Parametric Model Approach for Rapid SAR Measurements

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Abstract – Specific Absorption Rate (SAR) designates the electromagnetic power density deposited per unit mass of biological tissues. This paper presents a new approach where SAR calculation for mobile phones is based on a parametric reconstruction of the E-field distribution in the phantom to assess rapid SAR measurements, by means of an ellipsoidal model. The estimation of its parameters is achieved by two ways: using a reduced number of real data points exclusively, and using a combination of real data points and E-field extrapolation with a given expansion model.

Keywords – Dosimetry, Rapid SAR, Phaseless Near-Field technique, Parametric reconstruction, Optimization

I. INTRODUCTION

The experimental electromagnetic dosimetry of mobile phones has been very much developed recently. The relevant quantity to be measured is the Specific Absorption Rate (SAR), which designates the electromagnetic power density deposited per unit mass of biological tissue. SAR measurements are required to assess the compliance of mobile phones with existing standards and recommendations. Most of the existing dosimetric facilities utilize automatic positioning systems to move an isotropic E-field measuring probe in a head-like phantom [1] (Figure 1). The procedure to measure SAR is described in the European Standard prEN50361 [2], or the IEEE version [3]. Typically, no less than 300 measurement points in the phantom are prescribed for accurate SAR measurements. The electromagnetic properties of the liquid filling the phantom are similar to those of biological tissues, and are expected to provide a worst case, when compared to heterogeneous models of the head. According to the current standard procedure, a complete phone test will last about half a day. Indeed, the SAR induced by GSM mobile phones must be tested for different configurations, namely several frequencies in each band (900 MHz and 1800 MHz) and two positions with respect to the head, and for right and left hand users. Consequently, the rapidity of SAR measurement constitutes a key issue. Several attempts to reduce SAR measurement time are based, for example, on the use of a priori knowledge of the field decay in the phantom,

or on the observed correlation of the local peak SAR and the averaged SAR. In the first case, the number of measurement points can be reduced, the field probing being limited to a single plane, instead in a volume as imposed by the standard [4]. In the second one, the averaged SAR can be deduced from a quick manual search of the maximum field distribution in the phantom [5].

Fig. 1. Supélec dosimetric facility

This paper presents a new approach where SAR calculation is based on a parametric reconstruction of the E-field distribution in the phantom from a minimum number of sampling points. It has been shown that the field distribution in the phantom can be adequately described by means of an ellipsoidal model involving only 11 parameters. No more than 36 points are needed to determine these parameters from which the SAR

can be accurately deduced. As a consequence, the acquisition time is reduced from at least 10 min to only 1 min per configuration. The local SAR is given by the following formula:

$$SAR = \frac{\sigma E^2}{2\rho} \quad (\text{W} \cdot \text{kg}^{-1}) \quad (1)$$

where E (V/m) represents the electric field, σ (S/m) the conductivity and ρ (kg/m^3) the volumic mass. The averaged SAR in 1g and 10g of contiguous tissues results from the volumetric integration of formula (1). In the last section, the averaged 1g and 10g SAR results obtained from measurements conducted on a large number of commercial phones, with 36 sampling points, validate the parametric approach.

II. PHYSICAL CHARACTERISTICS OF THE E-FIELD

A. Decrease of the E-Field in function of depth

The construction of the ellipsoidal model representing the E-field distribution in a phantom is based on some physical assumptions. In a lossy homogeneous phantom, the decrease of the E-field is necessarily monotonous and follows *a priori* an exponential decay in function of depth. This has been verified with systematic SAR tests on a hundred of real mobile phones (at 2 frequencies - 900 MHz and 1800 MHz). The curves corresponding to each tested phone (Fig. 1) are distributed around the line corresponding to the theoretical propagation constant α of the plane wave:

$$E(x, y, z) = E(x, y, d) e^{-\alpha(z-d)} \quad (2)$$

For the standard materials used, and for a plane wave, $\alpha = -29.4$ Np/m at 900 MHz, and $\alpha = 41.4$ Np/m at 1800 MHz.

Fig. 2. Depth decrease of the maximum normalized E-Field, for 64 phones at 900 MHz.

B. Attenuation and propagation

In a homogeneous and isotropic media (inside the phantom), the E-field can be represented as a plane wave sum. When the media is conductive, it is however necessary to take into account the introduction of the losses. Furthermore, the presence of evanescent waves has to be studied. The E-Field inside the phantom can be seen as the integration of the angular plane wave spectrum $\mathbf{A}(\mathbf{u})$ in the considered domain Ω :

$$\mathbf{E}(\mathbf{r}) = \iint_{\Omega} \mathbf{A}(\mathbf{u}) e^{-j\mathbf{k}\mathbf{r} \cdot \mathbf{u}} d\Omega(\mathbf{u}) \quad (3)$$

with $\mathbf{u} = \{\alpha, \beta, \gamma\}$, $\alpha^2 + \beta^2 + \gamma^2 = 1$, and $\gamma = (1 - (\alpha^2 + \beta^2))^{-1/2}$.

Each plane wave can be represented with the following standard form, in which the term k , the propagation constant, is a unique complex scalar:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) e^{-j k z} \quad (4)$$

In function of the physical characteristics of the media $k^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_r$, with $\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$, the complex propagation “constant” k is expressed as follows,

$$k = \frac{\omega}{c} \left(\sqrt{\frac{|\varepsilon_r| + \varepsilon_r'}{2}} - j \sqrt{\frac{|\varepsilon_r| - \varepsilon_r'}{2}} \right) \quad (5)$$

with $k = k' - jk''$. The plane wave is represented in the homogeneous conductive media with the equation:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) e^{-j k' z} e^{-k'' z} \quad (6)$$

The imaginary part of k is responsible for the attenuation α of the wave, due to the losses in the equivalent liquid. We then introduce $\alpha = \Im(k)$, expressed in $\text{Np} \cdot \text{m}^{-1}$, which is the inverse of the penetration depth. k'' represents somewhat the skin thickness in the media. The real part k' of the wave vector allows defining the wavelength in this media. For a standard phantom and the plane wave, the values of α regarding the frequency are:

Plane wave	900 MHz	1800 MHz
α (Np/m)	-29,4	-41,4

TABLE I

PLANE WAVE ATTENUATION IN A BIOLOGICAL MEDIA

When the phantom is exposed to a mobile phone, the E-field inside is of course not a plane wave but a linear combination of plane waves. As we will see, the value of α depends strongly of the frequency, but also of the phone under test. The accuracy of the measurement, directly related to the strength of the E-field in the phantom, plays also a role in the determination of α . On the contrary, in order to describe the distribution of

the E-field in a phantom with a simple model, to limit the number of parameters, it is necessary to find a convenient way of determination for a unique propagation constant α , at least for each frequency band. The calculation of α from a few points distributed in function of depth on a line perpendicular to the planar phantom interface doesn't allow describing the exact behaviour of the electric field, but gives an interesting approach. This principle has still been applied to describe the decrease of the electric field in a phantom for a rapid SAR measurement [6]. To improve the calculation, α should be dependent of the plane wave spectrum of the E-field.

As a first approximation, the value of α is calculated for each phone as the slope value of the lines observed in the Figure 3. This figure shows the values of α obtained from the maximum amplitude of the electric field, and from the averaging of all the propagation constants calculated for each point of the measurement surface. The horizontal line represents the corresponding plane wave propagation constant value, at 900 MHz and 1800 MHz. The measurement is much less noisy in the 900 MHz band than in the 1800 MHz band, where the attenuation is stronger. The sensitivity of the E-field measuring probe (about 1 V/m) is unfortunately, for some phones, too low to give a good accuracy for the measurement.

Fig. 3. Absolute values of the mean and maximum propagation “constant” α (Np.m^{-1}) at 900 MHz for 64 phones

We have represented the distribution of the propagation constant in function of its values, by means of the histograms of the Figure 4. With the corresponding distribution functions, we then deduced the 10 % and 90 % quantiles. The q order quantile is the inverse function for the distribution function. It gives the value α_n so that $\text{Prob}(\alpha \leq \alpha_n) = q$. It allows obtaining an estimation of the upper bound of α with a probability q that this value will not be under-estimated.

The calculation of the parameter α gives a quasi gaussian distribution of the values at 900 MHz, that allows evaluating precisely, from the distribution function, the 10 % and 90 %

Fig. 4. Absolute values of the maximum propagation “constant” α (Np.m^{-1}) at 900 MHz for 64 phones represented as a histogram (a) and its corresponding distribution function (b)

quantiles. At 1800 MHz, the distributions are more difficult to characterize in terms of statistical laws. At 900 MHz, determining this value is not difficult, the different values are very close, but at 1800 MHz the measurement accuracy is poor, resulting in many different values for the propagation constant. The selected values of α representing the global decrease of the electric field in the phantom are equal to 23 Np/m at 900 MHz and 55 Np/m at 1800 MHz. The use of the penetration depth as an E-field depth decrease have been also recently used to build a fast method of averaged SAR estimation [4].

C. Geometrical E-field distribution

The characteristics of the propagation medium have an influence on the electromagnetic propagation, but also on the shape of the E-field. It is commonly accepted that the spatial distribution of the electric field is more or less of ellipsoidal type (or of “potatoe” shape, more exactly) for the large majority of the measurements done. With the benefits of an important data base of SAR and E-field measurements for about hundred commercial phones, we assume that the distribution of the E-field in the phantom is much more simple than it could appear. It can be observed, especially at 1800 MHz, but very occasionally, some distributions with double SAR spots, due to particular phones, like clam-shells for example. But even in this case, the double spot distribution can be separated in two ellipsoids. The Figure 5 shows an example of E-field shapes for typical commercial phones in the plane close to the bottom of the phantom (6 mm above it). The E-field data have been obtained with a meshing step of 7 mm, in 70 mm size squares. These examples are characteristic of those met the most often in our own data base.

III. DEGREES OF FREEDOM

The elliptic spatial distribution of the E-field and its propagation as an exponential function of depth are characteristics

Fig. 5. Classical E-field distribution in a plane normal to the surface phantom (a) and in function of depth (b)

relatively easy to model. These investigations let think that giving morphological and physical considerations on the E-field must allow introducing *a priori* information in the SAR calculation [7]. The number of data points required in the standard way to compute the SAR should be decreased, by considering that a large number of those points depend from each other from electromagnetic or morphological laws. The electric field in the phantom can then be seen, in that way, as a relatively simple system: the E-field generated by a spherical wave expansion, as a first approximation. A first estimation of the degree of freedom N of the distribution of the electric field in the phantom can be evaluated by the number of useful terms of its development in spherical modes [8]: $N = \lfloor kr \rfloor + 10$, where $k = \frac{2\pi f}{v_\phi}$ and where the square brackets indicate the largest integer smaller than or equal to kr , r is the radius of the domain of study *i.e.* the sphere of smallest radius that circumscribes the system, v_ϕ the velocity of light, f the frequency in the liquid. As an example, for $r = 3.5$ cm we obtain $N \approx 14$ for $f = 900$ MHz, and $N \approx 18$ for $f = 1800$ MHz. We can then think that about 10 parameters of a mathematical model should be enough to reconstruct the E-field in the entire volume from a reduced number of data points.

IV. PROPOSED APPROACH

It has been observed, for a large number of mobile phones, that the spatial E-field distribution, in the phantom, looks like an elliptical "spot" or "potato", as shown on Figure 2. This has led to consider an ellipsoidal model to represent such field profiles:

$$E(\mathbf{p}, \mathbf{r}) = E_0 e^{-k(1-\rho)\sqrt{(x-x_e)^2 + (y-y_e)^2 + (z-z_e)^2}} \quad (7)$$

where $\rho = \left(\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} \right)^{\frac{-1}{2}}$ and $\mathbf{r}' = \mathbf{R}_\alpha \mathbf{R}_\beta \mathbf{R}_\gamma \mathbf{r}$ with $\mathbf{R}_\alpha, \mathbf{R}_\beta, \mathbf{R}_\gamma$, the rotation matrices built from the Euler rotation angles α, β, γ , and $\mathbf{r}(x, y, z)$ the vector of positions of the E-field measurement points. $\mathbf{p}(E_0, k, x_e, y_e, z_e, a, b, c, \alpha, \beta, \gamma)$ is the vector of parameters, where E_0 represents the E-field amplitude, k is similar to the propagation term, x_e, y_e and z_e

are the coordinates of the center of the ellipsoid, and a, b and c are the semi-axes of the ellipsoid.

From the n measurement points chosen, the parameters $\mathbf{p}$ of the considered model are estimated in the sense of the least squares of the error $\xi(\mathbf{p})$ between the model $E(\mathbf{p}, \mathbf{r})$ and the measurement points $f(\mathbf{r})$:

$$\xi(\mathbf{p}, \mathbf{r}) = \|f(\mathbf{r}) - E(\mathbf{p}, \mathbf{r})\|^2 \quad (8)$$

The estimation procedure of the ellipsoidal parameters is a classical optimization resolution. The least-squares problem can be solved by using, for example, a Levenberg-Marquardt algorithm. In the estimation process, the parameters are normalized, in order to eliminate the scaling factors and then avoid the prevalence of some parameters on the others. When the parameters have been determined, the calculation of E-field at any point in the phantom is straightforward, and the computation of the maximum SAR integrated in 1 or 10 g can be performed. The choice of the measurement points in the initial mesh governs the correct estimation of the parameters. Extensive trials have shown that a good compromise consists both in one area scan in a plane close to the phantom interface and in 4 zoom scans in deeper planes (Fig. 3). The area scan allows localizing the maximum E-field area, while the zoom scans are expected to provide an information on the field decay. The area scan consists of 4×4 sampling points. The zoom scans are centered on the maximum E-field area and consist of only 2×2 sampling points. Globally 36 sampling points are required to determine the 11 model parameters. The depth of the first measurement plane (6 mm from the bottom of the phantom) is fixed for compromising sufficient sensitivity and low probe interaction with the phantom wall.

Fig. 6. standard cubic meshing of 300 E-field points (a), and fast SAR 36 E-field points meshing (b)

The first planar meshing, relatively close to the radiation source, allows to localize the maximum E-field zone, in what the maximum averaged SAR cube will necessary be, whereas the depth exploration in this area gives information on the decrease of the field. This crucial information is required for the correct estimation of the parameters. The depth of the first plane (6 mm from the bottom of the phantom) is fixed for setting sufficient sensitivity and low interaction with the phantom wall.

V. RESULTS

The parametric reconstruction technique has been assessed from 128 measurements conducted on 64 dual-band commercial GSM mobile phones, at 900 MHz and 1800 MHz. The following figures show the correlation between 1) the maximum SAR averaged on 10g and 1g obtained by the parametric reconstruction, from 36 measurement points, and 2) the same quantity deduced from the standard procedure involving 300 measurement points. The figure 7 presents the results obtained with 16 measured data points plus 5 extrapolated points calculated with the plane wave propagation constant α , while figure 8 show results obtained with 16 measured data points plus 5 extrapolated points calculated with the mean propagation value α . Those results are compared to the standards measurements obtained from 726 E-field data points. The accuracy of the method (the mean error is 10 % for the 21 measured data points) has to be compared with the complete error made with standard dosimetric facilities, which is commonly 30 % to 40 %. The reproducibility between two measurements for the same phone and with the same assessment is about 5 % to 10 %. The choice of an E-field propagation model based on real measurements is therefore better than the simplest description of the plane wave. This result confirms the previous observation of the relatively random decreases of the E-field as a function of depth. However, the approach using the plane wave propagation constant, modified by the use of lower propagation constants ensuring a maximized value for the calculated SAR, could nevertheless find an application in very fast measurements, for pre-compliance or systematic production tests for example.

Figure 9 shows SAR results obtained from 36 measurement points, near the surface and in depth of the phantom, without the use of the propagation constant. In this case, α is not explicitly used, but the measurements in depth give the information of the E-field decrease, for each phone measured. This is the only way to precisely match α for each case. The regression curve superposed to the measurement points shows the good accuracy of the parametric reconstruction approach. The mean relative error calculated is about 5 %, to be compared to the 30 % of uncertainty commonly observed for the standard measurement procedure. This configuration of 36 points is optimum to obtain a good accuracy and a fast measurement. Using only 21 measured points for example leads to a degradation of the accuracy to about 10 %.

The efficiency of the parametric reconstruction technique can be explained by the similarity existing between the actual field distribution and the model for most of the commercial phones, and by the fact that the averaged SAR involves an integration of this field distribution. On the contrary, more surprising is that the parametric reconstruction proved also very efficient, even in the case of multi-spot E-field distributions which are sometimes encountered for "flip-top" phones and/or in the upper GSM frequency band. An explanation is that, in

(a)

(b)

Fig. 7. Reconstructed SAR (W/kg) in function of the standard SAR, integrated in 1g (a) and in 10g (b), with 16 E-field points and 5 points calculated with the plane wave propagation constant.

such cases, the zoom scans are centered on the maximum spot area, which is predominantly responsible of the SAR value.

VI. CONCLUSIONS

This parametric approach applied to SAR measurement domain has already proved its efficiency with the robust estimation of other physical quantities [9]. The method proposed in this paper allows reducing the number of sampling points from 300, for the standard approach, to only 36. The error made on the 10g or 1g averaged SAR with this interpolation technique do not contribute much than a few percent to the global uncertainty of the measurement method. The acquisition time for a given configuration is about 1 minute (36 E-field points), including data processing, which is very fast. The method is fully compatible with popular instrumentation and, hence, can be directly implemented as option on most existing SAR measurement facilities using mechanical scanning of an E-field probe [10], [11]. Furthermore, the hybrid ellipsoidal-propagation constant approach, used in conjunction with an array of E-field probes, should be able to provide quasi real time

(a)

(b)

Fig. 8. Reconstructed SAR (W/kg) in function of the standard SAR, integrated in 1g (a) and in 10g (b), with 16 E-field points and 5 points calculated with the mean propagation constant.

(a)

(b)

Fig. 9. Maximum SAR averaged on 10g (a) and 1g (b), for the standard measured values (abscissa), and the values obtained from reconstructions with 36 points (ordinate), at 900 and 1800 MHz.

SAR measurement capabilities.

References

- [1] T. Schmid, O. Egger, N. Kuster, "Automated E-Field scanning system for dosimetric assessments," *IEEE Trans. MTT*, vol. 44, no. 1, pp. 105–113, Jan. 1996.
- [2] CENELEC TC211, "Basic standard for the measurement of Specific Absorption Rate related to human exposure to electromagnetic fields from mobile phones (300 MHz - 3 GHz)," *PrEN 50361 European Standard*, Jun. 2000.
- [3] IEEE 1528 Standard, "Recommended Practice for Determining the Peak Spatial-Averaged Specific Absorption Rate (SAR) in the Human Body Due to Wireless Communication Devices: Experimental Techniques," *IEEE SCC34 SC2 Standard 1528*, Jun. 2003.
- [4] M. Y. Kanda, M. Ballen, M. G. Douglas, A. Gessner, and C-K. Chou, "Fast SAR determination of gram-averaged SAR from 2-D coarse scans," *Proc. 25th Ann. Meeting of the Bioelectromagnetics Soc. (BEMS)*, pp. 45–46, Wailea, Maui, USA, Jun. 2003.
- [5] Indexsar, MapSAR system, www.indexsar.com.
- [6] M. Manning, P. Massey, "FRapid SAR Testing of Mobile Phone Prototype Using a Spherical Test Geometry," *Proc. IEE Technical Seminar on Antenna Measurements and SAR* (AMS 2002), Loughborough University, UK, May. 2002.
- [7] L. Sandu, J. Oksman, G. Fleury, "Information criteria for the choice of parametric functions for measurement," *IEEE Transactions on Measurement*, vol. 47, n4, pp. 920–924, 1998.
- [8] J. E. Hansen, J. Oksman, G. Fleury, "Spherical near-field antenna measurements," *IEEE Electromagnetic Waves Series*, n26, 1988.
- [9] G. Fleury, M.-E. Davoust, "Remote Field Eddy Current Inspection for Groove Sizing Choice of a Direct Model Structure," *Review of Progress in Quantitative Nondestructive Evaluation - American Institute of Physics*, vol. 19A, n 509, pp. 541–548, 1999.
- [10] O. Merckel, G. Fleury, J.-Ch. Bolomey, "Propagation model choice for rapid SAR measurement," *Proc. XIth European Signal Processing Conference (EUSIPCO 2002)*, pp. 80, Toulouse, France, Sept. 2002.
- [11] O. Merckel, J.-Ch. Bolomey, G. Fleury, "Extension of the parametric rapid SAR measurement to the SAM phantom," *Proc. 6th International Congress of the European Bioelectromagnetics Association (EBEA 2003)*, Budapest, Hungary, Nov. 2003.