derivatives $\dot{r}_m, \dot{r}_s \in \mathcal{L}_\infty[0, \infty)$. The second claim of Theorem 3 then gives us that

$$\lim_{t \to \infty} |r_s(t - T_2) - r_m| = \lim_{t \to \infty} |r_m(t - T_1) - r_s| = 0$$

State synchronization follows from the earlier discussion.

Remark 3. If the human-operator force is given as $F_h = -K_h\dot{q}_m$, then it is possible to show that in addition to state-synchronization, the master/slave velocities go the origin [21].

Remark 4. If the human operator and the environment models are given as

$$F_h = \alpha_o - \alpha_m r_m \quad ; \quad F_e = \alpha_s r_s \tag{15.25}$$

where $\alpha_o, \alpha_m, \alpha_s$ are bounded positive constants, then the trajectories are ultimately bounded independent of the constant delay [21].

15.4 Experiments

In this section, we test the proposed state synchronization scheme under time-varying delays and packet losses. The experiments were performed on two direct-drive, planar, two-degree-of freedom nonlinear robots exchanging information across a stochastic Internet model. Force sensors, located on the end-effectors, measure the force exerted by the operator/environment. The controllers and the Internet model were implemented using Wincon 3.3, which is a Windows application used for running real-time Simulink models.

In the experiments, time-varying delays from the master to the slave and vice versa, were fluctuating between (0.448, 0.544)s. The mean delay was 0.48s with a standard deviation of 0.022s. The packet loss rate in the experiments ranged from 45-55 percent. In the free motion experiment, as seen in Fig. 15.4, the tracking performance is good in the face of time-varying delays and packet losses. The steady state errors were 0.0853 rad and 0.1125 rad for the first and second link respectively.

In the next experiment, the motion of the slave (the second link) is constrained by an aluminium wall, approximately during the (8,17)s of motion. The trajectories of the

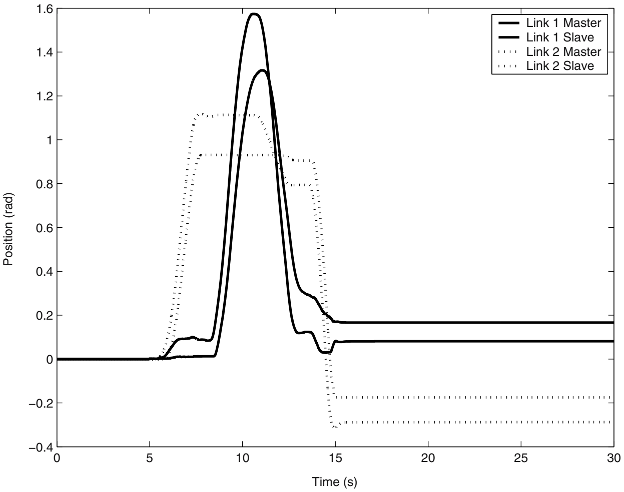

Fig. 15.4. Master and slave trajectories during free motion

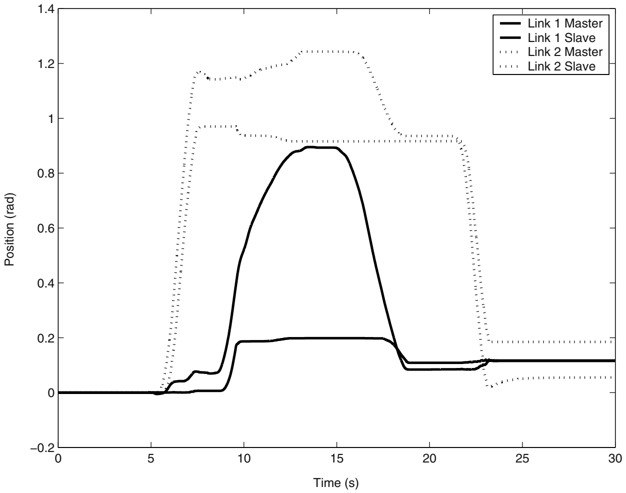

Fig. 15.5. Master and slave trajectories in the constrained motion experiment

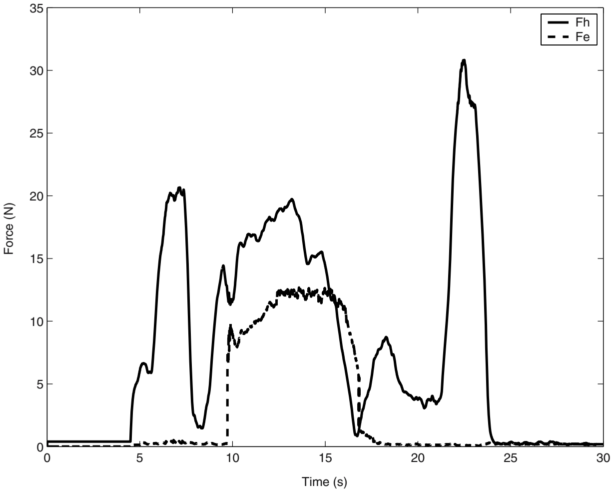

Fig. 15.6. Human operator and environment force in the constrained motion experiment

master/slave robots are shown in Fig. 15.5. The human operator/environment forces at the end effectors, are plotted in Fig. 15.6, and thus the proposed algorithm provides reasonable force tracking on contact with the environment.

15.5 Conclusions and Future Work

In this chapter we developed synchronization algorithms for bilateral teleoperators. The problem of passivity in feedback interconnection of passive systems with delays was studied. If the individual systems are output strictly passive, it was shown that their feedback interconnection is passive independent of the constant delays. This property was then exploited to output synchronize the individual systems independent of the constant delays.

The synchronization result was first applied to the problem of bilateral teleoperation to synchronize the master/slave velocities in free motion. To improve tracking performance, a second architecture was also proposed that guaranteed state synchronization of the master/slave robots in free motion independent of the constant delay. Experimental results were also presented to test the state-synchronizing architecture.

There is considerable work still to be done. The realistic case of time-varying delays needs to be addressed. The approach in [4] is likely to maintain passivity of the state-synchronizing teleoperator, provided the rate of change of the delay is bounded. However, state synchronization with time-varying delays is an open problem. The problem of static force reflection in the state-synchronizing architecture, with a non-passive human operator is also interesting. Finally, teleoperating teams of robots, rather than a single slave robot, will open up new challenges in the field.

Acknowledgements. The authors would like to thank Erick. J. Rodríguez-Seda for providing the experimental data.

References

1. P.F. Hokayem and M.W. Spong. Bilateral teleoperation: an historical survey. *Automatica*, 42(12):2035–2057, 2006.
2. R.J. Anderson and M.W. Spong. Bilateral control of teleoperators with time delay. *IEEE Transactions on Automatic Control*, 34(5):494–501, 1989.
3. G. Niemeyer and J.J.E. Slotine. Stable adaptive teleoperation. *IEEE Journal of Oceanic Engineering*, 16(1):152–162, 1991.
4. N. Chopra, M.W. Spong, S. Hirche, and M. Buss. Bilateral teleoperation over the internet: the time varying delay problem. In *American Control Conference*, pages 155–160, 2003.
5. N. Chopra, M.W. Spong, and R. Lozano. Adaptive coordination control of bilateral teleoperators with time delay. In *IEEE Conf. on Decision and Control*, pages 4540–4547, 2004.
6. N. Chopra, M.W. Spong, R. Ortega, and N.E. Barabanov. On tracking performance in bilateral teleoperation. *IEEE Transactions on Robotics*, 22(4):861–866, 2006.
7. S. Munir. *Internet-based tele-operation*. PhD thesis, Department of Mechanical Engineering, Georgia Institute of Technology, 2001.
8. K. Kosuge, H. Murayama, and K. Takeo. Bilateral feedback control of telemanipulators via computer network. In *IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 1380–1385, 1996.
9. D.J. Lee and M.W. Spong. Passive bilateral control of teleoperators under constant time delay. *IEEE Transactions on Robotics*, 22(2):269–281, 2006.
10. R. Lozano, N. Chopra, and M.W. Spong. Passivation of force reflecting bilateral teleoperation with time varying delay. In *Mechatronics'02*, Entschede, Netherlands, 2002.
11. G. Niemeyer and J.J.E. Slotine. Towards force-reflecting teleoperation over the internet. In *IEEE Conference on Robotics and Automation*, pages 1909–1915, 1998.
12. S. Stramigioli, A.V.D. Schaft, B. Maschke, and C. Melchiorri. Geometric scattering in robotic telemanipulation. *IEEE Transactions on Robotics and Automation*, 18(4):588–596, 2002.

13. R. Lozano, B. Brogliato, O. Egeland, and B. Maschke. *Dissipative Systems Analysis and Control: Theory and Applications*. Springer-Verlag, London, 2000.
14. R. Sepulchre, M. Janković, and P. Kokotović. *Constructive Nonlinear Control*. Springer-Verlag, London, 1997.
15. H.K. Khalil. Nonlinear systems. In *Upper Saddle River*. Prentice Hall, New Jersey, 2002.
16. P.J. Moylan. A connective stability result for interconnected passive systems. *IEEE Transactions on Automatic Control*, 25:812–813, 1980.
17. R.M. Lewis and B.D.O. Anderson. Necessary and sufficient conditions for delay-independent stability of linear autonomous systems. *IEEE Transactions on Automatic Control*, 25:735–739, 1980.
18. A.Y. Pogromsky. Passivity based design of synchronizing systems. *International Journal of Bifurcation and Chaos*, 8:295–319, 1998.
19. N. Chopra and M.W. Spong. Passivity-based control of multi-agent systems. In Sadao Kawamura and Mikhail Svinin, editors, *Advances in Robot Control: From Everyday Physics to Human-Like Movements*, pages 107–134. Springer Verlag, 2006.
20. M.W. Spong, S. Hutchinson, and M. Vidyasagar. *Robot Modeling and Control*. John Wiley & Sons, Inc., New York, 2006.
21. N. Chopra, M.W. Spong, and R. Lozano. Synchronization of bilateral teleoperators with time delay. Submitted, 2006.