A Pseudorandom Postfix OFDM Modulator—Semi-Blind Channel Estimation and Equalization

Markus Muck, *Member, IEEE*, Marc de Courville, *Member, IEEE*, and Pierre Duhamel, *Fellow, IEEE*

Abstract—This paper details a new orthogonal-frequency-division-multiplexing (OFDM) modulator based on the use of a pseudorandom postfix (PRP)-OFDM and discusses low-complexity equalization and channel estimation/tracking architectures. The main property of this new modulation scheme is the ability to estimate and track the channel variations semi-blindly using order-one statistics of the received signal. Compared with known cyclic prefix OFDM (CP-OFDM) pilot-symbol-assisted modulation (PSAM) schemes, the pilot overhead is avoided: The channel estimation is performed based on the exploitation of pseudorandomly weighted postfix sequences replacing the guard interval contents of CP-OFDM. PRP-OFDM is shown to be of advantage if the target application requires 1) a minimum pilot overhead, 2) low-complexity channel tracking (e.g., high mobility context), and 3) adjustable receiver complexity/performance trade-offs (available due to the similarities of PRP-OFDM to the zero-padded OFDM (ZP-OFDM) modulation scheme) without requiring any feedback loop to the transmitter.

Index Terms—Channel estimation, cyclic prefix, equalization, IEEE 802.11a, orthogonal frequency-division multiplexing (OFDM).

I. INTRODUCTION

ORTHOGONAL-frequency-division multiplexing (OFDM) nowadays seems the preferred modulation scheme for modern broad-band communication systems. Indeed, the OFDM inherent robustness to multipath propagation and its appealing low complexity equalization receiver make it suitable either for high speed modems over twisted pair [digital subscriber lines (xDSL)], terrestrial digital broadcasting [digital audio broadcasting (DAB) and digital video broadcasting (DVB)], and 5-GHz wireless local area networks [(WLANs): IEEE 802.11a and ETSI BRAN HIPERLAN/2] [2]–[6].

All these systems are based on a traditional cyclic prefix OFDM (CP-OFDM) modulation scheme. The role of the cyclic prefix is to turn the linear convolution into a set of parallel attenuations in the discrete frequency domain. Recent contributions have proposed an alternative: replacing this time-domain redundancy by null samples consisting in the so-called zero-padded

OFDM (ZP-OFDM) [7]–[11]. This solution relying on a larger fast Fourier transform (FFT) demodulator, has the merit to guarantee symbol recovery irrespective of channel null locations in absence of noise when the channel is known (coherent modulations are assumed).

In a quasi-static scenario, channel coefficients estimation is usually performed using known training sequences periodically transmitted (e.g., at the start of each frame), implicitly assuming that the channel does not vary between two training sequences. Wireless systems operating in a mobility context usually apply a combination of such initial training and pilot-symbol-assisted modulation (PSAM) schemes [12]–[14]. The pilots are exploited for channel tracking and estimation at the cost of a throughput reduction due to increased overhead. An alternative is to track the channel variations by refining the channel coefficients blindly using the training sequences as initializations for the estimator. Such semi-blind equalization algorithms based on second order statistics have already been proposed for the CP-OFDM and ZP-OFDM modulators [9], [10], [15], [16].

This paper introduces a new OFDM modulator that capitalizes on the advantages of ZP-OFDM. It is proposed to replace the null samples inserted between each OFDM modulated block by a known vector weighted by a pseudorandom scalar sequence: the pseudo random postfix (PRP)-OFDM. In this way, unlike former OFDM modulators, the receiver can exploit an additional information: the prior knowledge of part of the transmitted block [17]. It is explained how to build on this knowledge in order to perform an extremely low-complexity order-one semi-blind channel estimation and tracking. Moreover, several PRP-OFDM equalization architectures derived from the zero-padded transmission scheme are proposed, allowing implementations ranging from low-complexity/medium performance to increased-complexity/high performance. The least complex decoding approach presented in the paper is based on a suppression of the postfix sequence convolved by the channel followed by an overlap-add (OLA) operation. In a typical example, the additional receiver calculation complexity compared with CP-OFDM is approximately 1.2 complex additions in average per time-domain sample. Each channel estimation additionally requires one FFT operation and one complex multiplication per useful carrier.

Note that a constant prefix has been proposed in both, the single carrier [18] and multicarrier [19] context, but neither exploitation of this known sequence nor more advanced than classical equalization schemes are proposed. Moreover spectrumwise, it is important to avoid 1) highly variable carrier

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M. Muck and M. de Courville are with Motorola Laboratories, 91193 Gif-Sur-Yvette, France (e-mail: Markus.Muck@motorola.com; Marc.De.Courville@motorola.com).

P. Duhamel is with CNRS/LSS, Supelec, 91192 Gif-sur-Yvette, France (e-mail: pierre.duhamel@lss.supelec.fr).

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Fig. 1. Discrete model of the PRP-OFDM modulator.

amplitudes and 2) the insertion of the same training sequence at each block since the repetition generates peaks in the transmitted signal spectrum: The impact of channel fades on the system performance then depends on the affected frequency band. The pseudorandomly weighted sequence used in this contribution avoids this issue [20].

This paper is organized as follows. Section II introduces the notations and presents the new PRP-OFDM modulator. Section III reveals how to exploit the postfix for performing blind channel estimation in the context of both, low (Doppler is negligible) and high (Doppler is significant) mobility; it is further explained in Section IV how to combine estimates of different origins, e.g., preamble- and blind postfix-based results. Then suitable reception strategies are presented in Section V, including several equalization schemes [zero forcing (ZF) and minimum mean square error (MMSE)]. Section VI explains how to adapt the maximum-likelihood (ML) decoding metrics in the presence of bit-interleaved convolutional coded modulation. Section VII discusses design criteria for the choice of suitable postfixes. Finally, Section VIII presents a low-complexity implementation architecture, and simulations in Section IX illustrate the performances in both a static and a mobility scenario reminiscent of the IEEE 802.11a system.

II. NOTATIONS AND PRP-OFDM MODULATOR

Fig. 1 depicts the baseband discrete-time block equivalent model of an N carrier PRP-OFDM system. The i th $N \times 1$ input digital vector¹ $\tilde{\mathbf{s}}_N(i)$ is first modulated by the inverse fast Fourier transform (IFFT) matrix $\mathbf{F}_N^H := 1/\sqrt{N}(W_N^{kl})^H$, $0 \leq k < N$, $0 \leq l < N$ and $W_N := e^{-j(2\pi/N)}$.

¹Lower (upper) boldface symbols will be used for column vectors (matrices) sometimes with subscripts N or P emphasizing their sizes (for square matrices only); the tilde will denote frequency-domain quantities; argument i will be used to index blocks of symbols; ^{H} (^{T}) will denote Hermitian (transpose).

Then, a deterministic postfix vector $\mathbf{c}_D := (c_0, \dots, c_{D-1})^T$ weighted by a pseudorandom value $\alpha(i) \in \mathbb{C}$ is appended to the IFFT output $\mathbf{s}_N(i)$. With $P := N + D$, the corresponding $P \times 1$ transmitted vector is

$$\mathbf{s}_P(i) := \mathbf{F}_{ZP}^H \tilde{\mathbf{s}}_N(i) + \alpha(i) \mathbf{c}_P \quad (1)$$

where

$$\mathbf{F}_{ZP}^H := \begin{bmatrix} \mathbf{I}_N \\ \mathbf{0}_{D,N} \end{bmatrix}_{P \times N} \mathbf{F}_N^H \quad \text{and} \quad \mathbf{c}_P := (\mathbf{0}_{1,N} \mathbf{c}_D^T)^T.$$

Without loss of generality, the elements of $\tilde{\mathbf{s}}_N(i)$ are assumed to be independent identically distributed (i.i.d.) and zero-mean random variables of variance $\sigma_s^2 = 1$ which are independent of $\alpha(i) \mathbf{c}_D$. The samples of $\mathbf{s}_P(i)$ are then sent sequentially through the channel modeled here as a finite-impulse-response (FIR) filter of order $L-1$, $H(z) := \sum_{n=0}^{L-1} h_n z^{-n}$. The OFDM system is designed such that the postfix duration exceeds the channel memory $L \leq D$.

Let $\mathbf{H}_{\text{ISI}}(P)$ and $\mathbf{H}_{\text{IBI}}(P)$ be, respectively, the size $P \times P$ Toeplitz inferior and superior triangular matrices of first column $[h_0, h_1, \dots, h_{L-1}, 0, \dots, 0]^T$ and first row $[0, \dots, 0, h_{L-1}, \dots, h_1]$. As already explained in [21], the channel convolution can be modeled by $\mathbf{r}_P(i) := \mathbf{H}_{\text{ISI}}(P) \mathbf{s}_P(i) + \mathbf{H}_{\text{IBI}}(P) \mathbf{s}_P(i-1) + \mathbf{n}_P(i)$. $\mathbf{H}_{\text{ISI}}(P)$ and $\mathbf{H}_{\text{IBI}}(P)$ represent, respectively, the intrablock and interblock interference. $\mathbf{n}_P(i)$ is the i th AWGN vector of element variance σ_n^2 . Given the expression of (1), we have as illustrated by Fig. 2:

$$\mathbf{r}_P(i) = (\mathbf{H}_{\text{ISI}}(P) + \beta_i \mathbf{H}_{\text{IBI}}(P)) \mathbf{s}_P(i) + \mathbf{n}_P(i) \quad (2)$$

where $\beta_i := (\alpha(i-1)/\alpha(i))$. Note that $\mathbf{H}_{\beta i} := (\mathbf{H}_{\text{ISI}}(P) + \beta_i \mathbf{H}_{\text{IBI}}(P))$ is pseudocirculant: i.e., a circular matrix whose $(D-1) \times (D-1)$ upper triangular part is weighted by β_i .

Fig. 2. Circularization for PRP-OFDM.

The expression of the received block is thus

$$\mathbf{r}_P(i) = \mathbf{H}_{\beta_i} (\mathbf{F}_{\text{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i) \mathbf{c}_P) + \mathbf{n}_P(i) \quad (3)$$

$$= \mathbf{H}_{\beta_i} \begin{pmatrix} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i) \\ \alpha(i) \mathbf{c}_D \end{pmatrix} + \mathbf{n}_P(i). \quad (4)$$

Please note that (3) is closely related to the CP and ZP modulation schemes. With $\mathbf{H}_{\beta_i} = \mathbf{H}_{\text{ISI}}(P) + \mathbf{H}_{\text{IBI}}(P), \forall i$, indeed ZP-OFDM corresponds to $\alpha(i) = 0, \forall i$ and CP-OFDM is achieved for $\alpha(i) = 0 \forall i$ when $\mathbf{F}_{\text{ZP}}^H$ is replaced by $\mathbf{F}_{\text{CP}}^H$, as follows:

$$\mathbf{F}_{\text{CP}}^H := \left[\begin{array}{c|c} \mathbf{0}_{D, N-D} & \mathbf{I}_D \\ \hline \mathbf{I}_N & \end{array} \right]_{P \times N} \mathbf{F}_N^H.$$

A different modulator also introducing a known postfix sequence is proposed in the literature [19]: A $P \times 1$ frequency-domain vector $\tilde{\mathbf{s}}_P(i)$ is defined containing N frequency-domain data carrier amplitudes $\tilde{s}_N(i)$ and D pilot tones. The pilot tones are chosen for each symbol in function of $\tilde{s}_N(i)$ such that the last (or first) D time-domain samples of $\mathbf{F}_P^H \tilde{\mathbf{s}}_P(i)$ correspond to the predefined postfix sequence; note that there is in general no equivalence between both schemes: in general $\mathbf{F}_P^H \tilde{\mathbf{s}}_P(i) \neq \mathbf{F}_{\text{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i) \mathbf{c}_P$ for an identical $\tilde{s}_N(i)$. This approach has the advantage of simple inherent equalization schemes similar to the ones of CP-OFDM in combination with a larger FFT. However, the corresponding pilot amplitudes can vary significantly in amplitude which is in contradiction to a flat spectrum requirement: The impact of channel fades on the system performance then depends on the affected frequency band. This undesired effect is avoided with the previously presented modulator.

Considering (4), it is shown in the sequel that $\mathbf{H}_{\beta_i} \mathbf{c}_P$ can be retrieved by a simple averaging i.e., mean value calculation of the received samples if the OFDM data symbols $\mathbf{F}_N^H \tilde{\mathbf{s}}_N(i)$ are assumed to be zero mean. The channel can afterwards be extracted by deconvolution. Afterwards, it is shown that we can preserve the simple equalization/decoding properties of OFDM even with the introduction of the pseudorandom postfix.

III. ORDER-ONE SEMI-BLIND CHANNEL ESTIMATION BASED ON PRP-OFDM POSTFIXES ONLY

This section presents two different ways of estimating the CIR based on order-one statistics. The first one exploits the $D \times D$ circulant matrix containing the CIR convolved by the postfix yielding a low-complexity estimation scheme. Then, a more general and powerful estimator is proposed playing with the two frequency-domain grid resolutions: D and P . The second method leads to better results in cases where some frequency-

domain amplitudes of the postfix $\mathbf{c}_D$ are close to zero. A discussion on channel estimation in a mobility context finalizes this section.

A. Channel Estimation Using Minimum Dimension Circular Diagonalization

In this section, the channel impulse response (CIR) is assumed static. Define $\mathbf{H}_{\text{CIR}}(D) := \mathbf{H}_{\text{ISI}}(D) + \mathbf{H}_{\text{IBI}}(D)$ as the $D \times D$ circulant channel matrix of first row $\text{row}_0(\mathbf{H}_{\text{CIR}}(D)) = [h_0, 0, \dots, 0, h_{L-1}, \dots, h_1]$. Note that $\mathbf{H}_{\text{ISI}}(D)$ and $\mathbf{H}_{\text{IBI}}(D)$ contain, respectively, the lower and upper triangular parts of $\mathbf{H}_{\text{CIR}}(D)$.

Denoting by $\mathbf{s}_N(i) := [s_0(i), \dots, s_{N-1}(i)]^T$, extracting two subvectors $\mathbf{s}_{N,0}(i) := [s_0(i), \dots, s_{D-1}(i)]^T$ and $\mathbf{s}_{N,1}(i) := [s_{N-D}(i), \dots, s_{N-1}(i)]^T$, and performing the same operations for the noise vector $\mathbf{n}_P(i) := [n_0(i), \dots, n_{P-1}(i)]^T$, $\mathbf{n}_{D,0}(i) := [n_0(i), \dots, n_{D-1}(i)]^T$, $\mathbf{n}_{D,1}(i) := [n_{P-D}(i), \dots, n_{P-1}(i)]^T$, the received vector $\mathbf{r}_P(i)$ can be expressed as

$$\mathbf{r}_P(i) = \begin{pmatrix} \mathbf{H}_{\text{ISI}}(D) \mathbf{s}_{N,0}(i) + \alpha(i-1) \mathbf{H}_{\text{IBI}}(D) \mathbf{c}_D + \mathbf{n}_{D,0} \\ \vdots \\ \mathbf{H}_{\text{IBI}}(D) \mathbf{s}_{N,1}(i) + \alpha(i) \mathbf{H}_{\text{ISI}}(D) \mathbf{c}_D + \mathbf{n}_{D,1} \end{pmatrix}. \quad (5)$$

As defined earlier, the transmitted time-domain signal $\mathbf{s}_N(i)$ is zero-mean. Thus, the first D samples $\mathbf{r}_{P,0}(i)$ of $\mathbf{r}_P(i)$ and its last D samples $\mathbf{r}_{P,1}(i)$ can be exploited very easily to retrieve the channel matrices relying on the deterministic nature of the postfix, as follows:

$$\begin{aligned} \hat{\mathbf{r}}_{c,0} &:= \mathbb{E} \left[\frac{\mathbf{r}_{P,0}(i)}{\alpha(i-1)} \right] = \mathbf{H}_{\text{IBI}}(D) \mathbf{c}_D \\ \hat{\mathbf{r}}_{c,1} &:= \mathbb{E} \left[\frac{\mathbf{r}_{P,1}(i)}{\alpha(i)} \right] = \mathbf{H}_{\text{ISI}}(D) \mathbf{c}_D. \end{aligned} \quad (6)$$

Since $\mathbf{H}_{\text{ISI}}(D) + \mathbf{H}_{\text{IBI}}(D) = \mathbf{H}_{\text{CIRC}}(D)$ is circular and diagonalizable in the frequency domain $\mathbf{F}_D$ combining (6) and using the commutativity of the convolution yields

$$\hat{\mathbf{r}}_c := \hat{\mathbf{r}}_{c,0} + \hat{\mathbf{r}}_{c,1} = \mathbf{H}_{\text{CIRC}}(D) \mathbf{c}_D = \mathbf{C}_D \mathbf{h}_D = \mathbf{F}_D^H \tilde{\mathbf{C}}_D \mathbf{F}_D \mathbf{h}_D \quad (7)$$

where $\mathbf{C}_D$ is a $D \times D$ circulant matrix with first row $\text{row}_0(\mathbf{C}_D) := [c_0, c_{D-1}, c_{D-2}, \dots, c_1]$ and $\tilde{\mathbf{C}}_D := \text{diag}\{\mathbf{F}_D \mathbf{C}_D\}$. $\text{diag}\{\cdot\}$ denotes a diagonal matrix whose components are given by the vector argument.

Since in practice the expectation $\mathbb{E}[\cdot]$ in (6) is approximated by a mean value calculation over a limited number Z of symbols, we can model the estimation error as noise $\tilde{\mathbf{n}}_D$.

The covariance of $\tilde{\mathbf{n}}_D$ is

$$\mathbf{R}_{\tilde{\mathbf{n}}_D} = \mathbb{E}[\tilde{\mathbf{n}}_D \tilde{\mathbf{n}}_D^H] = 2(\sigma_n^2/Z)\mathbf{I}_D + (1/Z) \mathbb{E}[\mathbf{H}_{\text{ISI}}(D)\mathbf{s}_{N,0}\mathbf{s}_{N,0}^H\mathbf{H}_{\text{ISI}}^H(D) + \mathbf{H}_{\text{IBI}}(D)\mathbf{s}_{N,1}\mathbf{s}_{N,1}^H\mathbf{H}_{\text{IBI}}^H(D)].$$

Thus, an estimate of the CIR $\hat{\mathbf{h}}_D$ can be retrieved by either a ZF or MMSE approach [22]

$$\hat{\mathbf{h}}_D^{\text{ZF}} := \mathbf{C}_D^{-1}\hat{\mathbf{r}}_c = \mathbf{F}_D^H \tilde{\mathbf{C}}_D^{-1} \mathbf{F}_D \hat{\mathbf{r}}_c \quad (8)$$

$$\begin{aligned} \hat{\mathbf{h}}_D^{\text{MMSE}} &:= \mathbf{R}_{\mathbf{h}_D} \mathbf{C}_D^H (\mathbf{R}_{\tilde{\mathbf{n}}_D} + \mathbf{C}_D \mathbf{R}_{\mathbf{h}_D} \mathbf{C}_D^H)^{-1} \hat{\mathbf{r}}_c \\ &= \mathbf{F}_D^H \tilde{\mathbf{R}}_{\mathbf{h}_D} \tilde{\mathbf{C}}_D^H (\tilde{\mathbf{R}}_{\tilde{\mathbf{n}}_D} + \tilde{\mathbf{C}}_D \tilde{\mathbf{R}}_{\mathbf{h}_D} \tilde{\mathbf{C}}_D^H)^{-1} \mathbf{F}_D \hat{\mathbf{r}}_c \end{aligned} \quad (9)$$

where $\mathbf{R}_{\mathbf{h}_D} := \mathbb{E}[\mathbf{h}_D \mathbf{h}_D^H]$, $\tilde{\mathbf{R}}_{\mathbf{h}_D} := \mathbf{F}_D \mathbf{R}_{\mathbf{h}_D} \mathbf{F}_D^H$, $\mathbf{R}_{\tilde{\mathbf{n}}_D} := \mathbb{E}[\tilde{\mathbf{n}}_D \tilde{\mathbf{n}}_D^H]$ and $\tilde{\mathbf{R}}_{\tilde{\mathbf{n}}_D} := \mathbf{F}_D \mathbf{R}_{\tilde{\mathbf{n}}_D} \mathbf{F}_D^H$. Note that this ZF CIR estimation technique requires $\mathbf{c}_D$ to be designed in such a way that $\tilde{\mathbf{C}}_D$ is full rank. Since the channel power profile $\mathbb{E}[\mathbf{h}_D \mathbf{h}_D^H]$ is not known, it is convenient to approximate it by the unitary matrix. If the order of the channel is overestimated or the CIR contains zero contributions, $\tilde{\mathbf{R}}_{\mathbf{h}_D}$ is not invertible and (9) is only defined if the invertability is guaranteed by the properties of $\tilde{\mathbf{R}}_{\tilde{\mathbf{n}}_D}$.

B. Channel Estimation Using Frequency-Domain Carrier Grid Adaptation

When designing practical multicarrier systems, $\tilde{\mathbf{C}}_D$ is often chosen rank deficient to ease compliance with the spectrum mask. Indeed for IEEE 802.11a [2], implicit oversampling imposes null side carriers: Actually only 52 out of the specified $N = 64$ carriers are bearing information. As a consequence even if the channel estimation is performed using a frequency grid of resolution D , some side coefficient of $\tilde{\mathbf{C}}_D$ are not reliable. Thus, the derivation of the frequency-domain CIR on a frequency grid of resolution N or P (required for equalization) $\tilde{\mathbf{C}}_M$ where $M \in \{N, P\}$ from $\tilde{\mathbf{C}}_D$ using an oversampling matrix spreads the estimation error and noise yielding to a poor channel estimator. This subsection provides a way to overcome this issue.

Exploiting relations (5) and (6), we observe that

$$\begin{bmatrix} \hat{\mathbf{r}}_{c,0} \\ \mathbf{0}_{P-2D,1} \\ \hat{\mathbf{r}}_{c,1} \end{bmatrix} = \left((\mathbf{H}_{\text{IBI}}(D)\mathbf{c}_D)^T \mathbf{0}_{P-2D}^T (\mathbf{H}_{\text{ISI}}(D)\mathbf{c}_D)^T \right)^T. \quad (10)$$

It is possible to add or truncate any number of these zeros in order to adapt the grid resolution of the estimates. Contrarily to (7), this operation is not irreversible. Thus, a vector $\hat{\mathbf{r}}_M(i)$ is created with $M \geq 2D$ (e.g., $M \in \{N, P\}$): $\hat{\mathbf{r}}_M(i) := [(\mathbf{H}_{\text{IBI}}(D)\mathbf{c}_D)^T, \mathbf{0}_{M-2D}^T, (\mathbf{H}_{\text{ISI}}(D)\mathbf{c}_D)^T]^T$. As in the previous section, $\hat{\mathbf{r}}_M(i)$ can be expressed by a convolution of the postfix with the CIR

$$\hat{\mathbf{r}}_M = \mathbf{H}_{\text{CIRC}}(M) (\mathbf{0}_{M-D}^T \mathbf{c}_D^T)^T = \mathbf{F}_M^H \tilde{\mathbf{C}}_M \mathbf{F}_M \mathbf{h}_M \quad (11)$$

where $\mathbf{C}_M$ is an $M \times M$ circulant matrix with first row $\text{row}_0(\mathbf{C}_M) := [0, c_{D-1}, c_{D-2}, \dots, c_0, 0, \dots, 0]$, $\tilde{\mathbf{C}}_M = \text{diag}\{\mathbf{F}_M(\mathbf{0}_{M-D}^T \mathbf{c}_D^T)^T\}$. The CIR estimates are derived similarly to (8) and (9). Using this procedure, the frequency-domain channel coefficients are estimated for any desired carrier grid without requiring any upsampling or downsampling

after the postfix deconvolution. Particularly in combination with a ZF deconvolution approach, this is very convenient when some frequency-domain amplitudes of the postfix $\tilde{\mathbf{C}}_M$ are close to zero, since upsampling matrices of the type $\mathbf{F}_M^H [\mathbf{0}_{(M-M) \times M}^{\mathbf{I}_M}] \mathbf{F}_M^H$, $M > M$ are full matrices which in general spread more evenly the estimation errors onto the carriers in the upsampled domain. This effect can be avoided by directly estimating the CIR with the desired carrier grid.

We have detailed in these two sections very simple methods for blind estimation of the CIR only relying on a first order statistics: an expectation of the received signal vector.

Although the results presented above are based on the assumption that the channel does not vary, this method can be used to mitigate the effects of Doppler. Indeed, this approach can be combined with the initial channel estimates derived from the preambles usually present at the start of the frame for either refining the channel estimates or tracking the channel variations. For WLANs this enables to operate at a mobility exceeding the specification of the standard (3 m/s). In that case, MMSE channel estimates are usually more efficient than ZF ones.

C. Channel Estimation in Presence of Doppler

This section details a suitable channel estimator in the presence of a time varying channel. For all derivations, the channel is assumed to vary insignificantly over one OFDM symbol; however, the variation can be considerable over the data frame.

In the context of a Doppler scenario, the choice of the Doppler model plays an essential role. Jakes' commonly accepted Doppler model [23] is used throughout this paper stating that $\alpha_D = \mathbb{E}[h_l(n)h_l^*(n-1)] = J_0(2\pi f_D \Delta T) \mathbb{E}[|h_l(n)|^2]$ with $h_l(n)$ being the l^{th} component of the CIR $\mathbf{h}(n)$ at instant $T_n = n\Delta T$, $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind and f_D the Doppler frequency.

In the following, two different CIR estimation concepts are presented: 1) a Kalman-filtering-based approach relying on a order-one autoregressive (AR) channel evolution model and 2) a generic Wiener filtering approach.

1) *Symbol-Based CIR Update Based on Kalman Filtering:* Recent publications discuss the possibility to approximate Jakes' model by an order-one AR filter given by [24]

$$\mathbf{h}(n+1) = J_0(2\pi f_D T_1) \mathbf{h}(n) + \check{\mathbf{h}}(n+1). \quad (12)$$

$\check{\mathbf{h}}(n+1)$ is the so-called process noise. This model has several advantages; in particular, an MMSE estimator of the CIR $\mathbf{h}(0)$ based on noisy observations $\mathbf{h}(n) + \mathbf{n}(n)$ is straightforwardly derived by Kalman filtering [25]. Rewriting (12) as

$$\begin{cases} \mathbf{h}(0) = \check{\mathbf{h}}(0) \\ \mathbf{h}(1) = J_0(2\pi f_D T_1) \check{\mathbf{h}}(0) + \check{\mathbf{h}}(1) \\ \dots \\ \mathbf{h}(n) = \sum_{k=0}^n J_0^k(2\pi f_D T_1) \check{\mathbf{h}}(n-k) \end{cases}$$

clearly shows that the AR model inherently approximates $J_0(2\pi f_D k \Delta T)$ by $J_0^k(2\pi f_D \Delta T)$. The latter expression is justified by the Bessel function addition theorem stating $J_0(x+y) = \sum_{k=-\infty}^{\infty} (-1)^k J_k(x) J_k(y) \approx J_0(x) J_0(y)$ for small x and y . However, these approximations are not quite valid in the context of high Doppler frequencies which occur,

for example, in the context of high mobility WLANs or at high carrier frequencies (60 GHz).

This motivates the block-based Wiener filtering approach presented in the following.

2) *Block-Based CIR Update Based on Wiener Filtering*: Contrarily to the previous approaches designed for the static context, the CIR is not estimated based on the result of a mean-value calculation process; instead, the observations of the postfix sequences convolved by the channel are concatenated over $Z \in \mathbb{N}$ observations, as follows:

$$\bar{\mathbf{h}}_{ZD} = \left[(\mathbf{C}_D \mathbf{h}_D(0))^T, \dots, (\mathbf{C}_D \mathbf{h}_D(Z-1))^T \right]^T + \mathbf{n}_{ZD}. \quad (13)$$

Using a standard Wiener filtering approach [26], the optimum estimator of $\mathbf{h}_D(0)$ in the MMSE sense is given by the following theorem:

Theorem 3.1: The MMSE estimation of $\mathbf{h}_D(0)$ in (13) is given by

$$\begin{aligned} \hat{\mathbf{h}}_D &:= \mathbf{M}_R \left[(\mathbf{I}_Z \otimes \mathbf{C}_D) \mathbf{D}_{Rh} (\mathbf{I}_Z \otimes \mathbf{C}_D)^H + \mathbf{R}_n \right]^{-1} \bar{\mathbf{h}}_{ZD} \\ \mathbf{D}_{Rh} &:= \mathbb{E} \left[(\mathbf{h}(0)^T \dots \mathbf{h}(Z-1)^T)^T (\mathbf{h}(0)^H \dots \mathbf{h}(Z-1)^H) \right] \\ \mathbf{M}_R &:= [1, \dots, J_0(2\pi f_D T_{Z-1})] \otimes [\mathbf{R}_{h(0)} \mathbf{C}_D^H] \quad \blacksquare \end{aligned}$$

$\otimes$ is the Kronecker product. Contrarily, to approach presented in Section III-C-1) based on a AR-1 approximation, the block-based Wiener filtering approach does not require any approximation. The performance of these estimators are illustrated in Section IX.

IV. ORDER ONE SEMI-BLIND CHANNEL ESTIMATION BASED ON PRP-OFDM POSTFIXES AND PREAMBLES

Considering a practical system, CIR estimation is usually not performed based on PRP-OFDM postfix statistics only, but rather by combining several estimates affected by additive noise vectors of different covariance. These estimates can be derived for example from reference signals (preambles, pilot tones, etc.). A special case is well illustrated by semi-blind methods that rely on an initial CIR estimate as an initialization.

Let us consider the case of two channel estimates, both affected by noise contributions of known covariance

$$\bar{\mathbf{h}}_1 := \mathbf{h} + \mathbf{n}_1, \quad \bar{\mathbf{h}}_2 := \mathbf{h} + \mathbf{n}_2. \quad (14)$$

As derived in [26], the optimum estimator of $\mathbf{h}$ in the MMSE sense is thus given by Theorem 4.1.

Theorem 4.1: The MMSE estimator of $\mathbf{h}$ in (14) is given by

$$\begin{aligned} \hat{\mathbf{h}} &:= \Lambda_1 \bar{\mathbf{h}}_1 + \Lambda_2 \bar{\mathbf{h}}_2 \\ \Lambda_1 &:= [\mathbf{I} + \mathbf{R}_{n_1} \mathbf{R}_{n_2}^{-1} + \mathbf{R}_{n_1} \mathbf{R}_h^{-1}]^{-1} \\ \Lambda_2 &:= [\mathbf{I} + \mathbf{R}_{n_2} \mathbf{R}_{n_1}^{-1} + \mathbf{R}_{n_2} \mathbf{R}_h^{-1}]^{-1} \end{aligned} \quad (15)$$

with $\mathbf{R}_h := \mathbb{E}[\mathbf{h}\mathbf{h}^H]$, $\mathbf{R}_{n_1} := \mathbb{E}[\mathbf{n}_1\mathbf{n}_1^H]$, $\mathbf{R}_{n_2} := \mathbb{E}[\mathbf{n}_2\mathbf{n}_2^H]$ and $\mathbb{E}[\mathbf{n}_1\mathbf{n}_2^H] = \mathbf{0}$. $\blacksquare$

The results obtained in previous subsection are now applied to the PRP-OFDM case for which several estimates (direct or indirect ones) of the channel impulse response are available. The goal is to combine them in the MMSE sense.

For illustration purposes, the following case is considered below: the channel estimation is based on two observations.

- 1) The convolution of a learning symbol by the channel $\mathbf{h}$, expressed as

$$\mathbf{v}_1 := \mathbf{P}\mathbf{h} + \tilde{\mathbf{n}}_1. \quad (16)$$

$\mathbf{P}$ is a circular convolution matrix built from the learning symbol coefficients, $\tilde{\mathbf{n}}_1$ is a white Gaussian noise.

- 2) The pseudorandom postfix convolved by the channel. This data is extracted by mean-value calculation over K received PRP-OFDM symbols. The IBI and ISI contributions are added up as previously presented in order to obtain a circular convolution of $\mathbf{H}$. The noise contribution can be split into 2 terms: a white Gaussian noise $\tilde{\mathbf{n}}_2$ and the mean value of the OFDM data symbol interference over the K symbols considered for the estimation

$$\begin{aligned} \mathbf{v}_2 &:= \mathbf{C}\mathbf{h} + \frac{1}{K} \sum_{i=0}^{K-1} (\tilde{\mathbf{n}}_2(i) + \mathbf{H}_{\text{ISI}}^{D \times N} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i) \\ &\quad + \mathbf{H}_{\text{IBI}}^{D \times N} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i)) \\ \mathbf{H}_{\text{ISI}}^{D \times N} &:= [\mathbf{H}_{\text{ISI}}(D) \mathbf{0}_{D \times (N-D)}] \\ \mathbf{H}_{\text{IBI}}^{D \times N} &:= [\mathbf{0}_{D \times (N-D)} \mathbf{H}_{\text{IBI}}(D)]. \end{aligned} \quad (17)$$

Some reformatting of (16) and (17) is required in order to match expressions of (14). For that purpose (16) is premultiplied by $\mathbf{P}^{-1}$ and (17) by $\mathbf{C}^{-1}$, resulting in

$$\begin{aligned} \bar{\mathbf{h}}_1 &:= \mathbf{h} + \mathbf{P}^{-1} \tilde{\mathbf{n}}_1, \\ \bar{\mathbf{h}}_2 &:= \mathbf{h} + \mathbf{C}^{-1} \frac{1}{K} \\ &\quad \times \sum_{i=1}^{K-1} (\tilde{\mathbf{n}}_2(i) + \mathbf{H}_{\text{ISI}}^{D \times N} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i) + \mathbf{H}_{\text{IBI}}^{D \times N} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i)). \end{aligned}$$

In the following, the noise covariance expressions are computed for direct utilization of (15), as follows:

$$\begin{aligned} \mathbf{R}_{n_1} &:= \mathbb{E} [\mathbf{P}^{-1} \tilde{\mathbf{n}}_1 \tilde{\mathbf{n}}_1^H \mathbf{P}^{-H}] = \sigma_{\tilde{n}_1}^2 \mathbf{P}^{-1} \mathbf{P}^{-H}, \\ \mathbf{R}_{n_2} &:= \frac{\sigma_{\tilde{n}_2}^2}{K} \mathbf{C}^{-1} \mathbf{C}^{-H} \\ &\quad + \frac{1}{K} \mathbf{C}^{-1} \mathbb{E} [\mathbf{H}_{\text{ISI}}^{D \times N} \mathbf{s}_N \mathbf{s}_N^H (\mathbf{H}_{\text{ISI}}^{D \times N})^H] \mathbf{C}^{-H} \\ &\quad + \frac{1}{K} \mathbf{C}^{-1} \mathbb{E} [\mathbf{H}_{\text{IBI}}^{D \times N} \mathbf{s}_N \mathbf{s}_N^H (\mathbf{H}_{\text{IBI}}^{D \times N})^H] \mathbf{C}^{-H} \end{aligned}$$

with $\mathbb{E}[\mathbf{H}_{\text{IBI}}^{D \times N} \mathbf{s}_N \mathbf{s}_N^H (\mathbf{H}_{\text{IBI}}^{D \times N})^H] = \mathbf{0}_{D \times D}$ and $\mathbf{R}_{s_N} = \sigma_{s_n}^2 \mathbf{I}_N$. With these expressions, the optimum combination for estimating $\mathbf{h}$ in the MMSE sense is directly available from Theorem 4.1.

This finalizes the study of PRP-OFDM-based channel equalization. In the sequel, the presentation of suitable equalization approaches follows.

V. SYMBOL RECOVERY: EQUALIZATION

When the channel is known, two steps are usually performed in order to retrieve the data: 1) equalization of the received vector $\mathbf{r}_P(i)$ and 2) soft decoding when forward-error encoding is applied at the emitter. This section focuses on the equalization step and is followed by a section dealing with the soft decoding procedure.

Several equalization strategies can be proposed for the received vector $\mathbf{r}_P(i)$.

- One can first reduce (4) to the ZP-OFDM case by simple subtraction of the known postfix convolved by the pseudocirculant channel matrix $\mathbf{r}_P^{ZP}(i) := \mathbf{r}_P(i) - \alpha(i)\hat{\mathbf{H}}_{\beta_i}\mathbf{c}_P$, where $\hat{\mathbf{H}}_{\beta_i}$ is derived from the current channel estimate. In that case, all known methods related to the ZP-OFDM can be applied. Among others, let us recall the corresponding ZF and MMSE equalizers provided in [7], [9], and [10]

$$\mathbf{G}_{ZF} := \mathbf{F}_N \mathbf{H}_o^\dagger, \mathbf{G}_{MMSE} := \mathbf{F}_N \mathbf{H}_o^H (\sigma_n^2 \mathbf{I} + \mathbf{H}_o \mathbf{H}_o^H)^{-1} \quad (18)$$

where $(\cdot)^\dagger$ stands for the Moore–Penrose pseudoinverse [22], $\mathbf{H}_o$ is the $P \times N$ matrix containing $\mathbf{H}_{ISI}(P)N$ first columns. The frequency-domain symbols $\tilde{s}_N(i)$ are assumed uncorrelated and of unitary variance. Note that other alternatives exist [7] and that with an overlap–add (OLA) approach, ZP-OFDM-OLA can attain almost same performance and complexity as CP-OFDM.

- It is also possible to directly equalize (4) relying on the diagonalization properties of pseudocirculant matrices applied to $\mathbf{H}_{\beta_i}$. According to Appendix I, we have

$$\begin{aligned} \mathbf{H}_{\beta_i} &:= \mathbf{V}_P^{-1}(i) \mathbf{D}_i \mathbf{V}_P(i), \\ \mathbf{D}_i &:= \text{diag} \left\{ H \left(\beta_i^{-\frac{1}{P}} \right), \dots, H \left(\beta_i^{-\frac{1}{P}} e^{j2\pi \frac{P-1}{P}} \right) \right\} \\ \mathbf{V}_P(i) &:= \left[\frac{1}{P} \sum_{n=0}^{P-1} |\beta_i|^{-\frac{2n}{P}} \right]^{\frac{1}{2}} \mathbf{F}_P \text{diag} \left\{ 1, \beta_i^{\frac{1}{P}}, \dots, \beta_i^{\frac{P-1}{P}} \right\}. \end{aligned} \quad (19)$$

In order to preserve the overall block variance and allow further simplifications, in the sequel, β_i is chosen as a pure phase M -phase-shift-keying (PSK) symbol $\beta_i := e^{j2\pi(m_i/M)}$, $m_i \in \{0, 1, \dots, M-1\}$. Under that condition, (19) reduces to $\mathbf{D}_i = \text{diag}\{H(e^{-j2\pi(m_i/(MP))}), \dots, H(e^{j2\pi((P-1)M-m_i)/(PM)})\}$. Thus, diagonal $\mathbf{D}_i$ is obtained for all m_i by a FFT of size PM of vector $(h_0, \dots, h_{L-1}, 0, \dots, 0)^T$.

Since the ZP-OFDM case is entirely discussed by [7] *et al.*, we discuss in the following the second case only, i.e., the equalization based on pseudocirculant matrices.

A. Zero Forcing Equalizers

The equalization matrix for (4) verifying the ZF criterion for retrieving a minimum norm estimate of $\tilde{s}_N(i)$ in (4) is $\mathbf{G}_{ZF}^{\text{PRP}} = \mathbf{H}_o^\dagger$ applied to $\hat{\mathbf{r}}_P(i) = \mathbf{r}_P(i) - \mathbf{H}_{\beta_i}\mathbf{c}_P$. Following the idea presented in [7], when $\mathbf{D}_i^{-1}$ is available, a practical low-complexity, but suboptimum, equalizer is

$$\begin{aligned} \mathbf{G}_{ZF,\text{sub}}^{\text{PRP}} &= \mathbf{F}_N [\mathbf{I}_N \mathbf{0}_{N,D}] \mathbf{H}_{\beta_i}^{-1} \\ &= \mathbf{F}_N [\mathbf{I}_N \mathbf{0}_{N,D}] \mathbf{V}_P^H(i) \mathbf{D}_i^{-1} \mathbf{V}_P(i). \end{aligned}$$

Note that the simplified ZF equalization works well for Gaussian channel, but experiences severe drawbacks for frequency selective case. This can be shown by observing that $\mathbf{D}_i^{-1}$ deals with the parallel equalization in the $\mathbf{V}_P$ frequency

domain (i.e., when dealing with Fourier transforms of length $P > N$) which potentially enhances noise when a carrier undergoes deep attenuation. This becomes an issue when switching back to the original frequency domain ($\mathbf{F}_N$: grid of size N) of the OFDM symbol through the nondiagonal $\mathbf{F}_N [\mathbf{I}_N \mathbf{0}_{N,D}] \mathbf{V}_P^H(i)$ multiplication. Indeed, this has the effect of spreading the noise over all the carriers.

Thus $\mathbf{G}_{ZF,\text{sub}}^{\text{PRP}}$ leads to poor performance and since $\mathbf{G}_{ZF}^{\text{PRP}}$ is of considerable implementation complexity, this motivates the use of MMSE equalizers in order to mitigate this issue.

B. Minimum-Mean-Square-Error Equalizers

Using standard Wiener filtering approaches [26]–[28], the following theorem is obtained.

Theorem 5.1: The biased equalization matrix in the MMSE sense for retrieving an estimate of $\tilde{s}_N(i)$ in (4) is given by

$$\begin{aligned} \mathbf{G}_{MMSE}^{\text{PRP}} &:= \mathbf{F}_N [\mathbf{I}_N \mathbf{0}_{N,D}] \mathbf{R}_{\mathbf{s}_P} \mathbf{H}_{\beta_i}^H \mathbf{Q}^{-1} \\ &= \mathbf{F}_N [\mathbf{I}_N \mathbf{0}_{N,D}] \mathbf{R}_{\mathbf{s}_P} \mathbf{V}_P^H(i) \mathbf{D}_i^H \hat{\mathbf{Q}}^{-1} \mathbf{V}_P(i) \end{aligned}$$

with $\mathbf{Q} := \mathbf{R}_{\mathbf{n}_P} + \mathbf{H}_{\beta_i} \mathbf{R}_{\mathbf{s}_P} \mathbf{H}_{\beta_i}^H$, $\hat{\mathbf{Q}} := \mathbf{R}_{\mathbf{n}_P} + \mathbf{D}_i \hat{\mathbf{R}}_{\mathbf{s}_P} \mathbf{D}_i^H$, $\mathbf{R}_{\mathbf{n}_P} := \mathbf{E}[\mathbf{n}_P(i) \mathbf{n}_P^H(i)] = \sigma_n^2 \mathbf{I}_P$, $\mathbf{R}_{\mathbf{s}_P} := \mathbf{E}[\mathbf{s}_P(i) \mathbf{s}_P^H(i)]$, $\hat{\mathbf{R}}_{\mathbf{s}_P} := \mathbf{V}_P(i) \mathbf{R}_{\mathbf{s}_P} \mathbf{V}_P^H(i)$. ■

The wording *biased* indicates that the mean phase and amplitude offset of each carrier n after equalization is not necessarily zero. Please note that in combination with ML decoders, there is no disadvantage compared to *unbiased* equalizers as defined in [27], [28]: the resulting *unbiased* equalization matrix corresponds to the *biased* one combined with a premultiplication matrix. This leaves the ML decoding expression identical for both cases.

Contrary to the noise that has a diagonal autocorrelation in the $\mathbf{V}_P$ domain $\mathbf{R}_{\mathbf{n}_P} = \sigma_n^2 \mathbf{I}_P$, this is no longer the case for the time-domain vector $\mathbf{r}_P(i)$ since it contains the deterministic postfix. Thus, the expression of $\mathbf{G}_{MMSE}^{\text{PRP}}$ does not allow an easy hardware implementation. In order to overcome this issue, the following assumption can be made resulting in a suboptimal equalizer: $\mathbf{G}_{MMSE}^{\text{PRP}}(i) \approx \mathbf{F}_N [\mathbf{I}_N \mathbf{0}_{N,D}] \mathbf{V}_P^H(i) \mathbf{D}_i^H (\sigma_n^2 \mathbf{I} + \mathbf{D}_i \mathbf{D}_i^H)^{-1} \mathbf{V}_P(i)$. This amounts to approximate $\mathbf{E}[\mathbf{s}_P(i) \mathbf{s}_P^H(i)]$ by $\sigma_s^2 \mathbf{I}_P$. In the IEEE 802.11a context, one can check that with quadrature-phase-shift-keying (QPSK) constellations, this yields to almost identical results up to 10^{-3} BER.

Globally, PRP-OFDM leads to a very simple modulation scheme on the transmitter side. In the receiver, a variety of demodulation and equalization approaches are possible, each characterized by different complexity/performance tradeoffs. Note that the channel estimation and equalization schemes presented above can be adapted to a modulator that appends a postfix sequence not after each OFDM symbol, but after an ensemble of OFDM symbols. This allows to further reduce the overhead, but the possibility is lost to keep the standard CP-OFDM decoder in combination with the low-complexity OLA transformation.

Once the equalization is achieved, the next issue to solve is how to perform optimum decoding of the equalized data symbols which is covered in the next section.

VI. SYMBOL RECOVERY: METRIC DERIVATION

In this subsection we assume that a bit interleaved convolutionally coded modulation is used at the emitter and explain how to derive the Viterbi metrics. For example, for IEEE 802.11a, a rate $R = 1/2$, constraint length $K = 7$ convolutional code (CC) (o171/o133) is applied before bit interleaving over a single OFDM block followed by QAM mapping. First, the calculations are derived in general; then, a simplified low-complexity scheme is proposed for practical implementation purposes. Note that the approach detailed below is quite general and can be extended to other coding schemes.

A. Metric Derivation

According to (4), after equalization by any of the $N \times P$ matrices $\mathbf{G}$ presented above, the vector to be decoded can generally be expressed by $\hat{\mathbf{s}} := \mathbf{G}\mathbf{r}_P(i) = \mathbf{G}_d\tilde{\mathbf{s}}_N(i) + \hat{\mathbf{n}}_N$, where $\mathbf{G}_d$ is a diagonal weighting matrix and $\hat{\mathbf{n}}_N$ the total noise plus interference contribution, which for simplicity sake is approximated here as Gaussian and zero mean.

For ML decoding, usually a log-likelihood approach is chosen based on a multivariate Gaussian law leading to the following expression [29]:

$$\hat{\mathbf{d}} := \arg \max_{\hat{\mathbf{d}}} \left\{ - \sum_{i=0}^{S-1} \left(\mathbf{G}_d m_N \left(\hat{\mathbf{d}}(i) \right) - \hat{\mathbf{s}}(i) \right)^H \times \mathbf{R}_{\hat{\mathbf{n}}_N}^{-1} \left(\mathbf{G}_d m_N \left(\hat{\mathbf{d}}(i) \right) - \hat{\mathbf{s}}(i) \right) \right\} \quad (20)$$

where vector $\hat{\mathbf{d}}$ contains an estimation of the original uncoded information bits, $\hat{\mathbf{d}}(i)$ gathers the corresponding bits after encoding, puncturing, etc., within the i th OFDM symbol. S is the number of OFDM symbols in the sequence to be decoded, $m_N(\cdot)$ is an operator representing the mapping of encoded information bits onto the N constellations, one for each carrier of the OFDM symbol.

Thus, all what is needed for performing the decoding is an estimation of the noise covariance matrix $\mathbf{R}_{\hat{\mathbf{n}}_N}$, which requires the following derivations:

$$\hat{\mathbf{s}} = \mathbf{G}\mathbf{r}_P(i) = \mathbf{G}_d\tilde{\mathbf{s}}_N(i) + \mathbf{G}_f\tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{G}_p\mathbf{c}_D + \mathbf{G}\mathbf{n}_P(i) \quad (21)$$

where $\mathbf{G}_d$ is a $N \times N$ diagonal matrix and $\mathbf{G}_f$ a $N \times N$ full matrix with the main diagonal being zero such that $\mathbf{G}_d + \mathbf{G}_f := \mathbf{G}\mathbf{H}_{\beta_1}[(\mathbf{F}_N^H)^T \mathbf{0}_{D,N}^T]^T = \mathbf{G}\mathbf{H}_{\beta_1}[\mathbf{I}_N \mathbf{0}_{D,N}^T]^T \mathbf{F}_N^H$. $\mathbf{G}_p$ is an $N \times D$ matrix containing the last D columns of the matrix $\mathbf{G}\mathbf{H}_{\beta_1}$. Thus, $\mathbf{G}_f\tilde{\mathbf{s}}_N(i)$ represents the ISI. The total noise plus interference vector is $\hat{\mathbf{n}}_N = \mathbf{G}_f\tilde{\mathbf{s}}_N(i) + \mathbf{G}\mathbf{n}_P(i) + \alpha(i)\mathbf{G}_p\mathbf{c}_D$, and its covariance is

$$\mathbf{R}_{\hat{\mathbf{n}}_N} = \sigma_s^2 \mathbf{G}_f \mathbf{G}_f^H + \sigma_n^2 \mathbf{G} \mathbf{G}^H + \mathbf{G}_p \mathbf{c}_D \mathbf{c}_D^H \mathbf{G}_p^H. \quad (22)$$

Note that in order to deal with the non-Gaussian nature of the term $\mathbf{G}_p\mathbf{c}_D$, we could use a non linear equalization scheme that would suppress its contribution by substituting to (21) by $\hat{\mathbf{s}}^{(2)} := \mathbf{G}\mathbf{r}_P(i) - \alpha(i)\mathbf{G}_p\mathbf{c}_D$.

The trouble is that the overall noise covariance presented above is not diagonal which yields to a very high complexity

decoding scheme if no approximation is applied. One way to achieve a reasonable decoding complexity is to approximate $\mathbf{R}_{\hat{\mathbf{n}}_N}$ by a matrix containing only its main diagonal elements. Then, standard OFDM Viterbi decoding can be used. In that case, (20) reduces to the classical weighted summation of the Euclidian distances by the inverse noise variances. In the following, we call these weighted Euclidian distance the Viterbi metrics. Several further simplifications are discussed in the sequel.

B. Low-Complexity Metric Proposal

Even when approximating (22) by a diagonal matrix, the metric calculation complexity is still considerably increased compared to CP-OFDM. For low-complexity sake, we verify in the following that applying the standard CP-OFDM metrics (i.e., without taking into account the intercarrier interference) only incurs a moderate loss in performance.

This approach is applied to optimum pseudo circulant MMSE equalization given by Theorem 5.1. However, in the metric derivation, the noise covariance matrix is approximated as presented in (23), where H_n , $n = 0, \dots, N-1$ are the frequency-domain channel coefficients. Basically, this simplification amounts to use the standard CP-OFDM MMSE equalizer coefficients as the weights for the Viterbi algorithm metrics. Same simplification can be applied to matrix $\mathbf{G}_d$ as indicated by (24).

$$\mathbf{R}_{\hat{\mathbf{n}}_N} = \sigma_n^2 \mathbf{G} \mathbf{G}^H \approx \sigma_n^2 \mathbf{F}_N^H \text{diag} \left(\frac{|H_0|^2}{(|H_0|^2 + \sigma_n^2)^2}, \dots, \frac{|H_{N-1}|^2}{(|H_{N-1}|^2 + \sigma_n^2)^2} \right) \mathbf{F}_N \quad (23)$$

$$\mathbf{G}_d = \text{diag} \left(\mathbf{G}\mathbf{H}_{\beta_1} [\mathbf{I}_N \mathbf{0}_{D,N}^T]^T \mathbf{F}_N^H \right) \approx \text{diag} \left(\frac{|H_0|^2}{|H_0|^2 + \sigma_n^2}, \dots, \frac{|H_{N-1}|^2}{|H_{N-1}|^2 + \sigma_n^2} \right). \quad (24)$$

In practice, one can verify that the above approximations degrade the bit-error-rate (BER) performance by approximately 0.2 dB for a binary-phase-shift-keying (BPSK) and 0.7 dB for a QPSK constellation, which is acceptable in the WLAN context while granting for PRP-OFDM a low-cost equalization architectures.

By now, channel estimation in a static and mobility scenario has been discussed followed by the derivation of suitable ML decoding metrics. The next section discusses design constraints on the actual choice of the corresponding postfix sequences and their weighting factors.

VII. CONSIDERATIONS FOR A PROPER DESIGN OF THE POSTFIX

This section provides recommendations for the design of the PRP-OFDM postfix and the choice of the pseudorandom weighting sequence.

First, it is desirable that the introduction of the pseudorandom postfix results in a flat spectrum of the signal sent onto the channel. In order to analyze the spectral properties of the PRP-OFDM signal since the signal is obviously not stationary but cyclostationary with periodicity P (duration of the OFDM block)

Fig. 3. Discrete model of the PRP-OFDM OLA demodulator.

[30], the order 0 cyclo spectrum of the transmitted time-domain sequence $s(k)$, $k \in \mathbb{N}$ has to be calculated, as follows:

$$\mathbf{S}_{s,s}^{(0)}(z) = \sum_{k \in \mathbb{Z}} z^{-k} \frac{1}{P} \sum_{l=0}^{P-1} R_{s,s}(l, k)$$

with $R_{s,s}(l, k) = E[s_{l+k} s_l^*]$. Hereby, $R_{s,s}(l, k)$ is given for the symbol $s(k = 0 \dots P-1)$ as

$$R_{s,s}(l, k) = \begin{cases} E[s_{l+k} s_l^*] & \text{for } k+l \geq 0 \text{ and } k+l < P \\ s_{l+k} s_l^* E_{\alpha} & \text{for } k+l \geq mP \text{ and } k+l < mP+D, m \in \mathbb{Z}/\{0\} \\ 0 & \text{otherwise.} \end{cases}$$

with $E_{\alpha} = E[\alpha(\lfloor (l+n)/P \rfloor) \alpha^*(\lfloor l/P \rfloor)]$. Now it is clear that it is desirable to choose $\alpha(i)$, $i \in \mathbb{Z}$ such that $E_{\alpha} = 0$ in order to clear all influence of the deterministic postfix in the second order statistics of the transmitted signal. This is achievable by choosing $\alpha(i)$ as a pseudorandom, zero-mean value.

In order to specify the content of D samples composing the postfix we can consider the following criteria:

- 1) minimize the time-domain peak-to-average-power ratio (PAPR);
- 2) minimize out-of-band radiations, i.e., concentrate signal power on useful carriers;
- 3) maximize spectral flatness over useful carriers since the channel is not known at the transmitter (do not privilege certain carriers).

The resulting postfix is obtained through a multidimensional optimization involving a complex cost function. A suitable procedure is studied in detail in [20]. Note that if the PAPR criterion is not an issue, one can directly use the Kaiser window [31].

VIII. LOW-COMPLEXITY RECEIVER ARCHITECTURE

Section V presents various equalization strategies leading to improved performances at the cost of an increased complexity. This section presents a low arithmetic complexity PRP-OFDM receiver architecture. This structure based on the OLA [32] algorithm is shown to yield similar performances as the classical CP-OFDM with similar complexity.

Fig. 3 illustrates the final receiver architecture. Compared to a standard CP-OFDM system, the following steps are included here.

- Extract the postfix convolved by the channel, considering the IBI and the ISI part separately.
- Cancel the weighting by the pseudorandom values $\alpha(i)$ and $\alpha(i-1)$. In practice, all $\alpha(i)$ are chosen such that this operation is of minimum computational cost; with a typical $\alpha(i) \in (1, -1, j, -j)$, for example, only sign inversions and exchanges of real and imaginary parts are necessary. These operations are performed in combination with the following averaging steps.
- The *expectation* of the received values is calculated; in practice, this is done by simple averaging and takes $2 \times D$ additions per OFDM symbol (in mobility context: $4 \times D$ including subtraction of oldest contribution).
- The results of the *expectation* block are weighted again by the suitable pseudorandom weighting factors.
- The PRP-OFDM symbols are transformed to ZP-OFDM by subtraction of the postfix convolved by the channel. In the same time, the OLA operation is performed. This step takes $2 \times D$ additions per OFDM symbol.

Fig. 4. BER for IEEE 802.11a, BRAN channel model A, BPSK.

Fig. 6. BER for IEEE 802.11a, BRAN channel model A, QAM-16.

Fig. 5. BER for IEEE 802.11a, BRAN channel model A, QPSK.

Fig. 7. BER for IEEE 802.11a, BRAN channel model A, QAM-64.

- The resulting OFDM symbol corresponds to the CP-OFDM case after truncation of the guard interval. All standard equalization approaches (ZF, MMSE) can be applied. Since the equalization can be performed on each carrier separately, no noise correlation issue arises.

Finally, the mean value calculation, the OLA operation and the postfix suppression takes $6 \times D$ complex additions. Taking IEEE 802.11a as an example, the OFDM-plus-postfix block is of size $P = 5 \times D$ and thus an average of 1.2 complex additions are required per time-domain sample. Each channel estimation takes an FFT operation (in order to transform the estimated postfix sequence convolved by the CIR into frequency domain) and one complex multiplication per useful carrier (assuming that the corresponding estimation coefficients have been precalculated). In a high-mobility context, the calculation of the channel estimates requires a matrix multiplication: a number of received postfixes convolved by the channel are concatenated into a vector and multiplied by the (precalculated) estimator given in Section III-C.

IX. SIMULATION RESULTS

In order to illustrate the performances of our approach, simulations have been performed in the IEEE 802.11a [2] (equivalent to HIPERLAN/2 [3]) WLAN context: a $N = 64$ carrier 20 MHz bandwidth broad-band wireless system operating in the 5.2-GHz band using a 16 sample prefix or postfix. A rate $R = 1/2$, constraint length $K = 7$ CC (o171/o133) is used before bit interleaving followed by constellation mapping (BPSK, QPSK, QAM16, QAM64).

Monte Carlo simulations are run and averaged over 2500 realizations of a normalized BRAN-A [33] frequency selective channel without Doppler in order to obtain BER curves.

Figs. 4–8 present results where the CP-OFDM modulator has been replaced by a PRP-OFDM modulator. The curves compare the classical ZF CP-OFDM transceiver (standard IEEE 802.11a) and PRP-OFDM with the MMSE equalizers combined with a transformation of the received symbols to ZP-OFDM and ZF

Fig. 8. BER for IEEE 802.11a, BRAN channel model A, QPSK, different decoding approaches.

TABLE I
TIME-DOMAIN SAMPLES OF A SUITABLE POSTFIX

Sample nb	Amplitude	Sample nb	Amplitude
1	1.5649 - 0.0356i	9	-0.4027 - 0.5203i
2	1.1404 - 0.2923i	10	-0.0363 - 0.0561i
3	-1.1347 + 0.3148i	11	0.2141 + 0.4081i
4	1.5316 + 0.1681i	12	0.3389 + 0.1818i
5	1.6562 + 0.2440i	13	0.0789 + 0.4082i
6	0.0843 + 0.4842i	14	-0.0430 - 0.2456i
7	0.0058 - 0.5014i	15	-0.0926 - 0.1566i
8	-0.9751 - 0.1925i	16	-0.0587 - 0.2248i

equalization over the $P = N + D$ carriers. Each frame contains two known training symbols, followed by 72 OFDM data symbols.

For PRP-OFDM, the postfix is chosen as given by Table I; it has been derived following the method in [20]. The channel estimation is performed based on PRP-OFDM postfix sequences only using an averaging window over 20 and 40 OFDM symbols (BPSK and QPSK), 40 and 72 OFDM symbols (QAM-16), and 120 and 240 OFDM symbols (QAM-64), respectively. Preambles or pilot tones are not used for refining the estimates. When required by the equalization structure, only *a priori* knowledge of the time-domain channel confinement is used concerning its statistics $E[\mathbf{h}_D \mathbf{h}_D^H] \approx D^{-1} \mathbf{I}_D$. As highlighted Fig. 4, when performing MMSE equalization for a BPSK, semi-blind refinement brings a 1.8-dB gain for a BER of 10^{-3} over the reference CP-OFDM curve.

For QPSK, the performance results are similar except that when doing MMSE equalization, we are still 0.75 dB away from the optimum performance reached with a perfect CIR knowledge. This gap can further be reduced by increasing the averaging window. The OLA decoding approach (low arithmetical complexity) has approximately a 1-dB penalty compared with the MMSE equalizer, but still performs better than the standard CP-OFDM case by approximately 0.2 dB at a BER of 10^{-3} and an averaging window of 20 OFDM symbols. ZF equalization

Fig. 9. MSE of CIR estimates in Doppler scenario (channel power profile is assumed to be rectangular).

performs poorly due to the noise amplification issue when performing carrier grid adaptation (switching from $P = 80$ carriers to $N = 64$).

In the context of QAM-16 constellations, the MMSE equalization leads to a gain of approximately 1 dB over the CP-OFDM approach at a BER of 10^{-3} . The OLA approach has a BER slightly below the CP-OFDM case.

For QAM-64 a large averaging window must be used and the limitations of PRP-OFDM become visible. Here, a combination of the pure postfix-based estimation with pilot-based or preamble-based estimates is recommended in order to increase the performance.

The low-complexity metric proposal of Section VI-B and the MMSE equalizer based on the diagonalization of pseudocirculant metrics given by Theorem 5.1 are evaluated for QPSK constellation by Fig. 8. The low-complexity metrics lead to a loss in BER of approx. 0.75 dB and are still slightly superior to the performances of the OLA approach. The MMSE equalizer given by Theorem 5.1 leads to strictly the same simulation results as the transformation of PRP-OFDM to ZP-OFDM does, followed by the corresponding MMSE equalization.

Note that the improvements do not find their origin in the more robust equalization scheme but rather in an improved channel estimation brought by the exploitation of the knowledge of the postfix sequence in the receiver.

Fig. 9 presents the mean-square-error (MSE) performance of different channel estimators mobility of 30 m/s. In the receiver, the channel power delay profile is assumed to be a rectangular function over the postfix duration. Fig. 10 compares these results to the optimum case where the actual channel power delay profile is known. The estimates are obtained by choosing a 80-sample carrier grid. The stability of the block-based Wiener estimator is clearly illustrated compared to the AR-1 Kalman estimator. The block-based estimator stabilizes when the averaging window increases while the AR-1 Doppler model yields to a degradation of performance due to the inherent rough model approximation.

Fig. 10. MSE of CIR estimates in Doppler scenario (exact channel power profile is known).

Fig. 11. BPSK performances in a Doppler scenario.

Fig. 11 presents the corresponding BER results in a mobility scenario. For the PRP-OFDM, after initial acquisition, the channel estimate is then refined by the semi-blind procedure explained in the paper using an averaging window of 40 OFDM symbols.

As expected, in the case of standard IEEE 802.11a or BRAN HIPERLAN/2, stretching their design limits in order to cope with mobility exceeding the specified 3 m/s considerably degrades the performance. When replacing the standard CP-OFDM modulator by a PRP-OFDM modulator and performing an MMSE-based equalization in the receiver, only a slight performance loss compared to the static case occurs at a BER larger than 10^{-3} for a mobile environment where the terminal velocity is set to 72 m/s. The total performance degradation at a BER of $5 \cdot 10^{-4}$ is approximately 1 dB compared with the PRP-OFDM static case and quasi-equivalent to

the static CP-OFDM modulator. Below a BER of 10^{-4} , the performance degradation becomes significant. At a mobility of 36 m/s, there is no performance degradation up to a BER of 10^{-4} compared to the static case. Hence, PRP-OFDM proves to be a suitable approach in order to preserve the same BER at high mobility. Note that for BPSK constellations, the reasonable system working point should be chosen at approximately $\text{BER} = 5 \cdot 10^{-4} \dots 10^{-3}$.

The simulation results demonstrate 1) in a static scenario, PRP-OFDM typically leads to a moderate performance improvement (1.8 dB for the MMSE and 1 dB for the OLA low-complexity decoding approach in the given BPSK/QPSK example) compared to a CP-OFDM system and 2) in a high-mobility scenario, PRP-OFDM typically outperforms CP-OFDM (without channel tracking) by several decibels and a corresponding error floor is lowered by several orders of magnitude in BER (at a comparable complexity for the OLA decoding approach while avoiding the pilot overhead). An extended comparison of PRP-OFDM to CP-OFDM used in combination with rotating pilot tones is presented in [34]: in a IEEE 802.11a-like scenario, PRP-OFDM typically outperforms CP-OFDM up to an SNR of approx. 15 dB (i.e., for BPSK, QPSK, QAM-16). Beyond this level (constellation QAM-64 and higher), PRP-OFDM needs to be combined with pilot tones or more advanced interference cancellation algorithms need to be applied [35].

X. CONCLUSION

In this paper, a new OFDM modulation has been presented based on a pseudorandom postfix: PRP-OFDM, using known samples instead of random data. This multicarrier scheme has the advantage to inherently provide a very simple blind channel estimation exploiting this deterministic values. The same overhead as CP-OFDM is kept. Moreover several equalization approaches have been proposed with the same robustness granted by the ZP-OFDM receivers. Suboptimal arithmetic complexity yet efficient Viterbi decoding metrics have also been detailed. Simple channel estimates were shown to be feasible, leading to improved BER.

Due to the low additional complexity requirements for the simple decoding approaches derived in Section VIII, PRP-OFDM is of advantage compared to CP-OFDM and ZP-OFDM schemes if the target application requires 1) a minimum pilot overhead, 2) low-complexity channel tracking (e.g., a IEEE 802.11a like system in a high-mobility context), and 3) adjustable receiver complexity/performance tradeoffs.

APPENDIX I PSEUDOCIRCULANT MATRICES

This appendix details the proof of the diagonalization of pseudocirculant matrices [36]. These matrices correspond to standard circulant convolution matrices with the difference that the upper triangular part is weighted by a complex factor α . In the framework of this paper, it assumed that $|\alpha| = 1$.

Proposition: The matrix $\mathbf{H}_\alpha$ defined as

$$\mathbf{H}_\alpha := \begin{bmatrix} a_0 & \alpha \cdot a_{N-1} & \alpha \cdot a_{N-2} & \rightarrow & \alpha \cdot a_1 \\ a_1 & a_0 & \alpha \cdot a_{N-1} & \rightarrow & \alpha \cdot a_2 \\ \downarrow & \searrow & \searrow & \searrow & \downarrow \\ a_{N-1} & \rightarrow & \rightarrow & \rightarrow & a_0 \end{bmatrix}$$

$$= \mathbf{H}_{\text{ISI}} + \alpha \cdot \mathbf{H}_{\text{IBI}}$$

is diagonalized as follows:

$$\mathbf{H}_\alpha = \mathbf{V}_N^{-1} \text{diag} \left\{ H \left(\alpha^{-\frac{1}{N}} \right), \dots, H \left(\alpha^{-\frac{1}{N}} e^{j \frac{2\pi(N-1)}{N}} \right) \right\} \mathbf{V}_N$$

with

$$\mathbf{V}_N := \left(\frac{1}{N} \sum_{n=0}^{N-1} |\alpha|^{-\frac{2n}{N}} \right)^{\frac{1}{2}} \mathbf{F}_N \text{diag} \left\{ 1, \dots, \alpha^{\frac{N-1}{N}} \right\},$$

$$\mathbf{F}_N := \frac{1}{\sqrt{N}} \left(W_N^{ij} \right)_{0 \leq i < N, 0 \leq j < N}, \quad W_N = e^{-j \frac{2\pi}{N}},$$

$$H(z) := \sum_{n=0}^{N-1} a_n z^{-n} \quad \text{and} \quad z, \alpha, a_0, \dots, a_{N-1} \in \mathbb{C}.$$

Proof: $\mathbf{H}_\alpha$ can be rewritten as

$$\mathbf{H}_\alpha = a_0 \mathbf{I}_N + a_1 \mathbf{J}_N^1 + \dots + a_{N-1} \mathbf{J}_N^{N-1},$$

$$\mathbf{J}_N^n = \begin{bmatrix} \mathbf{0}_{n, (N-n)} & \alpha \mathbf{I}_n \\ \mathbf{I}_{(N-n)} & \mathbf{0}_{(N-n), n} \end{bmatrix}, \quad n = 1, \dots, N.$$

Denoting by $(\lambda_k, \mathbf{x}_k)$ the eigenvalues and associated eigenvectors of $\mathbf{J}_N$, $(\lambda_k^n, \mathbf{x}_k)$ are the corresponding ones for $\mathbf{J}_N^n$. λ_k is obtained by setting $\det(\mathbf{J}_N - \lambda_k \mathbf{I}_N) = (\alpha - \lambda_k^N)^N = 0$. Thus, $\lambda_k = \alpha^{1/N} = |\alpha|^{1/N} e^{j(\phi(\alpha)/N)} e^{-j2\pi(k/N)}$, $k = 0, \dots, N-1$ where $\phi(\alpha)$ is the phase of α . By setting $\mathbf{J}_N \mathbf{x}_k = \lambda_k \mathbf{x}_k$, the unitary eigenvectors of $\mathbf{J}_N$ can be verified to be

$$\mathbf{x}_k = \left(\sum_{n=0}^{N-1} |\alpha|^{-\frac{2n}{N}} \right)^{-\frac{1}{2}} \begin{bmatrix} 1 \\ |\alpha|^{-\frac{1}{N}} e^{-j\phi(\alpha)/N} e^{jk \frac{2\pi}{N}} \\ \vdots \\ |\alpha|^{-\frac{N-1}{N}} e^{-j\phi(\alpha)(N-1)/N} e^{jk \frac{2\pi(N-1)}{N}} \end{bmatrix}$$

with $k = 0, \dots, N-1$. Stacking all the $\mathbf{x}_k$ in matrix $\mathbf{X}_N$, we have

$$\mathbf{X}_N := [\mathbf{x}_0 \mathbf{x}_1 \dots \mathbf{x}_{N-1}]$$

$$= \left[\frac{1}{N} \sum_{n=0}^{N-1} |\alpha|^{-\frac{2n}{N}} \right]^{-\frac{1}{2}} \text{diag} \left\{ 1, \dots, \alpha^{-\frac{N-1}{N}} \right\} \mathbf{F}_N^H.$$

Since $\mathbf{x}_k$ is an eigenvector of $\mathbf{J}_N$ associated to λ_k , one can easily verify that $\mathbf{H}_\alpha \mathbf{x}_k = a_0 \mathbf{x}_k + a_1 (\mathbf{J}_N \mathbf{x}_k) + \dots + a_{N-1} (\mathbf{J}_N^{N-1} \mathbf{x}_k) = H(\lambda_k^{-1}) \mathbf{x}_k$. This shows that $(H(\lambda_k^{-1}), \mathbf{x}_k)$ are the eigenvalues of $\mathbf{H}_\alpha$. Thus, $\mathbf{X}_N^{-1} \mathbf{H}_\alpha \mathbf{X}_N = \text{diag} \{ H(\lambda_0^{-1}), \dots, H(\lambda_{N-1}^{-1}) \}$ which proves that

$$\mathbf{H}_\alpha = \mathbf{X}_N \text{diag} \left\{ H \left(\alpha^{-\frac{1}{N}} \right), \dots, H \left(\alpha^{-\frac{1}{N}} e^{j \frac{2\pi(N-1)}{N}} \right) \right\} \mathbf{X}_N^{-1}.$$

Since $\mathbf{V}_N = \mathbf{X}_N^{-1}$, we reach the desired result: QED.

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Markus Muck (M'03) was born in Ulm, Germany, in 1972. He received the Diplom-Ingenieur degree in electrical engineering from the University of Stuttgart, Stuttgart, Germany, and the Ingénieur Diplômé degree from the Ecole Nationale Supérieure des Télécommunications (ENST), Paris, France, both in 1999.

Since then, he has been a Staff Member of Motorola Laboratories, Paris, France. He is actively involved in developing studies in signal processing for next-generation wireless networks and is contributing to the standardization of future generation wireless local area networks (IEEE 802.11n).

Marc de Courville (M'96) was born in Paris, France, on April 21, 1969. He graduated from the Ecole Nationale Supérieure des Télécommunications (ENST, "Engineering University"), Paris, France, in 1993 and received the Ph.D. degree, from the ENST as well, in 1996.

Since 1996, he has been working for Motorola Laboratories, Paris, France, and is now a Research Team Manager involved in projects dealing with multicarrier and ultra-wide-band systems for current and future generations of wireless local and personal area networks (IEEE 802.11a/b/g/n, IEEE 802.15.3a, and beyond). His research interests include digital communications and digital signal processing.

Pierre Duhamel (M'87–SM'87–F'98) was born in France in 1953. He received the Eng. degree in electrical engineering from the National Institute for Applied Sciences (INSA) Rennes, France, in 1975 and the Dr.Eng. and the Doctorat ès Sciences degrees from Orsay University, Orsay, France, in 1978 and 1986, respectively.

From 1975 to 1980, he was with Thomson-CSF, Paris, France, where his research interests were in circuit theory and signal processing, including digital filtering and analog fault diagnosis. In 1980, he joined

the National Research Center in Telecommunications (CNET), Issy les Moulineaux, France, where his research activities were first concerned with the design of recursive charge-coupled device (CCD) filters. Later, he worked on fast algorithms for computing Fourier transforms and convolutions and applied similar techniques to adaptive filtering, spectral analysis, and wavelet transforms. From 1993 to September 2000, he has been Professor at the Ecole Nationale Supérieure des Télécommunications (ENST, National School of Engineering in Telecommunications), Paris, France, with research activities focused on signal processing for communications, where he headed the Signal and Image Processing Department from 1997 to 2000. He is now with CNRS/LSS (Laboratoire de Signaux et Systemes), Gif-sur-Yvette, France, where he is developing studies in signal processing for communications (including equalization, iterative decoding, multicarrier systems) and signal/image processing for multimedia applications, including source coding, joint source/channel coding, watermarking, and audio processing.

Dr. Duhamel was Chairman of the DSP committee from 1996 to 1998 and a Member of the SP for Com committee until 2001. He was an Associate Editor of the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 1989 to 1991, an Associate Editor for IEEE SIGNAL PROCESSING LETTERS, and a Guest Editor for the special issue on wavelets of the IEEE TRANSACTIONS ON SIGNAL PROCESSING. He was Distinguished Lecturer, IEEE, for 1999, and was Co-general Chair of the 2001 International Workshop on Multimedia Signal Processing, Cannes, France. The paper on subspace-based methods for blind equalization, which he coauthored, received the "Best Paper Award" from the IEEE TRANSACTIONS ON SIGNAL PROCESSING in 1998. He was awarded the "Grand Prix France Telecom" by the French Science Academy in 2000.