1. Introduction to Power Engineering

1.1 Power System Functions

The traditional power system is arranged as a hierarchy. Generators feed into a high voltage transmission system that facilitates bulk transfers of power over large distances. Connected to the transmission system are medium voltage distribution networks that take power from grid supply points and deliver it to the customers who are supplied a low voltage.

For a lot of its history, the electricity supply industry has operated as a set of vertically integrated monopolies within certain geographic areas. Many countries, notably but not only, the UK, have sought to bring competition to this industry in order to provide better value to consumers. The approach in the UK, has been to unbundled the various functions within the industry and then use a combination of market mechanisms and regulation to enhance economic efficiency.

There are four functions on the supply-side:
- Generation;
- Transmission;
- Distribution and
- Supply

And one function on the demand-side:
- Consumption

The various functions are summarized in the table overleaf.
In the UK, the unbundling of the supply-side is known as “ownership unbundling” in which a company can only operate in one of the functions. Some of the large energy companies have split their business into subsidiary companies to comply with this but then must ensure there is no leakage of commercial information or influence between the subsidiaries.

There are two ways to describe the relationships between the functions based on either the trading relationships or on the flow of energy.

The trading of electricity in the UK has two fully competitive markets. The first is the generation market between generators on the one hand and on the other hand large consumers and electricity supply companies (wholesalers of electricity). This market is bilateral and allows parties to trade energy in advance of its actual delivery on any timescale they wish (from half-hour blocks to yearly commitments for instance). The generation market is frozen one hour ahead of delivery so that the technical side of grid operation can be accommodated. The second market is the supply market between the energy supply companies and domestic and small commercial consumers. Consumers can change their suppliers as they choose but will generally have a simple tariff that is not subject to the half-hour price variations of the main generation market.

The trading view of the unbundled industry makes no mention of how the energy is actually transported. The second view addresses this. Most generation is large scale and connected directly to a bulk transmission network that covers the country. This network delivers the energy to “bulk supply points” which feed a lower voltage distribution network that delivers the energy to the final consumers. There are some small generators (and plans for many more) that connect directly to the distribution system. Similarly there are some very large consumers who connect directly to the transmission network.

The British transmission system which is operated by National Grid Electricity Transmission Limited, NGETL. NGETL also owns the transmission network in England and Wales but in Scotland it is owned by Scottish Power ETL and Scottish Hydroelectric Limited. The separation of ownership of transmission from operation of transmission is taken further in the USA where Independent Transmission Operators (ISOs) operate systems in which they have no ownership of the assets.

<table>
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<tr>
<th>Function</th>
<th>Method</th>
<th>Examples (UK)</th>
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<tr>
<td>Generation</td>
<td>Steam, gas, water or wind turbines driving alternators</td>
<td>nPower, E.On, British Energy, SELCHP, Barking Power</td>
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<tr>
<td>Transmission</td>
<td>275kV &amp; 400kV overhead lines – “the national grid”</td>
<td>National Grid (owner and operator), Scottish Power (owner)</td>
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<tr>
<td>Distribution</td>
<td>132kV, 33kV, 11kV overhead lines and cables</td>
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<td>Supply</td>
<td>Purchase of energy on wholesale market, resell and bill</td>
<td>EDF Energy, E.On, British Gas</td>
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<tr>
<td>Consumption</td>
<td>Motors, heaters, lighting &amp; supplies for electronic equipment</td>
<td>Industrial, commercial and domestic consumers</td>
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Participants in Wholesale and Retail Electrical Energy Markets and the Ancillary Services Market

Energy Flows Between Generators and Consumers via Transmission and Distribution Networks
The GB Transmission Network
There are 15 distribution network operators (DNOs) in the UK.

1.2 Transmission and Distribution

There are many ways in which a transmission and distribution system could be configured but systems throughout the world have the same features.

1.2.1 Voltage Source System

A cable or overhead line has a series impedance and a shunt admittance. The series impedance is the (small) resistance of the copper (or aluminium etc.) wire and the inductive reactance of the wire path. The shunt admittance is the (very small) conductivity of the insulators and the capacitive susceptance between wires.

\[ P_{Loss}^{\text{Ser}} = I^2 R \]

\[ P_{Loss}^{\text{Ins}} = V^2 G \]

Power losses occur in the wire resistance \((I^2 R)\) and the insulator conductance \((V^2 G)\). In general, insulators are closer to ideal than conductors and the power loss in the insulator is often negligible. Considering only conductor power loss, a voltage source (variable current) system has losses that
decrease as the load decreases. A current source system would have constant loss for any load (including no load)

### 1.2.2 Impedance Matching

Amplifiers are used with a load impedance matched to the output impedance in order to maximise power transfer. Maximum power transfer is not the same as maximum efficiency. In fact, with resistive impedances, resistance matching gives 50% efficiency. For instance, a 600MW power station generator would lose 300MW of its power in the (very hot) cables to the loads. The main consideration with a power system is to maintain the consumer’s voltage within a few percent of its nominal value. For this reason the network is run with low source impedance and high load impedance.

![Diagram of impedance matching](image)

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\[ V = I R + P R \]

\[ P = \frac{V^2}{2r} \]

\[ \eta = \frac{P_R}{P_R + P_L} \]

\[ 100\% \]

\[ 50\% \]

### 1.2.3 A Variety of Voltages

Power, dependent on the V.I product, can be transmitted with any value V and a corresponding value of I. The loss is related to \( I^2 \) and so higher voltages reduce power loss and increase efficiency. At very high voltages the \( V^2/G \) losses become a significant factor. Voltages of between 500kV and 100kV are used for transmission. Distribution involves lower power levels on each route. The capital cost of H.V. equipment is prohibitive compared with the small saving in running costs (losses). Medium voltages of the between 100kV and 10kV are used. Safety considerations require most users to be supplied at voltages of 250V or less.

### 1.2.4 Alternating Current (and Voltage)

The traditional way (and still the only cost effective way) of changing voltage levels in a power system is the transformer. Transformers require an AC system because voltage can only be supported by changing flux \( (E=N.d\Phi/dt) \) and AC is the only way of obtaining continuously changing, but finite, flux. There are exceptions, DC transmission is used to link AC systems of different characteristics and DC is used in some isolated distribution systems such as on large ships.

### 1.2.5 Sinewave Voltage

The voltages and currents of inductors and capacitors are related by differential equations. A sine wave is the only AC form that will ensure that all voltage drops and leakage currents in the system are the same form as the source waveform.

It is often supposed that generators naturally produce sinewave voltages. This is true of simplistic air-cored machines but not of realistic iron-cored machines. Considerable design effort is required to achieve sinewave voltages.
1.2.6 50/60Hz
The frequency of an AC power system is directly related to the rotational speed of the generators. 3,000 or 3,600 rpm was a sensible choice (considering the bearings and materials available) when systems evolved. Higher frequencies would reduce transformer size but cause larger inductive voltage drops and capacitive leakage currents. Lower frequencies cause noticeable lighting flicker and require larger transformers. Higher frequencies are used where weight and size are an issue such as on ships and aircraft.

1.2.7 Three-Phases
Generators are best built with a large number of coils distributed around the machine. So that a practicable number of connections can be used outside the machine, the coils are grouped into phases. The choice of three phases is a sensible compromise between number of connections and good utilisation of the machine. More than one phase is essential to facilitate smoothly rotating AC motors.

1.3 Electricity Grid Operation
A grid system is expected to provide a secure source of energy whenever a customer demands it. The grid operator must balance the demand with the supply of energy from various generators.

There is a daily pattern of load variation that is reasonably predictable. It is different at weekends and different from one season to the next. The highest demand is in the early evening of a winter weekday. The absolute peak varies slightly from year to year depending on the weather conditions. In 2009 it was 60.2 GW (in 2005 it was 61.7 GW and in 2006 it was 59.1 GW)

![Figure 2.2 - GB Summer and Winter Daily Demand Profiles in 2004/05](image)

The system operator must plan for unusual demand patterns such as "television pick-up". This is the sudden increase in load when a popular television programme ends. This can provoke a rise of 1000 MW in demand in a matter of a few minutes. Major television events create even larger changes. The largest events are over 2,000 MW
Most TV events can be predicted reasonably well based on previous experience. It is highly unusual events that cause problems. It would also seem that the more diverse TV and entertainment market that now exists means that the TV pick-up is less severe because the audience for each TV programme is less.

The largest ever TV pick-up in the UK was 2,800 MW of demand after the penalty shootout in an England v. Germany world cup semi-final in 1990.

An example of a highly unusual event causing the system operator to work hard to maintain a stable system was the two-minute silence at 11am on 14th September 2001 in remembrance of the deaths on 11th September.

In the minutes before and during the silence some 2,500 MW of load was switched off. This requires two large power stations to be removed from the system very quickly. We can tell that this was not achieved quickly enough because the grid frequency rose above its target range. When generation exceeds demand the excess energy builds up in the inertia of the generators and the frequency rises. The “missing” 2500 MW of load reappeared over the next 5 minutes and because there was now a shortage of generation the frequency briefly dipped below its target range.

A more recently, National Grid seems to have done a remarkably good job of managing some difficult demand pick-up and drop-off associated with the Royal Wedding. They would have looked back in the records more than 10 years for the last similar wedding but would have found direct comparison tricky because of the changes in TV viewing habits over ten years and the fact that this time a back holiday...
was declared and last time it was a normal work day. Nonetheless, they managed to stop any large frequency excursions occurring despite swings in demand of more than 3,000 MW over an hour (actually a lot slower than a 2-minute silence).

Faults in either generator or the transmission network itself cause system operators the greatest problems because these are very fast events. The GB system operators to a 1320 MW loss-of-in-feed standard. The large 2,000 MW coal fired plat (such as Radcliffe-on-Soar or Drax) have four 500 MW turbine-generators run from two boilers with a double circuit connection to the grid. No single fault can cause all 2,000 MW to be lost in one go. The Sizewell B pressurised water reactor has two 660 MW turbine generators but only one steam circuit so the whole lot can get shut down in one go if there is a fault in the steam circuit. So National Grid carries enough reserve to deal with this loss if Sizewell B is running. In May 2008, Sizewell B did shut down unexpectedly which would have been OK but for the fact that a 500 MW generator at Longannet had also been lost unexpectedly about 2 minutes earlier. The loss of 1,582 MW of actually generation in such a short time was not covered by the reserve and the frequency dropped below the operational limit (49.8 Hz) and then below the statutory limit (49.5 Hz) (a breach of National Grid’s licence agreement unless an exceptional event). The frequency reached about 49.15 Hz before its fall was arrested. Then an unfortunate thing happened. A lot of small wind turbines other small generators (adding up to about 300 MW) disconnected 3 minutes later. These had been fitted with protection relays that interpreted a 3 minute under-frequency event as break-up of the system and were shut down to prevent electrical islands forming. The result was that the frequency dropped even further and this lead to automatic under-frequency load shedding. Put more bluntly, supply to parts of certain towns were shut off.
Electricity in Great Britain is traded in bilateral contracts between generators and large customers (including the energy supply companies) under a system known as BETA. Each generator and purchaser must declare its future output or demand half an hour before real time (the “gate closure” time). These predictions of generation and load are never exactly realised when the time comes. Generators sometimes develop faults, wind turbines have too little or too much wind, the televised football match goes into extra time, the forecast early morning frost does not happen. The system operator (National Grid PLC) must run a second-by-second “balancing operation” using one power station to make up for losses in the system and the mismatch between supply and demand. It does this by monitoring the system frequency and adjusting its power accordingly. This is known as primary response. Additional, slower response, known as secondary response can be instructed to respond by the National Grid control centre. If the balancing operation fails, the voltage and frequency of the system will fall and loads will have to be shed in order to avoid system-wide collapse. Some of the other power stations around the system will be used to regulate the voltage of the grid through adjustment of their excitation. National Grid contracts certain generators to provide these services. To recover these costs, National Grid applies use-of-system charges to all users of the grid and out-of-balance charges to users who exceed or fall-short of their predicted use.

A grid system is an efficient means of transporting energy but there are some losses. The local distribution system is less efficient because of the lower voltages used. Of the approximately 10p per kWh charged to domestic consumers in the UK, about 50% is accounted for by costs of generating and the remainder by the losses and operating costs in transmission (5%), losses and operating costs in distribution (30%) and the metering & billing costs (15%).
1.4 Revision of AC Power Calculation

We chose the cosine as the standard sinusoidal signal and note a cosine is the real part of a complex exponential

\[
\begin{align*}
v &= \hat{V} \cos(\omega t + \phi) \\
e^{j\theta} &= \cos(\theta) + j \sin(\theta) \\
v &= \text{Re}(\hat{V} e^{j(\omega t + \phi)}) = \text{Re}(\hat{V} e^{j\phi} e^{j\omega t})
\end{align*}
\]

In this final representation, the signal has three parts:

- \( \hat{V} \) is the magnitude of the signal
- \( e^{j\phi} \) represents phase angle of the signal
- \( e^{j\omega t} \) represents the sinusoidal variation at the system frequency

We only need to retain the magnitude and phase angle which together form the phasor

\[
\hat{V} = \hat{V} e^{j\phi} = \hat{V} e^{j(\omega t + \phi)} / e^{j\omega t}
\]

The ratio of voltage and current phasors gives impedance, \( Z \), in its complex form. \( \text{Re}(Z) \) is resistance and \( \text{Im}(Z) \) is reactance; both measured in ohms. The resistance of a conductor under dc and ac conditions is the same unless factors such as skin effect are relevant (Skin depth in copper at 50Hz is 9.3 mm; only conductors thicker than this experience significant skin effect). Reactance is a feature of energy storage components, for example inductors and capacitors.

\[
\begin{align*}
\overline{Z}_L &= jX_L = j\omega L \\
\overline{Z}_C &= -jX_C = -j \frac{1}{j\omega C} = \frac{1}{\omega C}
\end{align*}
\]

Note how the complex impedance creates the expected phase difference between the voltage and current vectors.

\[
\overline{V} = \overline{I} \overline{Z} \\
\therefore \angle \overline{V} = \angle \overline{I} + \angle \overline{Z}
\]

For an inductor

\[
\angle \overline{Z} = \angle (j\omega L) = 90^\circ \\
\angle \overline{V} = \angle \overline{I} + 90^\circ
\]

For a capacitor

\[
\angle \overline{Z} = \angle \left( \frac{-j}{\omega C} \right) = -90^\circ \\
\angle \overline{V} = \angle \overline{I} - 90^\circ
\]

With the definitions of phasors and complex impedances in place, all the standard circuit analysis techniques for DC steady-state can be applied to sinusoidal steady-state provided vector (rather than scalar) arithmetic is used.
1.4.1 Power: Real and Otherwise

Power is energy flow and is measured in watts (equivalent to joules per second). Instantaneous power is the product of voltage and current at that instant.

The voltages and currents are defined using the “passive” convention shown in the figure. A positive value of $p$ represents energy flowing into the component and a negative value is energy flowing out.

$$p(t) = v_{AB}(t) i_{AB}(t)$$

The energy flow may be occurring for one (or both) of two reasons:

1. Energy storage, such as occurs in inductors and capacitors or
2. Dissipation, such as the conversion of electrical energy to heat, kinetic energy or $em$-radiated energy.

If the instantaneous power is averaged (under steady-state conditions) then the average includes only the dissipative terms because the energy storage components store and release equal amounts of energy over a cycle (as is necessary for steady-state to exist).

An average power is calculated by integrating the instantaneous power over one cycle and dividing by the period. The average power is also known as real power.

$$P = P^{Avg} = \frac{1}{T} \int_0^T p(t)dt = \frac{1}{T} \int_0^T i(t)v(t)dt$$
The stored and returned power is known as reactive power and given the symbol $Q$. Because it is not dissipative it should not have the units of watts but a new unit (of the same dimensions) known as volt-amperes reactive, VAr.

Most electrical equipment has a voltage limit and a current limit. These combine to give a volt-ampere rating or apparent power rating. The unit of apparent power is VA. It does not specify how much of the volt-ampere is real (dissipative) power and how much reactive (temporary storage) power.

1.4.2 Average Power

For a resistor, we expect a non-zero result for the average power.

$$P_R = P_R^{\text{avg}} = \frac{1}{2\pi} \int_{-\pi}^{\pi} i(\omega t) R \, d(\omega t) = \frac{1}{2\pi} \int \hat{I} \hat{V}_R \cos^2(\omega t) \, d(\omega t)$$

$$= \frac{1}{2\pi} \int \frac{1}{2} \hat{I} \hat{V}_R (1 + \cos(2\omega t)) \, d(\omega t)$$

$$= \frac{1}{2} \hat{I} \hat{V}_R \quad \text{or} \quad \frac{1}{2} \hat{I}^2 R$$

If we perform the average using the $\hat{I}R$ form of the power equation, we can see that the mean of the square of the current is an important definition.

$$P_R = \frac{1}{2\pi} \int_{-\pi}^{\pi} i^2(\omega t) R \, d(\omega t) = R \frac{1}{2\pi} \int_{-\pi}^{\pi} i^2(\omega t) \, d(\omega t)$$

$$= I^{\text{MS}} R$$

where $I^{\text{MS}} = \frac{1}{2\pi} \int_{-\pi}^{\pi} i^2(\omega t) \, d(\omega t)$

The mean square is an inconvenient term to deal with (and has units of $A^2$); it is better to define a root-mean-square form of average current (or voltage) for use in these calculations.

$$I_{\text{RMS}} = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} i^2(\omega t) \, d(\omega t)}$$

We can then state:

$$P_R = \frac{1}{2} \hat{I} \hat{V}_R$$

$$= I_{\text{RMS}} I_{\text{RMS}}^R \quad \text{or} \quad \left( I_{\text{RMS}} \right)^2 R$$

For a sinusoidal waveform there is a simple relationship between the peak and RMS measures.

$$I_{\text{RMS}} = \hat{i} \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} \cos^2(\omega t) \, d(\omega t)}$$

$$= \frac{1}{\sqrt{2}} \hat{i}$$

The use of RMS is so widespread in power engineering that we expect magnitudes of all voltages and currents to be quoted in RMS. The superscript RMS is dropped and we assume a quantity is RMS unless told otherwise.

The average power of the inductor is zero because all energy stored in one half of the cycle is released in the other.
1.4.3 General Case for Calculating Power

To calculate power for general components, not just the basic R, L and C, the real power is given by the component of voltage that is in phase with the current (the component that is resistor-like).

\[
P_L = p_L^{avg} = \frac{1}{2\pi} \int_{-\pi}^{\pi} P_L d(\omega t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} I V_L \sin(\omega t) \cos(\omega t) d(\omega t)
\]
\[
= \frac{1}{2\pi} \int_{-\pi}^{\pi} I V_I \frac{1}{2} \sin(2\omega t) d(\omega t)
\]
\[
= 0
\]

The real power can be expressed as a dot product.

\[
P = \bar{I} \cdot \bar{V}
\]
\[
= I V \cos(\phi)
\]
\[
= I V_P
\]

where \( \phi = \angle \bar{V} - \angle \bar{I} \) and

Note that the convention here is that the magnitude of a vector is shown by dropping the over-bar, \( |\bar{I}| = I \).

Because \( V_P = IZ\cos(\phi) \), the real power can also be written as \( P = I^2 \Re(\bar{Z}) \)

By analogy we define the reactive in terms of the component of voltage, \( V_Q \) in quadrature (90° out of phase) with the current.

\[
Q = I V_Q = I V \sin(\phi)
\]

or

\[
Q = I^2 \Im(\bar{Z})
\]

The apparent power is defined as the product of voltage and current without regard to the angle:

\[
S = I V
\]
\[
= I(V_R^2 + V_Q^2)^{\frac{1}{2}} = (P^2 + Q^2)^{\frac{1}{2}}
\]

The ratio of real power to apparent power is defined as the power factor. For the sinusoidal case it is related to the impedance angle.
\[ PF = \frac{P}{S} = \frac{IV \cos(\phi)}{IV} = \cos(\phi) \]

It can also be shown that the real power is given by the real part of the voltage vector multiplied by the conjugate of the current vector. This leads to a definition of complex power that is a convenient method of finding real and reactive powers from voltages and currents in vector (phasor) form.

\[
\bar{S} = \bar{V} \bar{I}^* = P + jQ
\]

\[
P = \text{Re}\{\bar{V} \bar{I}^*\}
\]

\[
Q = \text{Im}\{\bar{V} \bar{I}^*\}
\]

**Recap:** We have four types of power: instantaneous, real, reactive and apparent. Only the first two are really about converting energy from one form to another and are measured in watts <W>. Apparent power is not necessarily about energy conversion and so is measured in volt-ampere <VA>. Reactive power is measured in volt-amperes reactive <VAr>. 