MaxEnt 2010 Workshop,
Chamonix (France), July 4-9, 2010

Multichannel SAR Image Classification by Finite Mixtures, Copulas and Markov Random Fields

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Outline

- Introduction:
  - supervised classification of multichannel synthetic aperture radar (SAR) images
- The proposed method:
  - overall architecture;
  - probability density function modeling by finite mixtures and copulas;
  - bayesian contextual classification by Markov random fields.
- Experimental results:
  - experiments on RADARSAT-2 images
- Conclusions and future research

Synthetic aperture radar

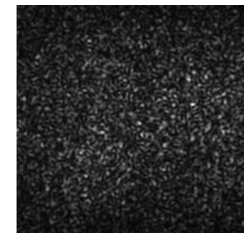
- Current high resolution (HR) satellite SAR missions (i.e., TerraSAR-X and RADARSAT-2) convey a huge potential for mapping and monitoring applications:
  - insensitive to Sun-illumination;
  - almost insensitive to atmospheric conditions;
  - resolution up to 1 m and very short revisit time (up to 12 h);
  - feasible multi-polarization (dual-pol/quad-pol) data.

Paris, RADARSAT-2

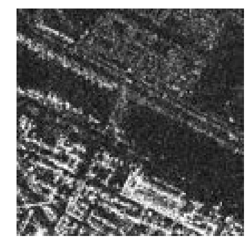
Multichannel SAR image classification

- Exploiting this potential requires automatic and accurate methods for the classification of multichannel SAR imagery.

- Multichannel SAR image classification is a difficult task:
  - very noisy spatial behavior (speckle in SAR).



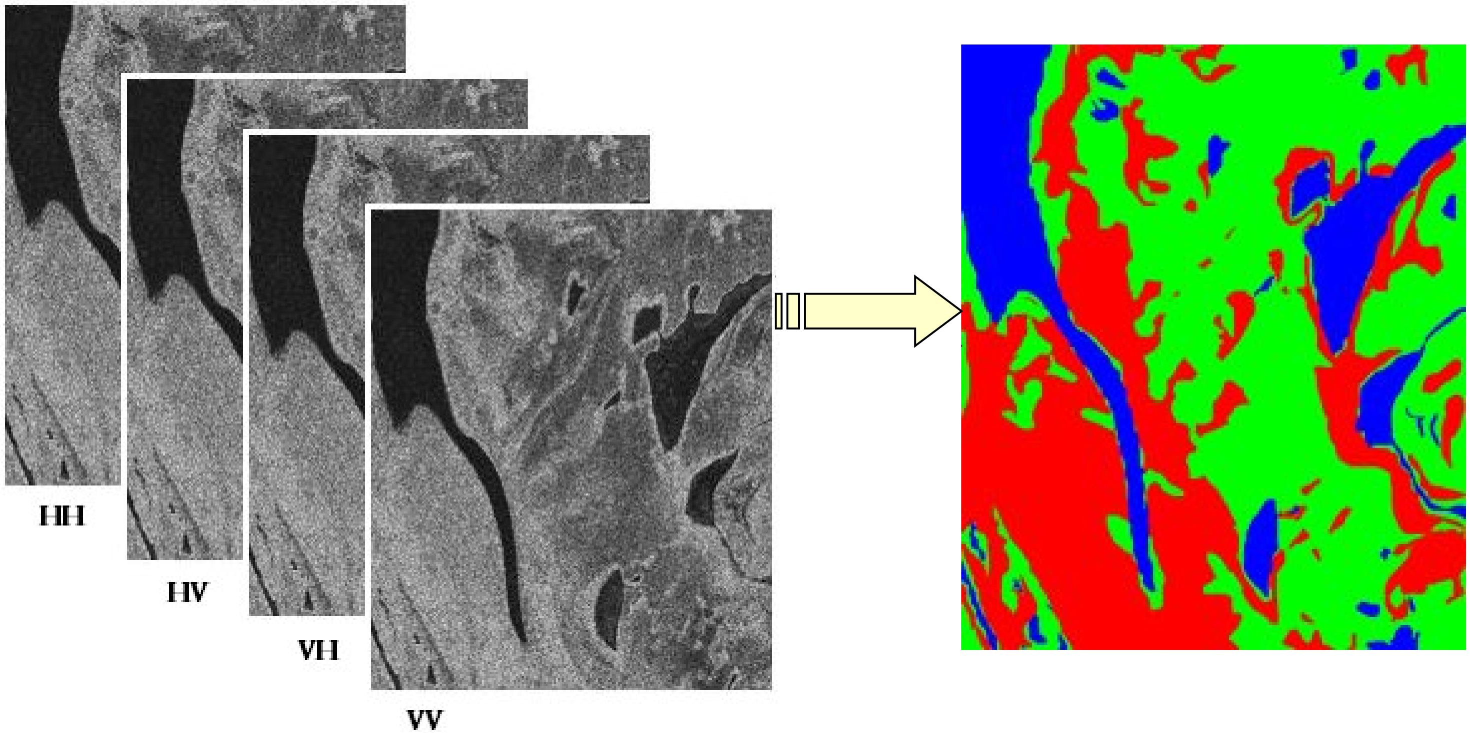
  - heavy heterogeneity, due to the appreciability of backscattering responses of distinct ground materials in HR data.



  - no accurate parametric model available for the joint statistics of SAR amplitude channels.

Multichannel SAR image classification

- Purpose of this work:
  - develop a novel supervised classification method for multichannel (HR) SAR amplitude images.



Sanchagang, China, TerraSAR-X, 2008

Proposed method

Key ideas

Methodology

– Model the heterogeneity by a finite-mixture model (FMM) for the marginal statistics of each marginal amplitude channel;
  - SAR-specific "dictionary-based stochastic expectation maximization" (DSEM) approach.

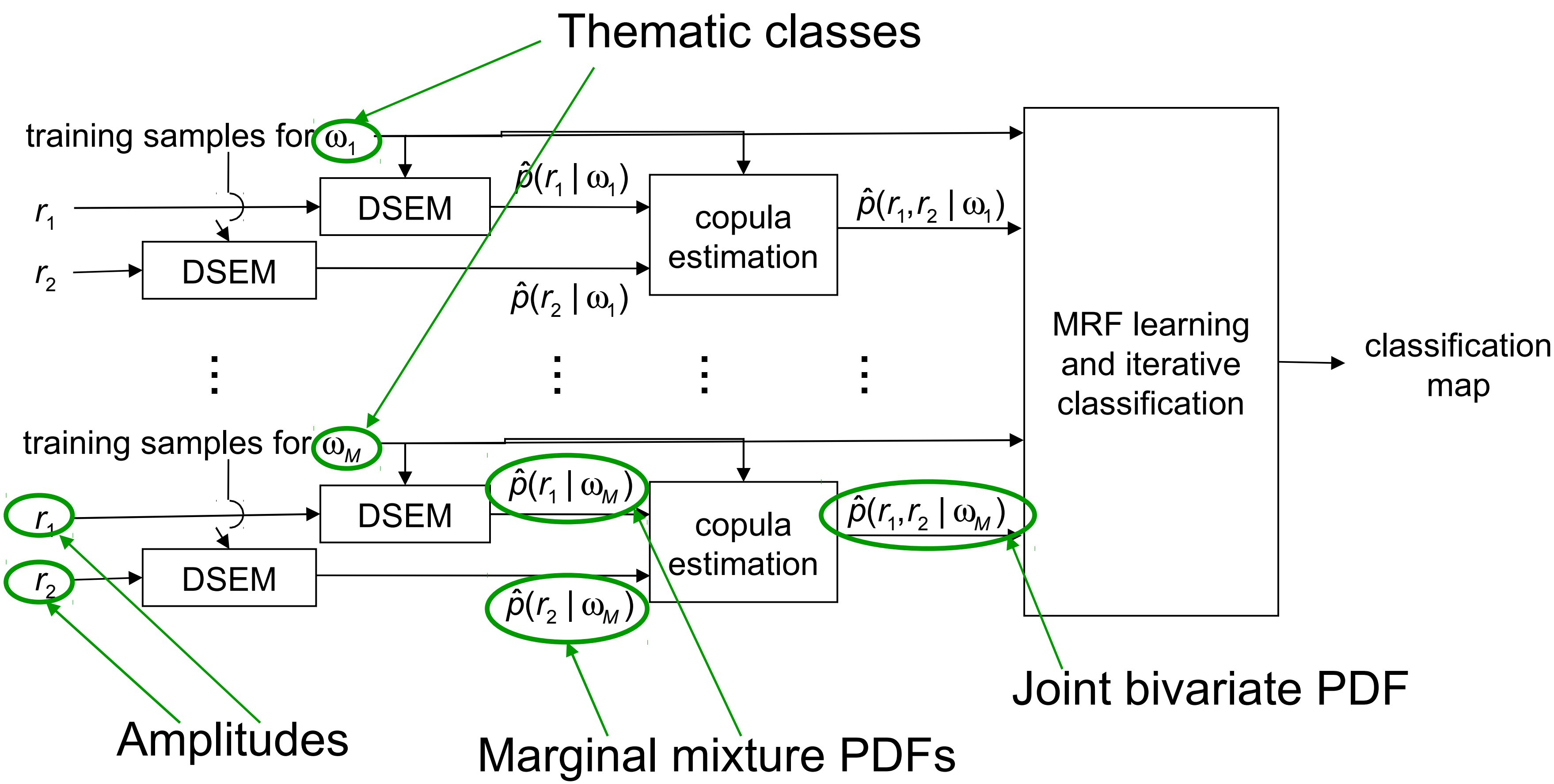
– Model the joint probability density function (PDF) of different amplitude channels by copula theory;
  - dictionary-based copula-selection approach.

– Model the spatial context by Markov random fields (MRFs);
  - Potts neighborhood model.

– Sample the labeling L^* that maximizes the posterior $P(L|Y)$;
  - Modified Metropolis dynamics (MMD) method.

Overall architecture of the method

Overview in 2 channel case:



Finite mixtures by DSEM

- DSEM was proposed to model the statistics of HR SAR:
  - mixture components drawn from a dictionary of SAR-specific parametric families;

$$p(z|\theta) = \sum_{i=1}^{K} P_i p_i(z|\theta_i), \; r \geqslant 0$$

  - parameter fitting by stochastic EM and method of log-cumulants (recent method based on Mellin transform);
  - automatic optimization of the choice of the family for each component and of the number of mixture components.

Family	Probability density function
Log-normal	$f_1(r) = \frac{1}{\sigma r\sqrt{2\pi}} \exp\left[-\frac{(\ln r - m)^2}{2\sigma^2}\right]$
Weibull	$f_2(r) = \frac{\eta}{\mu^\eta} r^{\eta-1} \exp\left[-\left(\frac{r}{\mu}\right)^\eta\right]$
Nakagami	$f_3(r) = \frac{2}{\Gamma(L)} (\lambda L)^L r^{2L-1} \exp\left(-\lambda L r^2\right)$
Generalized Gamma	$f_4(r) = \frac{\nu}{\sigma\Gamma(\kappa)} \left(\frac{r}{\sigma}\right)^{\kappa\nu-1} \exp\left\{-\left(\frac{r}{\sigma}\right)^\nu\right\}$

- Particularly attractive for modeling class-conditional marginal PDFs in HR SAR (intrinsically takes into account the heterogeneity).

Copulas

- A *D*-copula is a *D*-variate cumulative distribution function (CDF) *C* whose marginals are uniform in *[0, 1].*
  - Thanks to Sklar theorem, for each vector $(Y_1,\ldots,Y_d)$ of (absolutely continuous) random variables with CDFs $F_1,\ldots,F_d$, there exists a (single) copula *C* with the CDF *H(y)* such that

$$H(\mathbf{y}) = C(F(y_1),\ldots,F(y_D)),\ \forall \mathbf{y} \in \mathbb{R}^D.$$

- Parametric copula estimation:
  - for the considered copulas with one parameter θ a closed-form equation relates θ and Kendall's ranking coefficient:

$$\tau = \mathrm{Prob}\{(Z_1 - \hat{Z}_1)(Z_2 - \hat{Z}_2) > 0\} - \mathrm{Prob}\{(Z_1 - \hat{Z}_1)(Z_2 - \hat{Z}_2) < 0\}.$$

  - a sample estimate of τ can be obtained by rank statistics and an estimate of θ is derived by inverting $\tau = \tau(\theta)$.

Dictionary approach for copulas

- In order to maximize flexibility in the proposed method, a dictionary approach is adopted for copula modeling:
  - the copula related to each class-conditional joint PDF is drawn from a dictionary of three parametric copulas;
  - parameter estimation (based on Kendall's τ) is performed for each class and each parametric copula in the dictionary;
  - the optimal copula for each class is selected by a Pearson chi-square test of fitness.

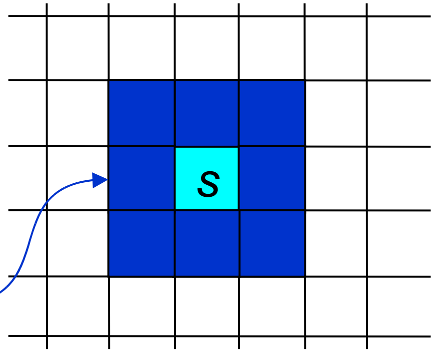
Family	$C(\mathbf{u})$	$\theta(\overline{\tau})$ dependence
Clayton	$C^1(\mathbf{u}) = (u_1^{-\theta} + \cdots + u_D^{-\theta} - D + 1)^{-1/\theta}$	$\theta = \frac{2\overline{\tau}}{1-\overline{\tau}}$
Gumbel-Hougaard	$C^2(\mathbf{u}) = \exp\left(-\left[(-\log(u_1))^{\theta} + \cdots + (-\log(u_D))^{\theta}\right]^{1/\theta}\right)$	$\theta = \frac{1}{1-\overline{\tau}}$
Frank	$C^3(\mathbf{u}) = -\frac{1}{\theta}\ln\left(1 + \frac{(e^{-\theta u_1}-1)\cdots(e^{-\theta u_D}-1)}{(e^{-\theta}-1)^{D-1}}\right)$	$\overline{\tau} = 1 - \frac{4}{\theta^2}\int_0^{\theta}\frac{t}{e^{-t}-1}dt$

Markov random fields

- MRFs formalize statistical interactions between distinct pixels by using only local relationships.
  - An MRF model is assumed for the random field *x* of the class labels x_s of the image pixels ($s \in S$ = pixel lattice):

$$P(x) > 0 \quad \forall x \in \Omega^{|S|}$$

$$P\{x_s = \omega_m | x_r, r \neq s\} = P\{x_s = \omega_m | C_s\}$$



conditioning only to the set C_s of the labels of the neighboring pixels ("local context" of the *s*-th pixel), according to a given neighborhood system (here 3 × 3)

  - Thanks to the Hammersley-Clifford theorem:

$$\max_{x \in \Omega^{|S|}} P(x | r_{1s}, r_{2s}, s \in S)$$

global "maximum *a-posteriori*": **intractable!**

$\Leftrightarrow$

minimization of a locally defined energy function for Gibbs distribution: **tractable!**

Energy function and parameter estimation

- Gibbs energy function:
  - linear combination of contributions related to spatial context and to estimated class-conditional pixelwise PDFs:

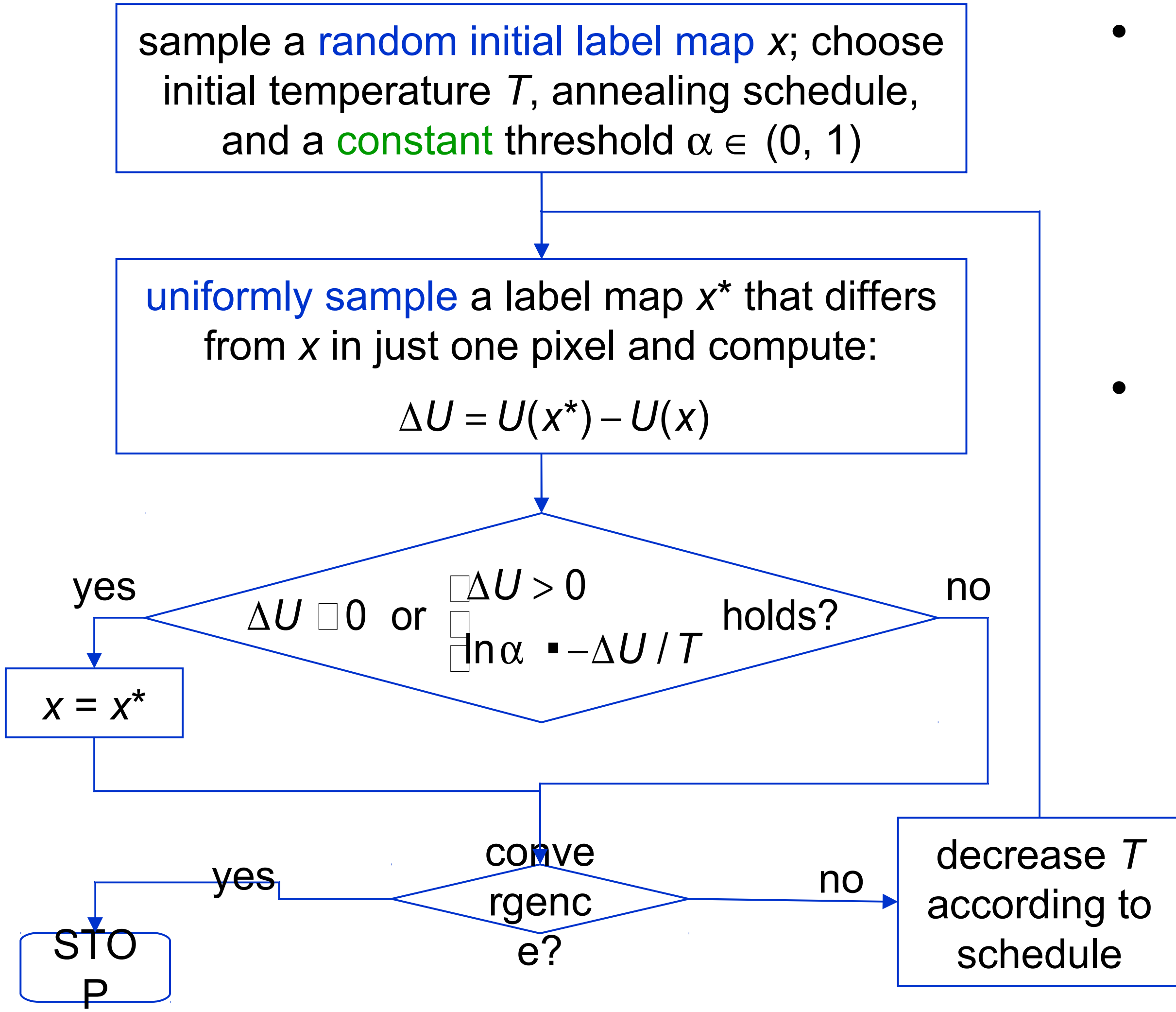
$$U(\omega_m \,|\, r_{1s}, \dots, r_{Ds}, C_s, \beta) = -\ln \hat{p}(r_{1s}, \dots, r_{Ds} \,|\, \omega_m) - \beta \sum_{x_r \in C_s} \delta_{x_r = \omega_m}$$

Potts model

- MRF parameter estimation:
  - the spatial regularization parameter β is estimated by a simulated annealing method, aimed at the maximization of a pseudo-likelihood function:

$$\max_{\beta>0} \sum_{s \in S} \left\{ -U(x_s \,|\, C_s, \beta) - \ln \sum_{m=1}^{M} \exp[-U(\omega_m \,|\, C_s, \beta)] \right\}$$

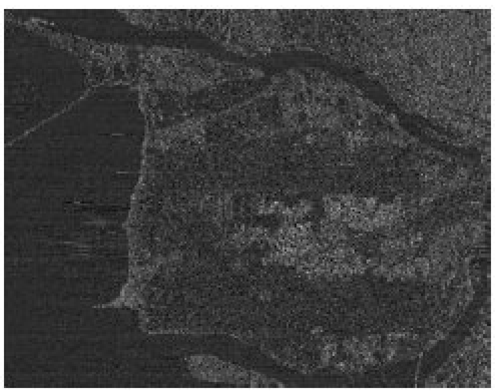
Energy minimization



- Energy minimization performed by a deterministic Modified Metropolis Dynamics (MMD).
- Tradeoff between deterministic iterated conditional mode and stochastic simulated annealing in terms of:
  - global/local minima;
  - computation time;
  - need for accurate initialization.

Data sets and experimental setup

TX



RS

- **The method was tested on two data sets:**
  - **TX, TerraSAR-X, 6-m resolution, 1.66-look, dual-pol HH/VV, 1200 × 1400 pixel, Sanchagang (China), ©Infoterra GmBH, 2008. Application: epidemiologic monitoring.**
  - **RS, RADARSAT-2, 7.5-m resolution, 1-look, Quad-pol HH/HV/VH/VV, 1000 × 700 pixel, Vancouver (Canada), ©MacDonald, Dettwiler and Associates Ltd., 2008. Application: urban area detection.**
- **Manually annotated training and test maps were available for 3 land-cover classes.**

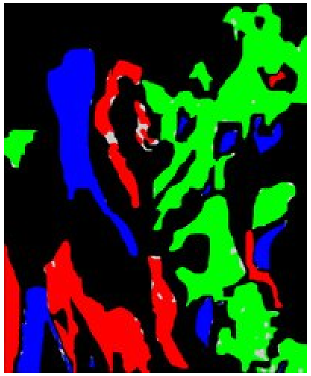
Experimental comparisons 2D

Method	by class			Overall
	water	wet soil	dry soil	
proposed method	99.37	96.83	95.73	97.07
bivariate Nakagami	98.75	94.63	75.50	90.22
K-NN	98.79	98.01	90.80	95.92
HH-only	99.01	94.18	65.65	87.12

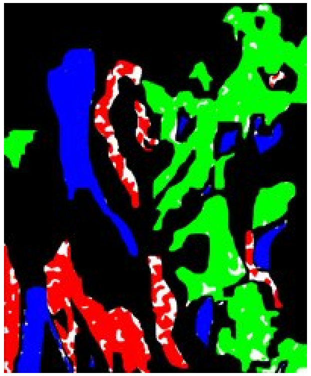
higher accuracies than by benchmark classifiers based on the combination of MRFs with a bivariate Nakagami PDF (for multilook dual-pol SAR amplitudes) and with *K*-NN ("*K* nearest neighbors")

correctly classified water
correctly classified wet soil
correctly classified dry soil
classification errors
outside test set

TX image



proposed method



bivariate Nakagami

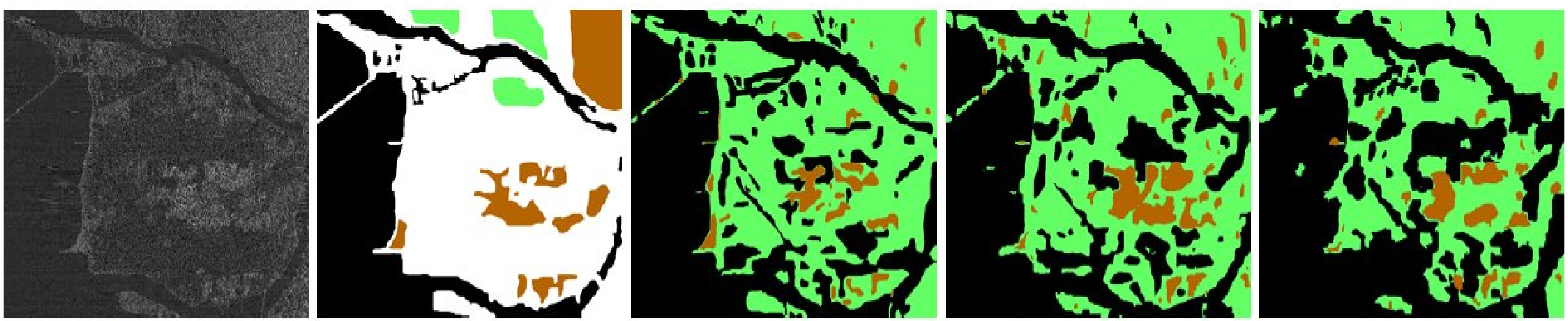
K-NN

Experimental comparisons 3D

Higher accuracies than by a benchmark classifier based on the combination of MRFs with a classical *K*-NN (“*K* nearest neighbors”) classifier and 2D version of the Copula-DSEM-MRF classifier

Color legend:
- water/wet
- vegetation
- urban
- outside GT

Method	Water	Vegetation	Urban	Average	Overall
Copula-DSEM-MRF in 3D	98.04%	90.33%	71.49%	86.61%	**88.69%**
K-NN-MRF in 3D	98.85%	89.02%	62.89%	83.59%	**86.42%**
Copula-DSEM-MRF in 2D	98.97%	87.16%	58.74%	81.62%	**84.78%**



RS image, VV pol — manual GT — *K*-NN-MRF — Copula-DSEM-MRF — Copula-DSEM-MRF 2

Conclusions

- The experimental results suggest the effectiveness of the proposed novel supervised multichannel SAR image classification technique:
  - high accuracy on humid and urban area mapping;
  - general multichannel model;
  - outperforms benchmark contextual classifier;
  - significant accuracy gain compared to single-pol/dual-pol classification.
- Future extensions:
  - integrating geometrical information in the MRF;
  - introduction of non-symmetric copulas.

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Finite mixtures by DSEM

- maximum-likelihood approach by Stochastic Expectation-Maximization (SEM) :
  - avoid local maxima;
  - feasibility for SAR-specific pdfs.

- SEM is an iterative estimation scheme for the problem of data incompleteness. At each iteration it involves:
  - E-step: compute posterior probability of each component

$$\tau_i^t(z) = \frac{P_i^t p_i^t(z)}{\sum_{j=1}^{K} P_j^t p_j^t(z)}, \quad \text{where } i = 1, 2, \cdots, K,$$

  - S-step: randomly sample component labels for every pixel according to the posterior probabilities $\tau_i^t(z), \; i = 1, 2, \cdots, K,$

  - M-step: update the ML estimate of the mixture

$$p(z|\theta) = \sum_{i=1}^{K} P_i^t p_i^t(z|\theta_i)$$