Entropic Time

Ariel Caticha
Department of Physics
University at Albany - SUNY

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Chamonix

Question:

Do the laws of Physics reflect Laws of Nature? Or...

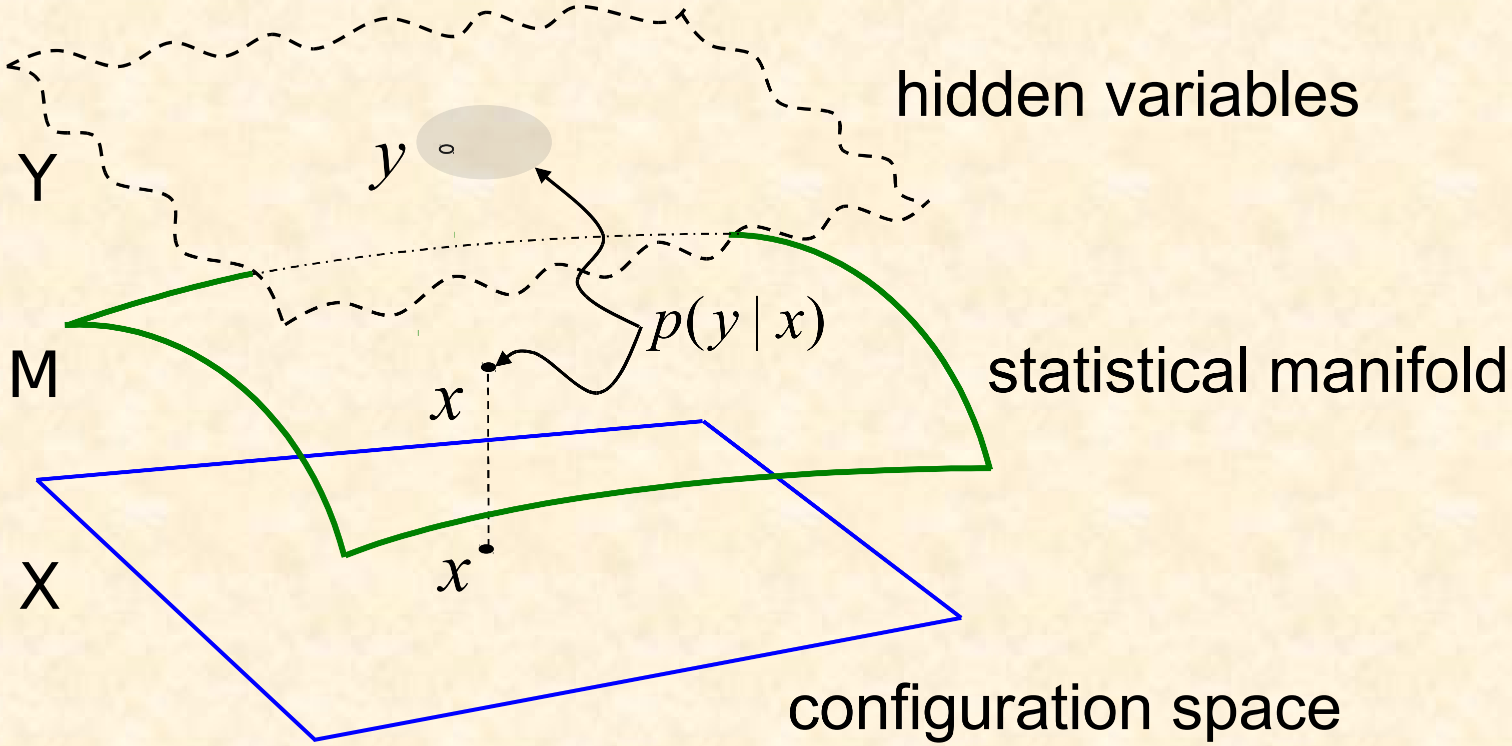
Are they rules for processing information about Nature?

Our objective:

To derive Quantum Theory as Entropic Dynamics

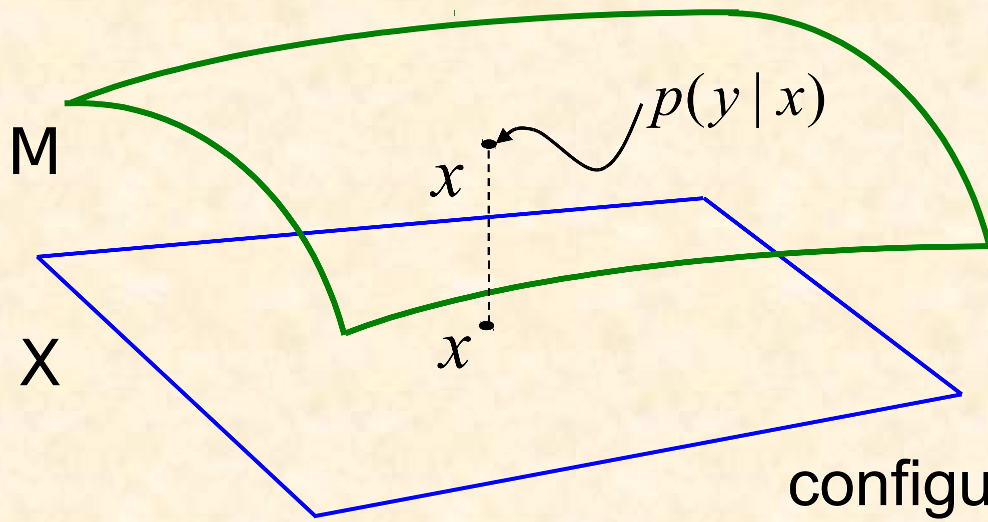
and focus on the implications for the theory of time.

Step 1: The Statistical Model



$$\gamma_{ab} = \frac{\delta_{ab}}{\sigma^2}$$

Step 1: The Statistical Model



statistical manifold

configuration space with metric $\gamma_{ab} = \frac{\delta_{ab}}{\sigma^2}$

Step 2: Entropic Dynamics

Maximize

$$S_J[P,Q] = -\int dx'dy'\, P(x',y'|x) \log \frac{P(x',y'|x)}{Q(x',y'|x)}$$

uniform

$$P(x'|x)\,P(y'|x',x)$$

$\in$ M

Changes happen gradually.

Short steps: $\langle \Delta \ell^2 \rangle = \langle \gamma_{ab} \Delta x^a \Delta x^b \rangle = \kappa$

The transition probability:

$$P(x' \mid x) = \frac{1}{\zeta} \exp[S(x') - \frac{1}{2}\alpha(x)\Delta\ell^2]$$

where $$S(x') = -\int dy' \, p(y' \mid x') \log \frac{p(y' \mid x')}{q(y')}$$

For short steps:

$$P(x' \mid x) \propto \exp[-\frac{\alpha}{2\sigma^2}\delta_{ab}(\Delta x^a - \Delta\bar{x}^a)(\Delta x^b - \Delta\bar{x}^b)]$$

Displacement: $\Delta x = \Delta\bar{x} + \Delta w$

Expected drift : $\Delta\bar{x}^a = \frac{\sigma^2}{\alpha}\delta^{ab}\,\partial_b S(x)$

Fluctuations: $\langle \Delta w^a \Delta w^b \rangle = \frac{\sigma^2}{\alpha}\delta^{ab}$

Step 3: Entropic Time

The foundation of any notion of time is dynamics.

Time is introduced to keep track of the accumulation of many small changes.

$$P(x') = \int dx\, P(x',x) = \int dx\, P(x'|x)P(x)$$

(1) Introduce the notion of an instant

$$\rho(x',t') = \int dx\, P(x'|x)\rho(x,t)$$

The Arrow of Entropic Time

A time-reversed evolution:

$$\rho(x,t) = \int dx' \, P(x \mid x') \rho(x',t')$$

Bayes' theorem $$P(x \mid x') = \frac{P(x)}{P(x')} P(x' \mid x)$$

There is no symmetry between prior and posterior.

Entropic time only goes forward.

Entropic time *vs.* physical time?

(2) Introduce the notion of interval between instants

For large α the dynamics is all in the fluctuations:

$$\langle \Delta w^a \Delta w^b \rangle = \frac{\sigma^2}{\alpha}\delta^{ab} \quad = \Delta t \frac{\sigma^2}{\tau}\delta^{ab}$$

Define duration so that motion looks simple: $\alpha = \dfrac{\tau}{\Delta t}$

Fokker-Planck equation: $\partial_t \rho = -\partial_a(\rho v^a)$

$$v^a = \frac{\sigma^2}{\tau}\partial^a \phi \qquad \phi(x,t) = S(x) - \log \rho^{1/2}(x,t)$$

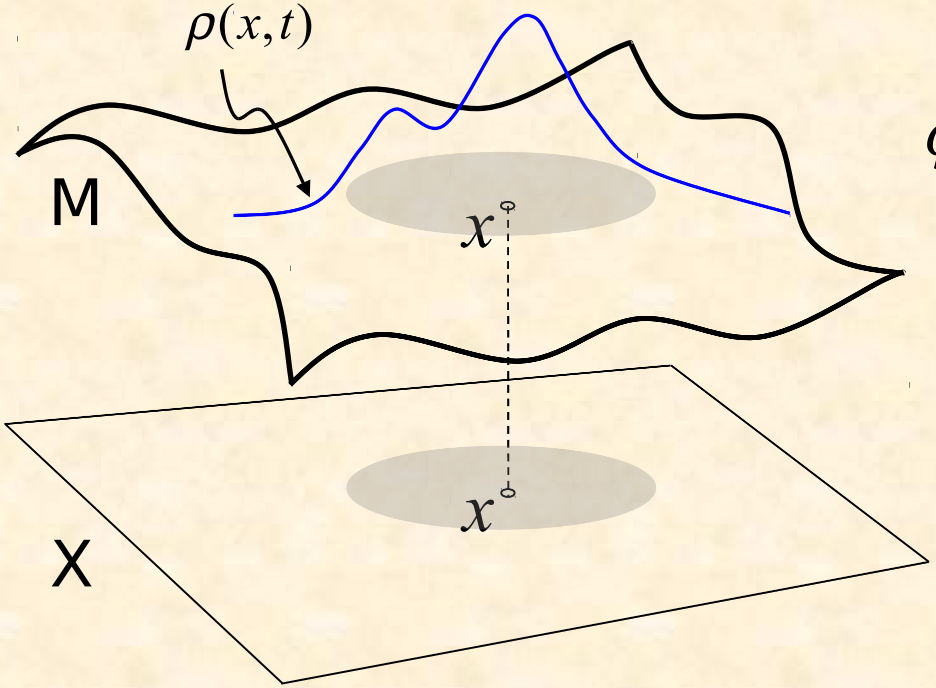
two components: $v^a = b^a + u^a$

drift velocity: $b^a = \frac{\sigma^2}{\tau}\partial^a S$

osmotic velocity: $u^a = -\frac{\sigma^2}{\tau}\partial^a \log\rho^{1/2}$

But this is just diffusion, not quantum mechanics!

A wave function requires two degrees of freedom.



$$\phi(x,t) = S(x,t) - \log \rho^{1/2}(x,t)$$

Step 4: Manifold dynamics?

"Energy" conservation [Nelson (1979), Smolin (2006)]

$$E = \int d^3x \, \rho \left[\frac{1}{2} m v^2 + \frac{1}{2} m u^2 + V(x) \right]$$

where $m = \frac{\square\tau}{\sigma^2}$

The result: two coupled equations

1) Fokker-Planck/diffusion equation

$$\dot{\rho} = -\frac{\hbar}{m}\partial^a\left(\rho\,\partial_a\phi\right)$$

2) energy conservation + diffusion

$$\hbar\dot{\phi} + \frac{\hbar^2}{2m}(\partial_a\phi)^2 + V - \frac{\hbar^2}{2m}\frac{\nabla^2\rho^{1/2}}{\rho^{1/2}} = 0$$

Combine ρ and ϕ into $\Psi = \rho^{1/2} e^{i\phi}$

to get Quantum Mechanics, $i\hbar \dfrac{\partial \Psi}{\partial t} = -\dfrac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$

and also Classical Mechanics $F = ma$

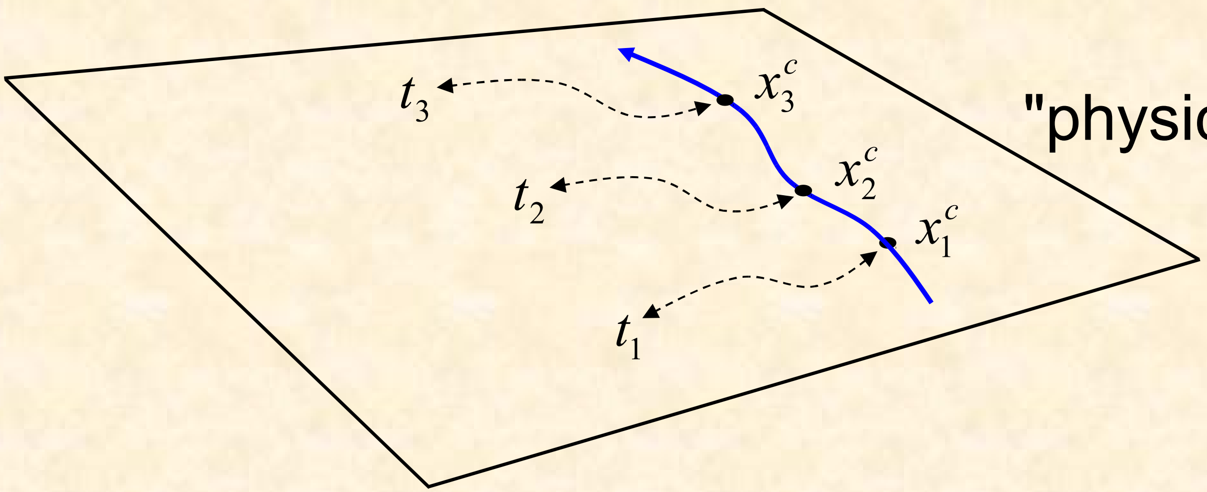
Let $S_{HJ} = \hbar\phi$ then

$$\dot{S}_{HJ} + \frac{1}{2m}(\partial_a S_{HJ})^2 + V = 0$$

$$\langle \Delta w^a \Delta w^b \rangle = \Delta t \frac{\hbar}{m}\delta^{ab} \to 0$$

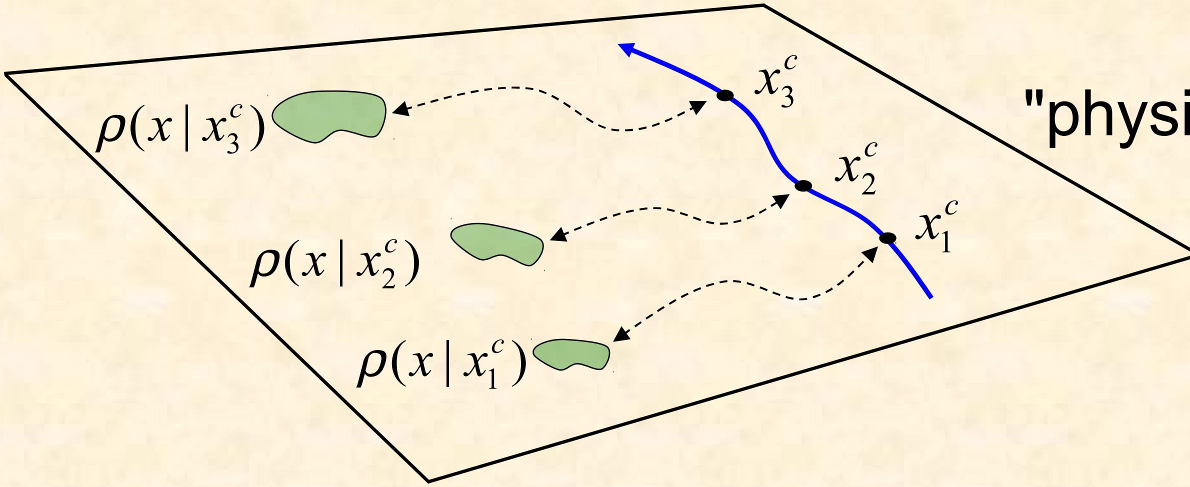
More on Entropic Time

A clock follows a classical trajectory.



"physical" space

For the composite system of particle and clock:

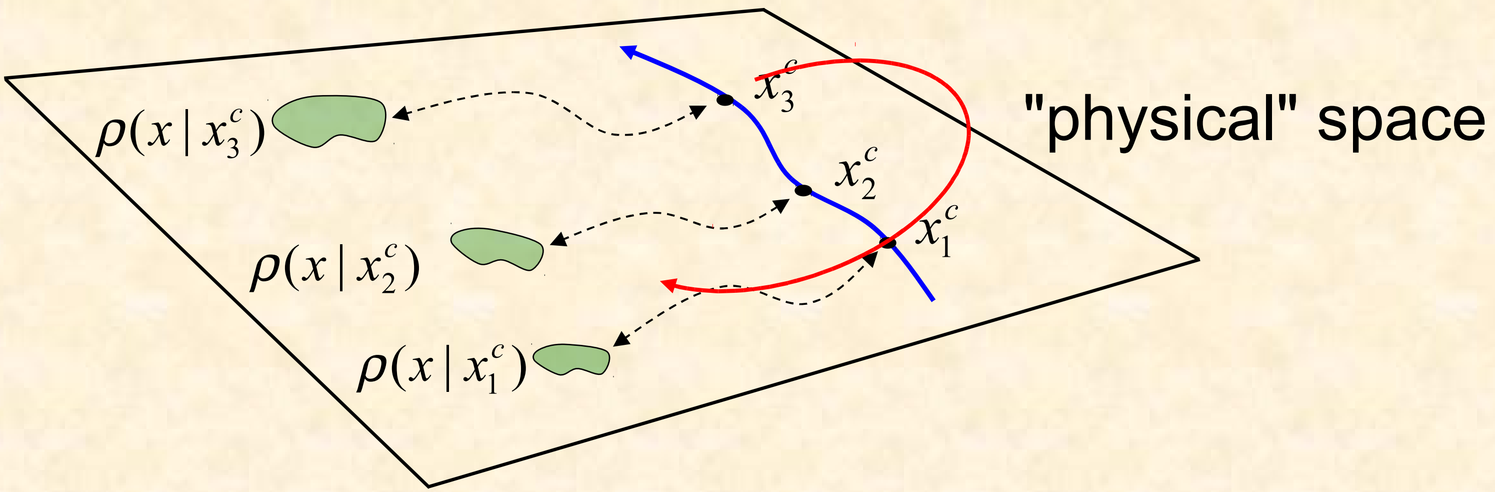


"physical" space

Entropic time *vs* "physical" time?

We observe correlations at an instant.

We do not observe the "absolute" order of the instants.



Entropic time is all we need.

Conclusion:

Entropic inference leads to Laws of Physics.

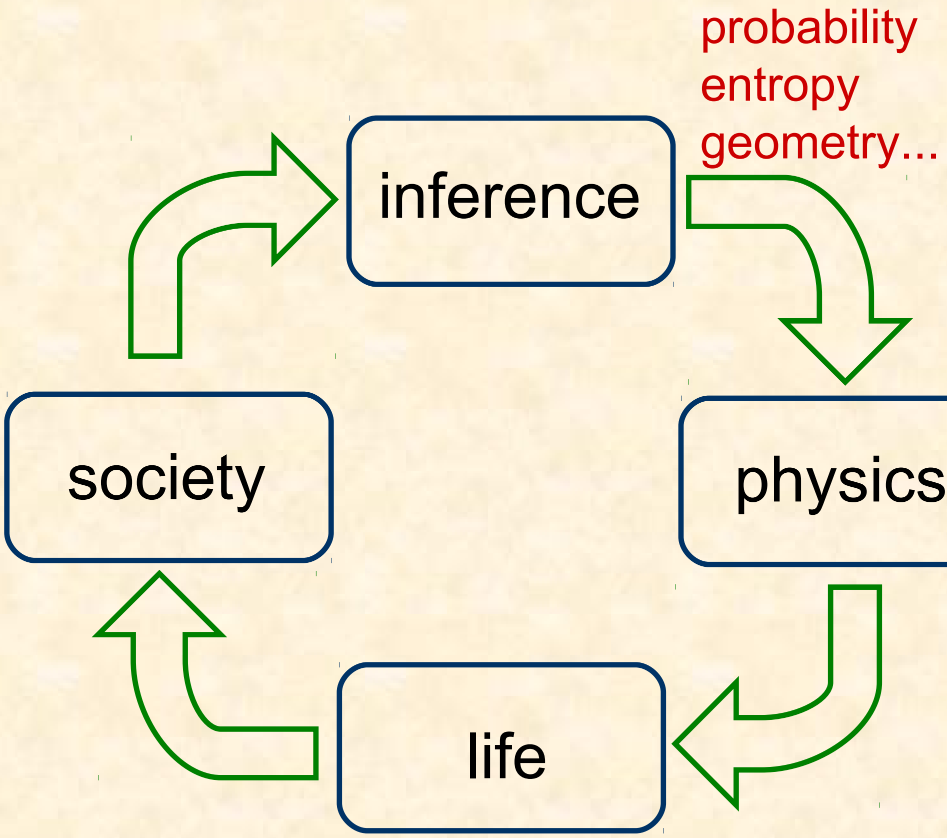
It provides an alternative to Action Principles.

The t in Laws of Physics is entropic time.

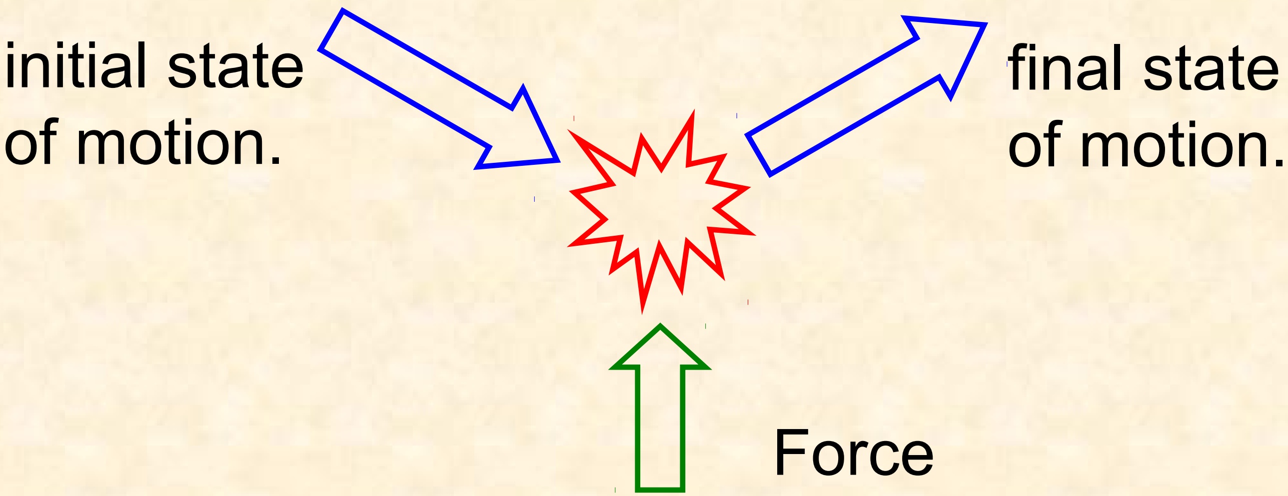
The natural order/arrow of inference is entropic time...

...and this is the only time we need.

The Big Picture:

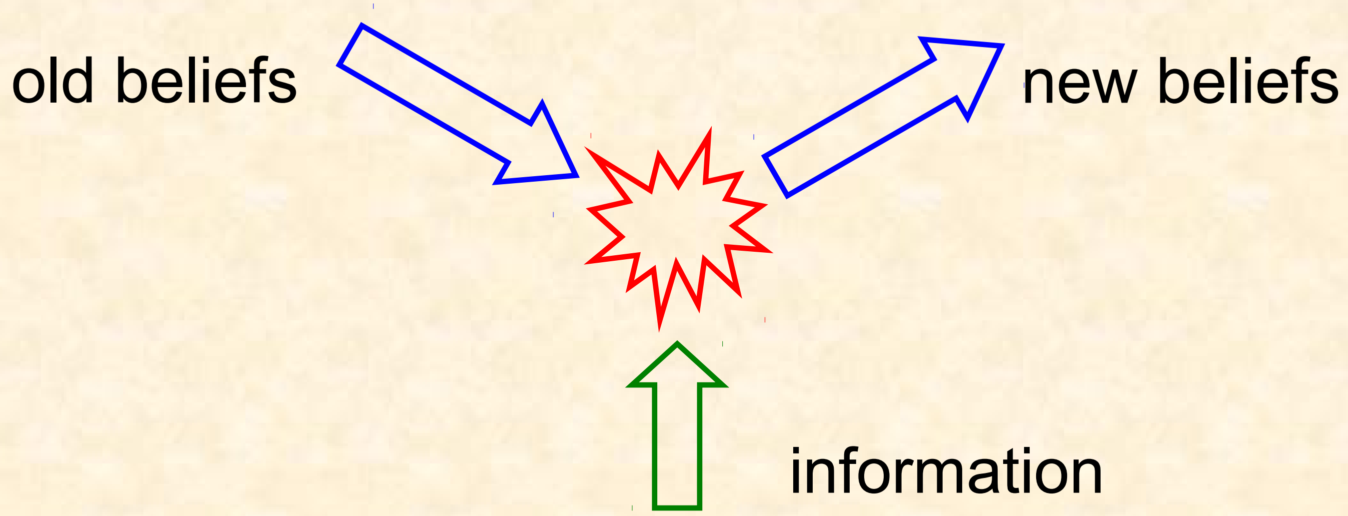


An analogy from physics:



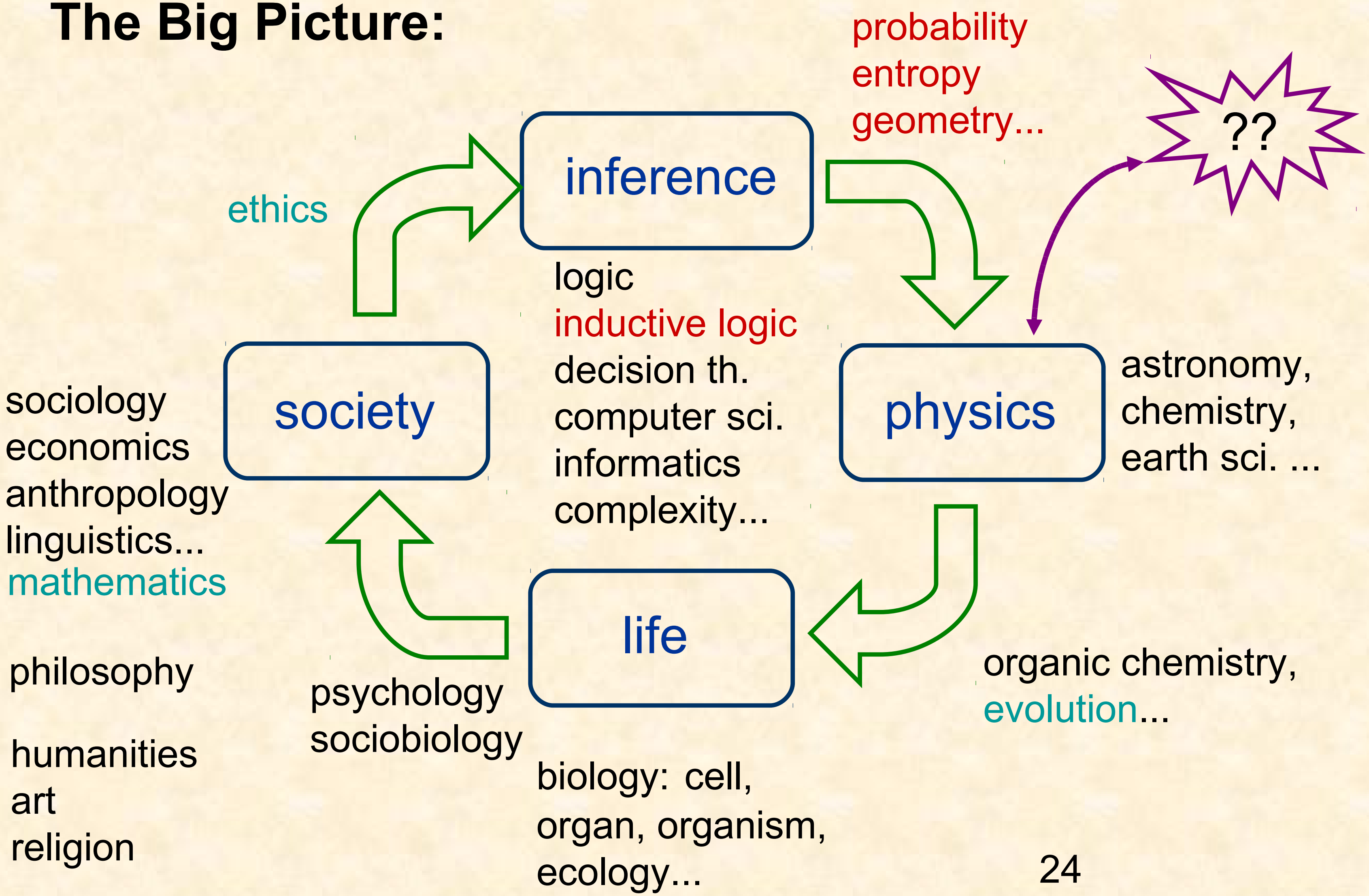
Force is whatever induces a change of motion: $\vec{F} = \frac{dp}{dt}$

Inference is dynamics too!



Information is what induces the change in rational beliefs.

The Big Picture:



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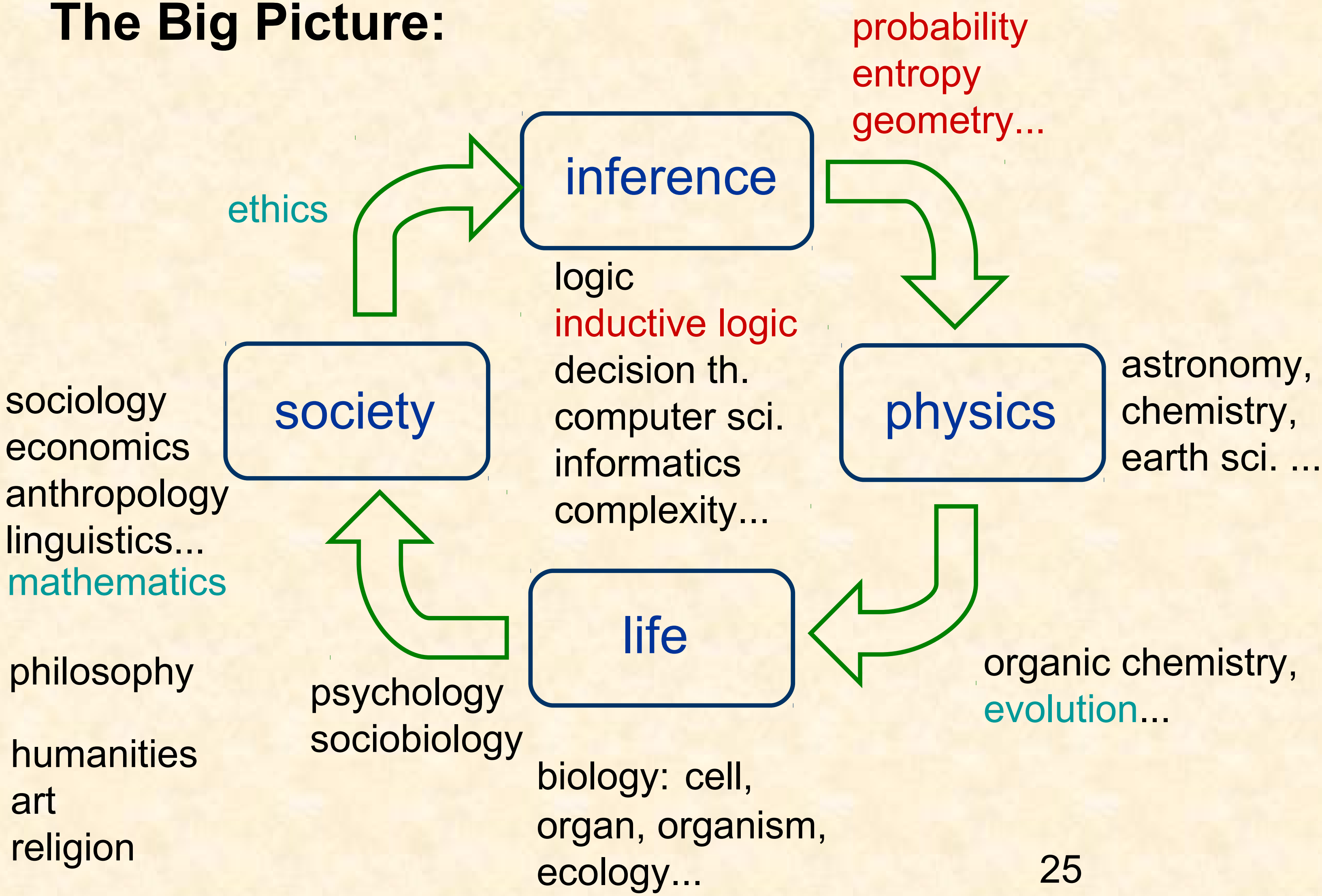
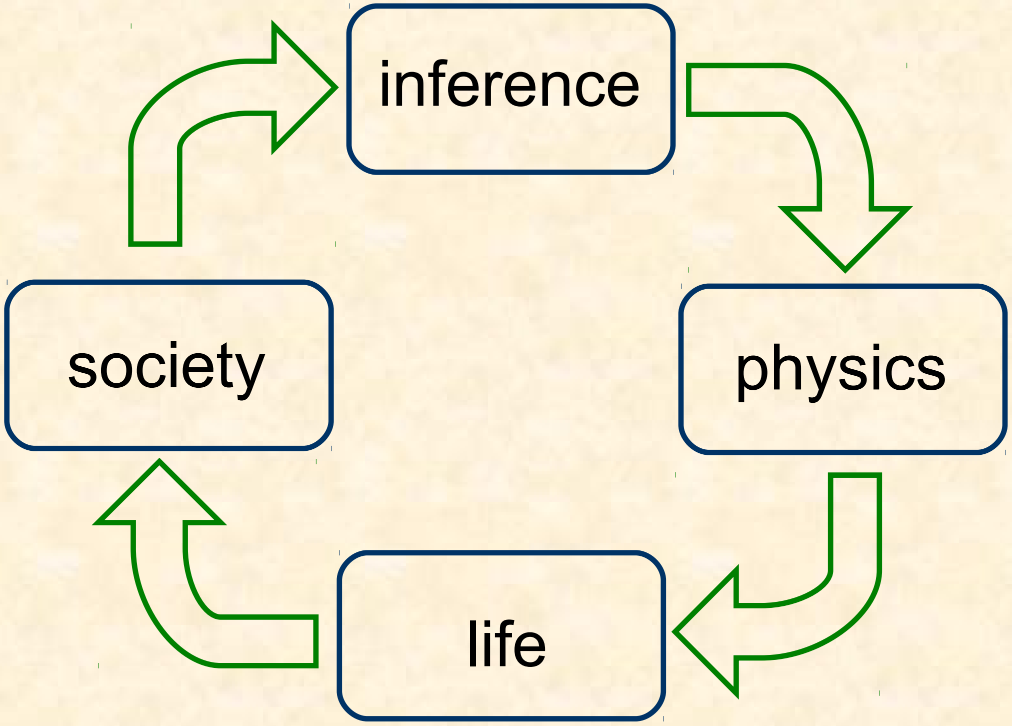
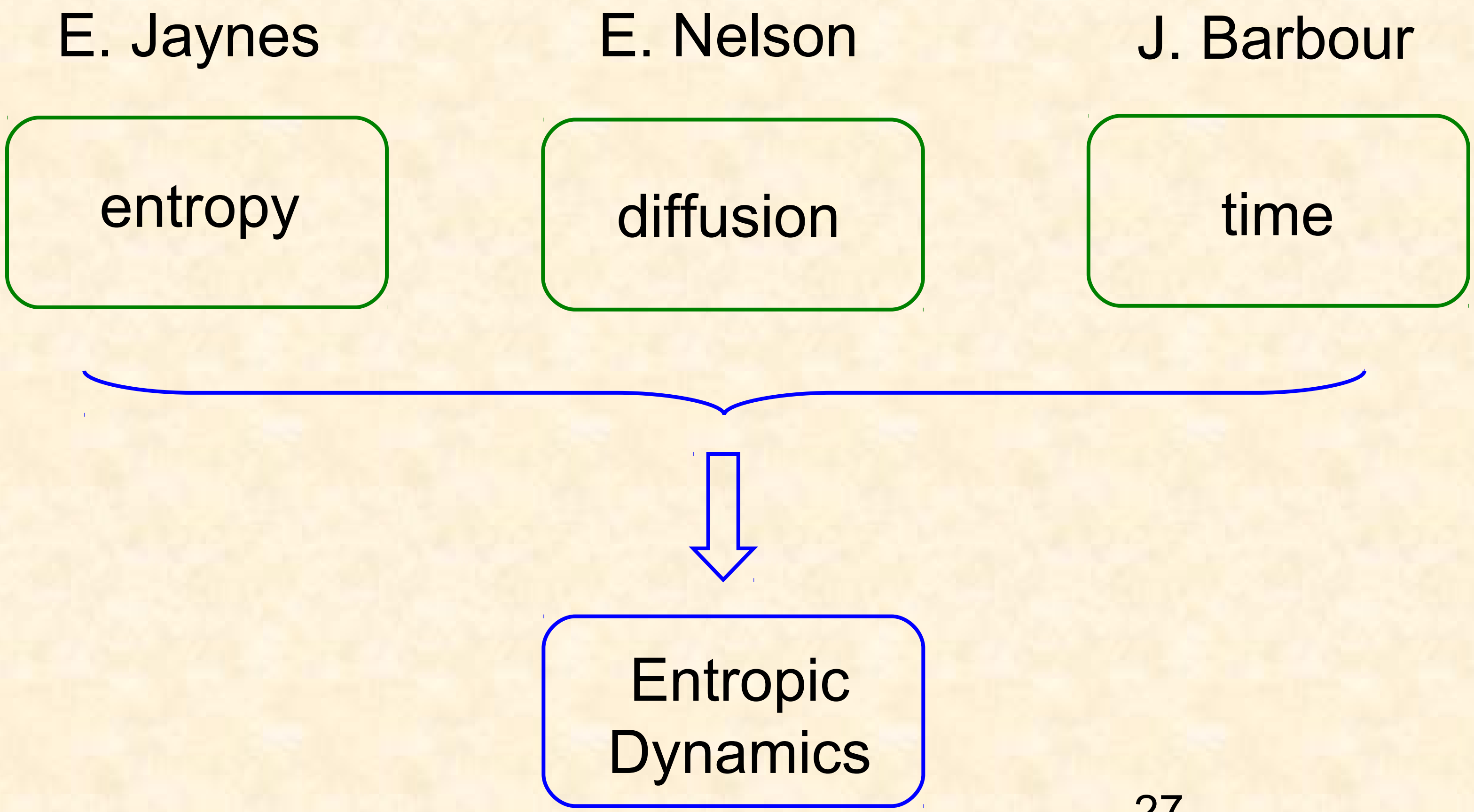


The Big Picture:



Three ingredients:



Summary:

1) Hidden variables: no dynamics; just $p(y\,|\,x)$ and $S(x)$.

2) Changes happen gradually.

3) Entropic time: instants, interval, ordering.

4) The statistical manifold is dynamical:
energy conservation.

Overview

1) The statistical model: Hidden variables.

2) The dynamical principle: Changes happen gradually.

3) A book-keeping device: Entropic time.

4) Statistical manifold dynamics: Conservative diffusion.

5) Entropic *vs.* "physical" time.

Further reading and references to the literature:
go to arxiv.org and search under A.C.

The Gravitational Equivalence Principle:

We accept the equivalence of gravitational with the "fictitious" forces that arise in accelerated frames because

it explains the equality of inertial and gravitational masses

and allows a geometrical explanation of gravity.

A Quantum Equivalence Principle?

We accept the equivalence of quantum and "epistemic" probabilities because

it explains the equality of inertial and osmotic masses

it explains linearity, superposition, complex numbers,

and allows an inferential explanation of quantum theory.

$$S_J[P,Q] = -\int dx'dy'\, P(x',y'\,|\,x) \log \frac{P(x',y'\,|\,x)}{Q(x',y'\,|\,x)}$$

Prior: $Q(x',y'\,|\,x) \propto q(x') \times q(y') \propto \underbrace{\gamma^{1/2} \times q(y')}_{\text{uniform}}$

First constraint:

$$P(x',y'\,|\,x) = P(x'\,|\,x)\underbrace{P(y'\,|\,x',x)}_{\in \mathrm{M}} = P(x'\,|\,x)\underbrace{p(y'\,|\,x')}_{\in \mathrm{M}}$$

Second constraint:

Short steps: $\langle \Delta\ell^2 \rangle = \langle \gamma_{ab} \Delta x^a \Delta x^b \rangle = \lambda^2(x)$

Step 4: Manifold dynamics?

Energy conservation [Nelson (1979), Smolin (2006)]

$$E = \int d^3x\, \rho \left[A\gamma_{ab}\, v^a v^b + B\gamma_{ab}\, u^a u^b + V(x) \right]$$

$$E = \int d^3x\, \rho \left[\frac{1}{2} m\, v^2 + \frac{1}{2} \mu\, u^2 + V(x) \right]$$

where $m = \frac{2A}{\sigma^2} = \frac{\Box\tau}{\sigma^2}$ $\mu = \frac{2B}{\sigma^2} \approx m$

mass osmotic mass

But we can always *regraduate* to a more *convenient* description.

new units: $\eta = \kappa\eta', \quad \tau = \frac{\tau'}{\kappa}, \quad \phi = \frac{\phi'}{\kappa}$

New wave function: $\Psi' = \rho^{1/2} \exp i\phi'$

Schrödinger equation:

$$i\eta'\dot{\Psi}' = -\frac{\eta'^2}{2m}\nabla^2\Psi' + V\Psi' + \frac{\eta'^2}{2m}\left(1 - \frac{\mu\kappa^2}{m}\right)\frac{\nabla^2\left(\Psi'\Psi'^*\right)^{1/2}}{\left(\Psi'\Psi'^*\right)^{1/2}}\Psi'$$