Nonlinear Excitation Control for Transient Stability of Multi-Machine Power Systems using Structure-Preserving Models^{*}

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Abstract: The approach adopted in this paper for the problem of transient stability of multi-machine power systems sees the entire network as the (structure-preserving) interconnection of the network components, described by well known models. These *structure-preserving models* preserve the identity of the network components and allow for a more realistic treatment of the loads. Our main contribution is the *explicit computation* of a control law which provides transient stabilization, *i.e.*, prevents the system from loss of synchronism after a large perturbation. The theory is illustrated by means of simulation on a 3-machine power system.

Keywords: Nonlinear control, power system, differential and algebraic system, stability.

1. INTRODUCTION

Classical research on transient stabilization of power systems has relied on the use of aggregated *reduced network* models that represent the system as an n -port described by a set of ordinary differential equations. Several excitation controllers that establish Lyapunov stability of the desired equilibrium of these models have been reported. These nonlinear controller design techniques include feedback linearization Wang (1993), damping injection, as well as, the more general, interconnection and damping assignment passivity-based control, see Ortega (2005).

Aggregated models erase the identity of the network components and impose an unrealistic treatment of the loads. In this paper, we abandon the aggregated n -port view of the network and consider the more natural and widely popular structure-preserving models (SPM), first proposed in Bergen et al. (1981). Since these models consist of differential algebraic equations (DAE) they require the development of some suitably tailored tools for controller synthesis and stability analysis. Another original feature of the present work is that we do not aim at Lyapunov stability, but establish instead a global convergence result. In Giusto (2006) SPM were used to identify—in terms of feasibility of a LMI—a class of power systems with nonlinear (so-called ZIP) loads and leaky lines for which a *linear time-invariant* controller renders the overall *linearized* system dissipative with a (locally) positive definite storage function, thus ensuring stability of the desired equilibrium. Unfortunately, a full-fledged *nonlinear* analysis of the problem was not possible due to the difficulty in handling the complicated interdependence of the variables appearing in the algebraic constraints of the DAEs. The

Lyapunov function in that paper is obtained by adding a quadratic term in the rotor angle to the classical energy function of Varaiya (1985). This quadratic term is needed to compensate for a linear term (in rotor angle) appearing in the energy function of Varaiya (1985) and render the new storage function positive definite. To obtain our global convergence result we observed in Dib (2008) that removing the linear term from the energy function of Varaiya (1985) and increasing the quadratic term in bus voltages yields a function whose time derivative can be *arbitrarily assigned* with a globally defined static state feedback. Furthermore, although this new function is not positive definite, it is *bounded from below* and has some suitable radial unboundedness properties—features that are essential to establish boundedness of trajectories. However due to the complexity of the calculations, the energy function used describes only the synchronous machines.

Goal of this work is to show that, by exploiting the results in Dib (2008), it is possible to construct a globally convergent controller that renders attractive the ball centered on the stable equilibrium point, with an energy function that describes the entire electric network. The only critical assumption required to establish this result is that the loads are constant impedances, see Dib (2009).

It is worth mentioning that the transient stabilization problem (in the sense of the definition given in Willems (1974)) does not coincide with the problem of stabilization of power system at the *desired* operating point. From a practical point of view the solution of the latter problem is preferable but its realization just after some fault can cause the large control efforts acting on the system. Therefore, one can use the controller which provides transient stabilization, *i.e.*, prevents the system from loss of synchronism, as an intermediate control law and then solve the problem

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of power system stabilization at the desired operating point which better fits the current electric power demand and supply.

The structure of the paper is as follows. Section 2 presents the mathematical model of the various elements comprising the power system. Then, we formulate the control problem in Section 3. Section 4 contains our main “global” convergence result. Section 5 includes the application of the proposed technique to a classical example. We wrap up the paper with some concluding remarks in Section 6.

Caveat Due to space limitations, we have omitted the proofs of the Lemmas that can be found in Dib (2008).

Notation All vectors in the paper are *column* vectors, even the gradient of a scalar function: $\nabla_x = \frac{\partial}{\partial x}$. For any function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, we define $\nabla_{z_j} f(z) := \frac{\partial f}{\partial z_j}(z)$. To simplify notation we introduce the sets

$\mathbb{M}^n := \mathbb{S}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{S}^n \times \mathbb{R}_{>0}^n$, $n \in \bar{n} := \{1, \dots, n\}$, where $\mathbb{R}_{>0}^n := \{x \in \mathbb{R}^n \mid x_j > 0\}$ and $\mathbb{S}$ is the unitary circle.

2. STRUCTURE-PRESERVING MODELLING

In this section we recall the well-known *structure-preserving* model reported in Varaiya (1985). To simplify the presentation of our results we assume a simplified network topology where attached to each bus there is a machine *and* a load.¹ Each bus, and their corresponding machine and load, have an associated identifier $j \in \bar{n} := \{1, \dots, n\}$. Buses are interconnected through transmission lines that are identified by the double subindex $jk \in \Omega \subset \bar{n} \times \bar{n}$, indicating that the line jk connects the bus $j \in \bar{n}$ with the bus $k \in \bar{n}$; the set avoids obvious repetitions, e.g., if $jk \in \Omega$ then $kj \notin \Omega$. All elements share as port variables the angle θ_j and the magnitude V_j of the bus voltage phasor $y_j = \text{col}(\theta_j, V_j) \in \mathbb{S} \times \mathbb{R}_{>0}$. Associated to each bus are the active and reactive powers entering the machine, the load or the transmission lines, that are denoted

$$u_j^M = \begin{bmatrix} P_j^M \\ Q_j^M \end{bmatrix}, u_j^L = \begin{bmatrix} P_j^L \\ Q_j^L \end{bmatrix}, u_{jk} = \begin{bmatrix} P_{jk} \\ Q_{jk} \end{bmatrix}, \quad (1)$$

respectively. Following standard convention, we take active and reactive powers as positive when entering their corresponding component.

2.1 Synchronous machines model

Each synchronous machine is described by a set of third order DAE's, Varaiya (1985):

$$\begin{aligned} \dot{\delta}_j &= \omega_j, \\ M_j \dot{\omega}_j &= P_{mj} - D_j \omega_j + P_j^M, \\ \tau_j \dot{E}_j &= -\frac{x_{d_j}}{x'_{d_j}} E_j + \frac{x_{d_j} - x'_{d_j}}{x'_{d_j}} V_j \cos(\delta_j - \theta_j) + E_{F_j}, \\ P_j^M &= -\frac{E_j V_j}{x'_{d_j}} \sin(\delta_j - \theta_j) - Y_{2j} V_j^2 \sin(2(\delta_j - \theta_j)), \\ Q_j^M &= (Y_{V_j} - Y_{2j} \cos(2(\delta_j - \theta_j))) V_j^2 - \frac{E_j V_j}{x'_{d_j}} \cos(\delta_j - \theta_j), \end{aligned} \quad (2)$$

¹ As will become clear below the derivations are also applicable for other network topologies—at the expense of a more cluttered notation.

where the *state variables* $x_j := \text{col}(\delta_j, \omega_j, E_j) \in \mathbb{S} \times \mathbb{R} \times \mathbb{R}$ denote the rotor angle, the rotor speed and the quadrature axis internal e.m.f., respectively, and E_{F_j} is the field voltage. The latter is split in two terms, $E_{F_j}^* + v_j$, the first is constant and fixes the equilibrium value, while the second one is the *control action*. The parameters are denoted as in Varaiya (1985), and are fairly standard. To simplify notation, we define the constants

$$\begin{aligned} Y_{2j} &:= \frac{x'_{d_j} - x_{q_j}}{2x_{q_j} x'_{d_j}}, \quad Y_{E_j} := \frac{x_{d_j}}{x'_{d_j} (x_{d_j} - x'_{d_j})}, \quad L_{v_j} := \begin{bmatrix} 0 & 0 & \frac{1}{\tau_j} \end{bmatrix}^\top, \\ Y_{V_j} &:= \frac{x'_{d_j} + x_{q_j}}{2x_{q_j} x'_{d_j}}, \quad Y_{F_j} := \frac{1}{x_{d_j} - x'_{d_j}}, \quad B_u(y_j) := \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{V_j} \end{bmatrix}, \\ J_j &:= \begin{bmatrix} 0 & \frac{1}{M_j} & 0 \\ -\frac{1}{M_j} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -J_j^\top, \quad R_j := \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{D_j}{M_j^2} & 0 \\ 0 & 0 & \frac{1}{\tau_j Y_{F_j}} \end{bmatrix} \geq 0. \end{aligned}$$

Notice that, in the operating region of interest, the magnitude of the bus voltages V_j is bounded away from zero hence the division by V_j is well defined.

Fact 1. The synchronous machine model (2) defines an operator $\Sigma_j^M : (u_j^M, y_j)$ described by the implicit perturbed port-Hamiltonian system

$$\begin{cases} \dot{x}_j = (J_j - R_j) \nabla_{x_j} S_j^M(x_j, y_j) + L_{v_j} v_j + \xi_j, \\ 0 = -\nabla_{y_j} S_j^M(x_j, y_j) + B_u(y_j) u_j^M - \Psi_j^M(y_j), \end{cases} \quad (3)$$

with storage function $S_j^M : \mathbb{M}^1 \rightarrow \mathbb{R}$,

$$\begin{aligned} S_j^M &:= \frac{1}{2} M_j \omega_j^2 + \frac{1}{2} Y_{E_j} E_j^2 + \frac{1}{2} [\Delta_j + Y_{V_j}] V_j^2 - Y_{F_j} E_{F_j}^* E_j \\ &\quad - \frac{Y_{2j}}{2} \cos 2(\theta_j - \delta_j) V_j^2 - \frac{E_j V_j}{x'_{d_j}} \cos(\theta_j - \delta_j), \end{aligned} \quad (4)$$

where $\xi_j := [0, \frac{P_{mj}}{M_j}, 0]^\top$, $\Psi_j^M(y_j) := [0, -\Delta_j V_j]^\top$, and $\Delta_j \geq 0$ is a key *design parameter*.

Remark 1. The functions S_j^M defined in (4) should be contrasted with the energy functions used in Tsolas (1985), see also Giusto (2006). On one hand, the latter includes an additional term $-P_{mj} \delta_j$.² On the other hand, we have included a term $\Delta_j V_j^2$ that, as will come clear below, is essential for the construction of the control law.

Remark 2. To handle the linear term $-P_{mj} \delta_j$ in a Lyapunov-like analysis we must take care of some delicate theoretical issues that have, unfortunately, been overlooked in the literature and we discuss in detail here—see also discussion in Ortega (2005). Since this function is not defined in $\mathbb{S}$, but in $\mathbb{R}$, if we look at the system as evolving in $\mathbb{M}^n$ it will be a *discontinuous* function and (standard) Lyapunov arguments will not hold true. To avoid this difficulty, we should consider that δ_j evolves in $\mathbb{R}$, instead of $\mathbb{S}$. In this case, the function S_j^M is not lower bounded anymore, stymying the establishment of the

² Removing this term induces the appearance of the constant term ξ_j in (3), but we show in the sequel that its effect can be compensated with a suitable selection of the control law.

property of trajectory boundedness needed for LaSalle-based arguments.³

2.2 Loads model

Loads are described by the standard ZIP model:

$$\begin{aligned} P_j^L &= P_{Z_j} V_j^2 + P_{I_j} V_j + P_{0_j}, \\ Q_j^L &= Q_{Z_j} V_j^2 + Q_{I_j} V_j + Q_{0_j}, \end{aligned} \quad (5)$$

which explicitly represent the contribution of each type of load, *i.e.*, constant impedance, current and power.

Fact 2. The ZIP load model (5) defines an operator $\Sigma_j^L : (u_j^L, y_j)$ described by the implicit PCH system

$$0 = -\nabla_{y_j} S_j^L(y_j) + B_u(y_j) u_j^L - \Psi_j^L(y_j), \quad (6)$$

with storage function $S_j^L : \mathbb{R} \times \mathbb{R}_{>0} \rightarrow \mathbb{R}$,

$$S_j^L(y_j) := P_{0_j} \theta_j + \frac{Q_{Z_j}}{2} V_j^2 + Q_{I_j} V_j + Q_{0_j} \ln(V_j), \quad (7)$$

where

$$\Psi_j^L(y_j) := [P_{Z_j} V_j^2 + P_{I_j} V_j \quad 0]^\top.$$

2.3 Transmission lines model

The transmission lines are modeled with the standard lumped Π circuit, Anderson and Fouad (1977):

$$\begin{aligned} P_{jk} &= G_{jk} V_j^2 + B_{jk} V_j V_k \sin(\theta_j - \theta_k) \\ &\quad - G_{jk} V_j V_k \cos(\theta_j - \theta_k), \\ Q_{jk} &= (B_{jk} - B_{jk}^c) V_j^2 - B_{jk} V_j V_k \cos(\theta_j - \theta_k) \\ &\quad - G_{jk} V_j V_k \sin(\theta_j - \theta_k), \end{aligned} \quad (8)$$

where $jk \in \Omega$, while G_{jk} , B_{jk} and B_{jk}^c denote the lines conductance, series and shunt susceptance, respectively.

Fact 3. The transmission line model (8) defines an operator $\Sigma_{jk} : (u_{jk}, y_j, y_k)$ described by an implicit (memory-less) PCH system

$$0 = -\nabla_{y_j} S_{jk}(y_j, y_k) + B_u(y_j) u_{jk} - \Psi_{jk}(y_j, y_k), \quad (9)$$

with storage function $S_{jk} : \mathbb{R} \times \mathbb{R}_{>0} \times \mathbb{R} \times \mathbb{R}_{>0} \rightarrow \mathbb{R}$,

$$S_{jk}(y_j, y_k) := \frac{(B_{jk} - B_{jk}^c)}{2} (V_j^2 + V_k^2) - B_{jk} V_j V_k \cos(\theta_j - \theta_k), \quad (10)$$

where

$$\Psi_{jk}(y_j, y_k) := -G_{jk} \begin{bmatrix} -V_j^2 + V_j V_k \cos(\theta_j - \theta_k) \\ V_k \sin(\theta_j - \theta_k) \end{bmatrix}.$$

2.4 Bus equations

From Kirchhoff's laws, at each bus we have

$$\begin{aligned} 0 &= \sum_{k \in \Omega_j} P_{jk} + P_j^M + P_j^L, \\ 0 &= \sum_{k \in \Omega_j} Q_{jk} + Q_j^M + Q_j^L, \end{aligned} \quad (11)$$

where $\Omega_j := \{k \in \bar{n} \mid \exists jk \in \Omega\}$, the set of buses that are linked to the bus j through some transmission line.

³ This unfortunate mistake is made in many papers. For instance, in Proposition 1 of Hao (2007), where the interesting idea of damping injection for structure preserving models is proposed, boundedness of trajectories is never established—nor assumed.

3. CONTROL PROBLEM

To obtain the overall model, it is convenient to group all the algebraic constraints (3), (6), (9), (11) into the function $g : \mathbb{M}^n \rightarrow \mathbb{R}^{2n}$:

$$g(x, y) := \nabla_y S_0(x, y) + \Psi(y), \quad (12)$$

where $\Psi : \mathbb{S}^n \times \mathbb{R}_{>0}^n \rightarrow \mathbb{R}^{2n}$ is given by $\Psi(y) := \text{col}(\Psi_j(y))$, with

$$\Psi_j(y) = \sum_{k \in \Omega_j} \Psi_{jk}(y_j, y_k) + \Psi_j^L(y_j) + \Psi_j^M(y_j),$$

and $S_0 : \mathbb{M}^n \rightarrow \mathbb{R}$

$$S_0(x, y) := \sum_j S_j^M(x_j) + \sum_j S_j^L(y_j) + \sum_{jk \in \Omega} S_{jk}(y_j, y_k). \quad (13)$$

A compact description of the system is obtained defining the block-diagonal matrices

$$L_v := \text{diag}\{\text{col}(0, 0, \frac{1}{\tau_j})\}, \quad J := \text{diag}\{J_i\}, \quad R := \text{diag}\{R_i\}.$$

This allows us to rewrite the overall system as

$$\begin{cases} \dot{x} = (J - R) \nabla_x S_0(x, y) + L_v v + \xi \\ 0 = \nabla_y S_0(x, y) + \Psi(y), \end{cases} \quad (14)$$

where $x := \text{col}(x_j)$, $y := \text{col}(y_j)$, $v := \text{col}(v_j)$ and $\xi := \text{col}(\xi_j)$.

3.1 Problem formulation

Assumption A1. There exists an isolated asymptotically stable *open loop equilibrium* (x^*, y^*) of the system (14).

Transient Stabilization Problem. Consider the system (14) satisfying Assumptions A1. Find a control law $v = \hat{v}(x, y)$ such that there exists $\epsilon > 0$ with :

$$B_\epsilon = \{(x(t), y(t)); \| (x(t), y(t)) - (x^*, y^*) \| \leq \epsilon\}$$

is globally attractive in closed-loop for all $(x(t), y(t)) \in \mathbb{M}^n$, $\forall t \geq 0$.

3.2 Proposed solution strategy

The solution to the problem stated in section 3.1 proceeds along the following steps:

- (1) Give an explicit solution of the power balance equations $g(x, y) = 0$.
- (2) Construction of a control signal that, assigning the derivative of the Hamiltonian function $S_0(x, y)$, ensures the boundedness of all trajectories.
- (3) Prove that the resulting controller is well defined and convergence is guaranteed.

The second step can be carried out for the model with the general ZIP loads (5). Invoking the existence of an isolated local minimum of Assumption A1 and using some continuity arguments we can, therefore, conclude that the proposed controller renders the neighborhood of equilibrium locally attractive. This kind of local results are easily obtained using linearization, and known in the power systems community as small-signal stability. In this paper we are interested in the nonlinear transient stability phenomenon, *i.e.*, the large-signal stability problem,

therefore, the last step is indispensable. To complete it, the first step is essential—unfortunately, this imposes the restrictive requirement of constant impedance loads.

Assumption A2. Assume that the loads is described only by a constant impedance, therefore a simplified model of the loads is:

$$\begin{aligned} P_j^L &= P_{Z_j} V_j^2, \\ Q_j^L &= Q_{Z_j} V_j^2. \end{aligned} \quad (15)$$

Remark 3. This simplification allows us to transform the algebraic constraints into a set of *linear* equations for which we can give conditions for solvability.

4. MAIN RESULT

This section contains our main “global” convergence result, which is derived proceeding along the steps delineated in Subsection 3.2.

4.1 Solution of $g(x, y) = 0$

In this subsection we present an explicit solution to the algebraic constraints $g(x, y) = 0$ satisfying Assumption A2, a result which is of interest on its own. To simplify the presentation we define, for $j \in \bar{n}$, the complex variables

$$\mathbf{V}_j := V_j e^{i\theta_j} \in \mathbb{C}, \quad \mathbf{V} := \text{col}(\mathbf{V}_j)_{j \in \bar{n}} \in \mathbb{C}^n, \quad (16)$$

and

$$E := \text{col}(E_j)_{j \in \bar{n}} \in \mathbb{R}^n, \quad \delta := \text{diag}\{\delta_j\}_{j \in \bar{n}} \in \mathbb{R}^{n \times n}.$$

Lemma 1. Consider the algebraic equations $g(x, y) = 0$ of the power systems model (14) defined by (2), (8), (11), and (15). If

$$\frac{1}{x'_{d_j}} + Q_{Z_j} > \sum_{k \in \Omega_j} B_{jk}^c, \quad j \in \bar{n}, \quad (17)$$

$g(x, y) = 0$ has a “globally” defined solution. That is, there exists a function $\hat{y} : \mathbb{S}^n \times \mathbb{R}^n \rightarrow \mathbb{S}^n \times \mathbb{R}_{>0}^n$ such that $g(x, \hat{y}(x)) = 0$. Furthermore, this function can be written in the form

$$\mathbf{V} = W(\delta)E, \quad (18)$$

where $W : \mathbb{R}^{n \times n} \rightarrow \mathbb{C}^{n \times n}$ is *bounded and invertible*, with elements are rational functions of $\cos(\delta_j)$ and $\sin(\delta_j)$. See Dib (2009) for more details.

Remark 4. Note that condition (17) is always verified and realistic, since we are considering the low voltage terminals of the generators, which is usually connected through a single step-up transformer to the network. Also, it is clear that the construction of $\hat{y}$ directly follows from (16) and (18), and is omitted for brevity.

Remark 5. Since the solution of the algebraic equations $g(x, y) = 0$ is globally defined, and it will be used in the design of the controller, we overcome the classical assumption made about the invertibility of $\nabla_y g(x, y)$, *i.e.* the continuity of the trajectories restricted to $g(x, y) = 0$ Hao (2007).

4.2 “Global” assignment of $\dot{S}_0(x, \hat{y}(x))$

Consider the system (14) satisfying Assumptions A1 and A2. Therefore, one important property of the energy function $S_0(x, y)$ is that it is *quadratic* in $Z_{jk} := \text{col}(\omega_j, E_j, V_j, V_k)$ and, furthermore, *bounded from below*. (Consequently, if $S_0(x, y)$ is non-increasing, we can conclude that all signals are bounded—because Z_{jk} will be bounded and θ_j and δ_j live in compact sets.) To prove this fact, let us define the function

$$S_{0_j}(x, y) := S_j^M(x_j) + S_j^L(y_j) + \sum_{k \in \Omega_j} S_{jk}(y_j, y_k)$$

which describes $S_0(x, y)$ at bus j and write it in the form

$$S_{0_j} = \frac{1}{2} Z_{jk}^\top T_j (\theta_j - \delta_j, \{\theta_j - \theta_k\}_{k \in \Omega_j}) Z_{jk} + Z_{jk}^\top b_j, \quad (19)$$

where we have defined

$$T_j := \begin{bmatrix} M_j & 0 & 0 & 0 \\ 0 & Y_{E_j} & c_j & 0 \\ 0 & c_j & a_j & d_j \\ 0 & 0 & d_j & B_j^d \end{bmatrix},$$

$$b_j := \text{col}(0, -Y_{F_j} E_{F_j}^*, 0, 0), \quad B_j^d := \sum_{k \in \Omega_j} (B_{jk} - B_{jk}^c) > 0,$$

$$c_j := \frac{-\cos(\theta_j - \delta_j)}{x'_{d_j}}, \quad d_j := -\sum_{k \in \Omega_j} B_{jk} \cos(\theta_j - \theta_k),$$

$$\text{and } a_j := \Delta_j + Y_{V_j} + Q_{Z_j} + B_j^d - Y_{2j} \cos 2(\theta_j - \delta_j).$$

Using the fact that (for all $\Delta_j \geq 0$)

$$\Delta_j + Y_{V_j} + Q_{z_j} + B_j^d > Y_{2j},$$

it is possible to show that, *uniformly* in $\theta_j - \delta_j$, and $\{\theta_j - \theta_k\}_{k \in \Omega_j}$, there exists $\epsilon_j > 0$ such that $T_j \geq \epsilon_j I$. Consequently, after some basic bounding, we prove that

$$S_{0_j} \geq -\frac{(Y_{F_j} E_{F_j}^*)^2}{2\epsilon_j}. \quad (20)$$

Moreover, besides being lower bounded and quadratic (in Z_{jk}) we prove in the paper another fundamental property of this function, namely, that its derivative restricted to the set $g(x, y) = 0$, can be *arbitrarily assigned* with a suitable selection of the control v . Towards this end, compute

$$\dot{S}_0(x, y) = -\nabla_x^\top S_0 R \nabla_x S_0 + \tilde{\xi}(x, y) + \nabla_E^\top S_0 \tau^{-1} v + \nabla_y^\top S_0 \dot{y} \quad (21)$$

where $\tilde{\xi}(x, y) := \sum_{j \in \bar{n}} \omega_j P_{mj} \in \mathbb{R}$ and $\tau := \text{diag}\{\tau_j\}_{j \in \bar{n}} \in \mathbb{R}^{n \times n}$. Lemma 1 shows that the set $g(x, y) = 0$ is equivalent to $\mathbf{V} = W(\delta)E$. Therefore, to evaluate $\dot{y}$ it is convenient to express the Hamiltonian function (13) in terms of the complex variables $\mathbf{V}$ defined in (16). Hence, noticing that

$$V_j^2 \cos 2(\theta_j - \delta_j) = \text{Re}\{e^{-2i\delta_j} \mathbf{V}_j^2\},$$

we define

$$\begin{aligned} S_{0c}(x, \mathbf{V}) &:= \frac{1}{2} (w^\top M w + E^\top Y_E E + \mathbf{V}^\mathcal{H} (Y_V + \Delta) \mathbf{V}) \\ &\quad - \text{Re}\{\mathbf{E}^\mathcal{H} X \mathbf{V}\} - \frac{1}{2} \text{Re}\{\mathbf{V}^\top Y_2 e^{-2i\delta} \mathbf{V}\} - E^\top Y_F E_F \\ &\quad + \frac{1}{2} \mathbf{V}^\mathcal{H} Q_z \mathbf{V} + \frac{1}{2} \mathbf{V}^\mathcal{H} (B^d - B) \mathbf{V}, \end{aligned}$$

where

$$\begin{aligned}
M &:= \text{diag}\{M_j\}, Y_E := \text{diag}\{Y_{E_j}\}, Y_V := \text{diag}\{Y_{V_j}\}, \\
\Delta &:= \text{diag}\{\Delta_j\}, X := \text{diag}\left\{\frac{1}{x_{dj}}\right\}, Y_2 := \text{diag}\{Y_{2j}\}, \\
Y_F &:= \text{diag}\{Y_{F_j}\}, E_F := \text{col}(E_{F_j}^*), Q_Z := \text{diag}\{Q_{Z_j}\}, \\
B^d &:= \text{diag}\{B_j^d\}, B := \{B_{jk}\}, G := \{G_{jk}\},
\end{aligned}$$

are defined in $\mathbb{R}^{n \times n}$ with $j, k \in \bar{n}$, and $B_{jk} = G_{jk} = 0$ if $k \notin \Omega_j$.

The following lemma is instrumental to compute the required derivative.

Lemma 2. Consider the quadratic function $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$,

$$f(\mu_1, \mu_2) = c_1 \mu_1^2 + 2c_3 \mu_1 \mu_2 + c_2 \mu_2^2,$$

with $c_i \in \mathbb{R}$ and $\mu_1, \mu_2 : \mathbb{R} \rightarrow \mathbb{R}$. Define $z = \mu_1 + i\mu_2 \in \mathbb{C}$ and the function $f_{\mathbb{C}} : \mathbb{C} \rightarrow \mathbb{R}$ such that

$$f_{\mathbb{C}}(z) = f(\mu_1, \mu_2).$$

Then,

$$\dot{f} = \text{Re} \left\{ \left(\frac{\partial f_{\mathbb{C}}}{\partial z} \right)^* \dot{z} \right\},$$

where $\frac{\partial f_{\mathbb{C}}}{\partial z} := \frac{\partial f_{\mathbb{C}}}{\partial \mu_1} + i \frac{\partial f_{\mathbb{C}}}{\partial \mu_2}$.

From the lemma it is clear that, to compute the time derivative of S_{0c} , we require the term $\text{Re} \left\{ (\nabla_{\mathbf{V}} S_{0c})^{\mathcal{H}} \dot{\mathbf{V}} \right\}$. It is easy to see that

$\nabla_{\mathbf{V}} S_{0c} = (Y_V + \Delta + Q_z + B^d - B) \mathbf{V} - Y_2 e^{2i\delta} \mathbf{V}^* - X \mathbf{E}$, where $(\cdot)^*$ denotes complex conjugation. From Dib (2008), we know that the algebraic constraints $g(x, y) = 0$ written in terms of the complex variables are equivalent to the following equation

$$A_0 \mathbf{V}^* - X \mathbf{E}^* - Y_2 e^{-2i\delta} \mathbf{V} = 0.$$

with $A_0 := Y_V + Q_Z - iP_Z + B^d - iG^d - B + iG \in \mathbb{C}^{n \times n}$, $P_Z := \text{diag}\{P_{Z_j}\}$, and $G^d := \text{diag}\{G_j^d\}$. Substituting the complex conjugate of the latter in $\nabla_{\mathbf{V}} S_{0c}$ above we get

$$\nabla_{\mathbf{V}} S_{0c} = (Y_V + \Delta + Q_z + B^d - B - A_0^*) \mathbf{V},$$

and by definition of A_0 , we get $\nabla_{\mathbf{V}} S_{0c} = D \mathbf{V}$, where

$$D := \Delta + i(-P_Z - G^d + G) \in \mathbb{C}^{n \times n}.$$

Let us now compute $\dot{\mathbf{V}}$. In Lemma 1 it is shown that $\mathbf{V} = WE$, where $W : \mathbb{R}^{n \times n} \rightarrow \mathbb{C}^{n \times n}$ is *bounded and invertible*. Therefore,

$$\dot{\mathbf{V}} = \dot{W}E + W\dot{E}$$

The function $\dot{W}$ depends on δ and ω , but is independent of v , while $\dot{E}$ will bring along terms on v . We now come back to $\dot{S}_0$, that takes the form

$$\dot{S}_0 = -\nabla_x^\top S_0 R \nabla_x S_0 + \tilde{\xi} + \nabla_E^\top S_0 \tau^{-1} v + \text{Re} \left\{ (\nabla_{\mathbf{V}} S_{0c})^{\mathcal{H}} \dot{\mathbf{V}} \right\}.$$

That, replacing the computations above, can be compactly written as

$$\dot{S}_0 = -\nabla_x^\top S_0 R \nabla_x S_0 + \Xi(\delta, w, E, \mathbf{V}) + \mathcal{L}^\top(\delta, w, E, \mathbf{V}) \tau^{-1} v \quad (22)$$

where we defined the (real valued) functions

$$\begin{aligned}
\Xi &:= \tilde{\xi} + \text{Re} \left\{ (\nabla_{\mathbf{V}} S_{0c})^{\mathcal{H}} \left[\dot{W}E - W(\tau Y_F)^{-1} \nabla_E S_0 \right] \right\}, \\
\mathcal{L}^\top &:= \left\{ \nabla_E^\top S_0 + \text{Re} \left\{ (\nabla_{\mathbf{V}} S_{0c})^{\mathcal{H}} W \right\} \right\}. \quad (23)
\end{aligned}$$

Let us take a brief respite to analyze (22). It is clear that, wherever the vector $\mathcal{L}(\delta, w, E, \mathbf{V})$ is bounded away from

zero, we can easily select a control law v to cancel the sign indefinite term Ξ and make $\dot{S}_0 \leq 0$ or even assign an arbitrary function to $\dot{S}_0$.

Proposition 1. Consider the power systems model (14) with Assumptions A1 and A2 and the Hamiltonian function (13). There exists $\Delta_j^{\min} > 0$ such that, for all $\Delta_j \geq \Delta_j^{\min}$ we have

$$\mathcal{L}^\top(x, \hat{y}(x))E > 0 \quad \text{for all } x \in \mathbb{S}^n \times \mathbb{R}^n \times \mathbb{R}_{>0}^n,$$

where $\mathcal{L}^\top$ is given in (23). Therefore, the globally defined control law

$$v = -2 \frac{M}{\mathcal{L}^\top E} \tau E \quad (24)$$

ensures $\dot{S}_0 \leq -\nabla_x^\top S_0 R \nabla_x S_0 - \beta$ where $M \geq |\Xi|$ belongs to class $\mathcal{K}$ and $\beta > 0$.

Proposition 2. Consider the power systems model (14) with Assumptions A1 and A2 and the Hamiltonian function (13). There exists $\Delta_j^{\min} > 0$ such that, for all $\Delta_j \geq \Delta_j^{\min}$ we have

$$\mathcal{L}^\top(x, \hat{y}(x))E > 0 \quad \text{for all } x \in \mathbb{S}^n \times \mathbb{R}^n \times \mathbb{R}_{>0}^n,$$

where $\mathcal{L}^\top$ is given in (23). Therefore, for any function $\alpha : \mathbb{M}^n \rightarrow \mathbb{R}$, the globally defined control law

$$v = \frac{1}{\mathcal{L}^\top E} \left[-\lambda [S_0(x, y) - S_0^*] + \nabla_x^\top S_0 R \nabla_x S_0 - \Xi \right] \tau E \quad (25)$$

ensures

$$\frac{d}{dt} [S_0(x, y) - S_0^*] = -\lambda [S_0(x, y) - S_0^*],$$

where $S_0^* := S_0(x^*, y^*)$, and $\lambda > 0$.

Proof. By definition,

$$\mathcal{L}^\top E = \text{Re} \left\{ (\nabla_{\mathbf{V}} S_{0c})^{\mathcal{H}} W(\delta) E \right\} + \nabla_E^\top S_0 E. \quad (26)$$

Let us consider the first term. Since $\mathbf{V} = W(\delta)E$ and $\nabla_{\mathbf{V}} S_{0c} = D \mathbf{V}$ we have

$$\text{Re} \left\{ (\nabla_{\mathbf{V}} S_{0c})^{\mathcal{H}} W E \right\} = \text{Re} \left\{ \mathbf{V}^{\mathcal{H}} D \mathbf{V} \right\} = \mathbf{V}^{\mathcal{H}} \frac{D + D^*}{2} \mathbf{V}$$

The matrix D is symmetric (not Hermitian self conjugate). Therefore,

$$\frac{D + D^*}{2} = \Delta,$$

The quadratic form above can then be made *arbitrarily large* by choosing a large $\Delta > 0$.

Using again $\mathbf{V} = WE$ and invertibility of W we see that the second term in (26) is also a quadratic function of $\mathbf{V}$, that can be written in the form

$$\nabla_E^\top S_0 E = \mathbf{V}^{\mathcal{H}} S(\delta) \mathbf{V} + \text{Re}(\mathbf{V}^{\mathcal{H}} s(\delta) E_F^*),$$

for some suitable matrices $S, s : \mathbb{R}^{n \times n} \rightarrow \mathbb{C}^{n \times n}$. From boundedness of W^{-1} we have that S and s are also *bounded* and we can conclude that, throughout $\mathbb{M}^n$, the first term in (26) can be made strictly greater than the second. Therefore, the denominator in (24) and (25) is always larger than zero, completing the claim.

Remark 6. Proposition 1 ensures that $\dot{S}_0 \leq 0$, therefore since S_0 is bounded from below, this implies that all signals are bounded and the aim is achieved. In addition, a practically interesting property of the control law (24) is that it is “almost” *decentralized*. Moreover, although all signals are bounded with Proposition 1, we propose in Proposition

2 the controller (25) in order to force all trajectories to converge to the level set of the Hamiltonian function that contains the stable equilibrium point, and henceforth all trajectories will be closer to the stable equilibrium point.

Remark 7. An interesting interpretation concerning the design parameter Δ is that it can be assimilated to Q_z in the energy function. This means that by increasing the reactive power in the network, we are able to stabilize the system after a large perturbation using the specific control law (24) or (25).

5. A BENCHMARK SIMULATION EXAMPLE

We consider here the classical 3-machines, 9-buses system considered in Anderson and Fouad (1977). We assume that the active and reactive components of the loads have constant impedance characteristics. Computations were done with the software package PSAT Milano (2005). Using the controller of Proposition 2, we analyze the response of (14) to a short circuit which consists of a zero-impedance three phase fault on the transmission line. The fault is introduced at $t = 0.5$ s and removed after a certain time (called the clearing time and denoted t_{cl}), after which the system is back to its pre-disturbance topology.

To tune the controller there is a compromise between the choices of Δ that, as indicated in Proposition 2, should be big enough to ensure that the denominator of the controller will stay away from zero, and λ that determines the speed of convergence to the desired level set. Indeed, Δ appears in S_0 as ΔV^2 where V represents, in some way, the perturbation. Then, the bigger Δ is, the bigger S_0 will be in the transient phase, and we have to decrease λ to eliminate impulsive responses in the controller during the perturbation.

Fig. 1 depicts the transient response of the nonlinear systems with $t_{cl} = 0.1$ s, $\Delta_j = 10$, $j = 1, 2, 3$, and $\lambda = 3$. As it can be seen, the controller is able to significantly improve the system's stability and provide damping.

Fig. 1. Speed w_1 . Open loop (dashed), closed loop (continuous).

6. CONCLUSIONS AND FUTURE WORK

We have presented in this paper an excitation controller to improve the transient stabilization properties of multi-machine power systems described by SPM with leaky lines including capacitive effects. Our main contribution is the explicit computation of a control law that ensures

boundedness of all trajectories. Although the proposed controller does not provide asymptotic convergence to the desired equilibrium point the goal of transient stabilization problem is successfully achieved. To the best of our knowledge, no equivalent result is available in the literature at this level of generality. The usefulness of the technique for synthesis was illustrated with its application to a classical example. Similarly to most developments reported by the control theory community on the transient stability problem, it is clear that the complexity of the proposed controller—as well as its high sensitivity to the system parameters and the assumption of full state measurement—severely stymies the practical application of this result. This kind of work pertains, however, to the realm of fundamental research where basic issues like existence of solutions are addressed.

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