Logarithm ubiquity in statistics, information, acoustics, …

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Abstract

Here the central figure is the logarithm function because its numerous properties in mathematics and its interventions in information theory, statistics, acoustics, functional equations, … Among all the possibilities of the logarithm and also the exponential, here we introduce a new statistic named the expo-dispersion well suited for logarithmic data as the levels of magnitudes. It is a statistic which characterizes the dispersion of data as do variance and entropy, and they have many common behaviours and properties.

I - Introduction

This presentation is related to the logarithm application, also named and tabulated by John Napier in 1614, and introduced in Calculus and Analysis by the Leibniz integral formula more than sixty years later (around 1670-1680). It is a central application in mathematics and in several other scientific fields such as Statistics, Information theory and also Acoustics, very generally dealing with levels of magnitudes instead of magnitudes themselves. It has many analytical and functional pleasant properties and we meet it in many circumstances (Maurin 2003). Here we examine some other features and properties of it, such as a new statistic adapted to the data's dispersion and then suited for levels, and also sharing a fine property with entropy.

II - Some reminders and definitions

One may first recall the Shannon information $I_S(A) = - \ln_2 P(A)$ about the P(A) probability of A event, following the previous Hartley information $I_H = - \log P(A)$ with a change of logarithm, (Rényi 1966) ;

- the Boltzmann entropy $S = k \ln W$ in physics applied to gas molecules' populations (Bruhat 1968), a name due to the hellenist physicist Clausius ; and the Shannon entropy as the expectation $E(I_S(A_i)) = - \sum_i P(A_i) \ln_2(P(A_i))$ of his information for the probabilistic distribution $P(A_i)$ of events A_i, (Rényi) ; one remembers also the Wiener quip to Shannon "*call it entropy, nobody knows what it is*" ;

- the Fisher information in Statistics ; when the $P_\theta(A_i)$ depends on a real parameter θ, one has $\sum_i \frac{d}{d\theta} P_\theta(A_i) = \sum_i P_\theta(A_i) \frac{d}{d\theta} \ln P_\theta(A_i) = 0$. Then the distribution for the derivatives of information is centered, the Fisher information is the variance $\sum_i P_\theta(A_i)\,[\frac{d}{d\theta}\ln P_\theta(A_i)]^2$ of this distribution ;

- the Kullback information when one has two probability distributions P_1 and P_2 ; there are classical distributions ratios in Statistics, as for example in the Neyman and Pearson Lemma or the Fisher likelihoods ratio, and one takes many times their logarithm (Kendall Stuart 1963). The Kullback information is the expectation of $\ln P_1(A_i)/P_2(A_i)$: $I_K(P_1, P_2) = \sum_i P_1(A_i) \ln P_1(A_i)/P_2(A_i)$, (Kullback 1967), and the Jeffreys divergence is rendered symmetric in P_1 and P_2 : $I_J = I_K(P_1, P_2) + I_K(P_2, P_1)$;

- acoustics deserves a special development. This physical and specific domain has conventionaly introduced the notion of "level". A level of a positive magnitude g (for French "grandeur") is given by the formula $L = a \ln g/g_0$ with an accompanying reference value g_0 (which is also due to a conventional and consensual choice). Acoustics retained the decimal logarithm with the numerical multiplying coefficient 10, and then an acoustic level is $L = 10 \log g/g_0$ (Beranek 1971, Liénard 1978) ; moreover, as for the unit bit or shannon in information theory, there is a specific unit for all these levels, the decibel, in order to recall G. Bell.
This is recognized as a ”*manière de compter*” which may be applied to “*n’importe quoi*” as said Liénard (1978), who added “*l’expression d’une telle échelle (*understood “logarithmic”*) permet d’appeler tout cela des « décibels » ; tous effectivement sont engendrés à partir de la (*we add “*même*, same”) *définition initiale* …”. The reasons for the introduction of the logarithm in acoustics are not quite clear and consensual (Liénard 1978, Maurin 1994, 1999), but sometimes one evokes a mimetism on the “telegraphists” in the Bell Laboratories during the twenties because they rendered additive their multiplicative loss of power in line (Josse 1973).

- another rather "forgotten" level in acoustics ; it is related to the range between two f_1 and f_2 frequencies which is defined as $1000 \log f_2/f_1$ (Matras 1990) in the unit of savarts, or again as $1200 \ln_2 f_2/f_1$ in cent unit. Then a musical octave has a range of 301,03 savarts (= 1000 log 2) or 1200 cents ; the range of the half-tone is roughly 25 savarts and exactly 100 cents in the Bach tempered scale.

- one also meets this definition in chemistry where one has $pH = -\log [H^+]$ in relation with the mass action law for aqueous solutions $[H^+]\,[OH^-]/[H_2O] = k(t)$ with $[H^+]\,[OH^-] \# 10^{-14}$; and in econometry the elasticity coefficient's definition $e = -\frac{dx/x}{dy/y}$ implicitly implies the logarithms ln x and ln y, and then the levels for x and y.

III - A new statistic, the "expo-dispersion"

III.1 - Another recall, some convexity properties
The logarithm is an application h(x) on R^+, and of course one knows its reciprocal function the exponential on R. Logarithm function is increasing concave while the exponential function is increasing convex. We note m_x the arithmetic mean $1/n \sum x_i$; in a general way one has the

h-mean $h^{-1}(1/n \sum h(x_i)) = h^{-1}(m_{h(x)})$ for the transformed h(x) with every bijective function h, which is homogeneous to the x data, and one has the classical inequality $h^{-1}(m_{h(x)}) \geq m_x$ for every increasing convex h function (Hardy, Littlewood & Pólya 1934).

III.2 - A formal construction for the "expo-dispersion"

Then the difference $T_2 = h^{-1}(m_{h(x)}) - m_x$ is positive (or null), and one notes that this is the case for the exponential. One notes also that it is a basic property of the variance. Below we examine under which conditions the T_2 difference has other common properties with variance.

Because a variance only depends on differences $\Delta x_i = x_i - m_x$ one poses $T_2 = S(\Delta x_i)$, then we have $\sum h(m_x + \Delta x_i) = n\, h[m_x + S(\Delta x_i)]$, and with $m_x = 0$ one gets $\sum h(\Delta x_i) = n\, h(S(\Delta x_i))$.
It results necessarily $S(\Delta x_i) = h^{-1}(1/n \sum h(\Delta x_i))$, that is to say $S(\Delta x_i)$ is the h-mean of the Δx_i ; it results also a functional equation for h

$$1/n \sum h(m_x + \Delta x_i) = h[m_x + S(\Delta x_i)]$$

or again $$h^{-1}(1/n \sum h(m_x + \Delta x_i)) = m_x + h^{-1}(1/n \sum h(\Delta x_i)).$$

In this relation m_x and Δx_i arguments are separated in the right member, it has to be necessarily the same on the left, say into the sum $\sum h(m_x + \Delta x_i)$ and into every term $h(m_x + \Delta x_i)$.

* First we envisage the additive decomposition $h(m_x + \Delta x_i) = k(m_x) + g(\Delta x_i)$. Consequently one has $n\, k(m_x) + \sum g(\Delta x_i) = n\, k(m_x) + n\, g[h^{-1}(1/n \sum h(\Delta x_i))]$, that is to say $\sum g(\Delta x_i) = n\, g[h^{-1}(1/n \sum h(\Delta x_i))]$, or again $g^{-1}(1/n \sum g(\Delta x_i)) = h^{-1}(1/n \sum h(\Delta x_i))$.
It results that we have necessarily $g = h$, $h(m_x + \Delta x_i) = k(m_x) + h(\Delta x_i)$ and by symmetry $h(m + \Delta x_i) = h(m_x) + h(\Delta x_i)$. This is a Cauchy functional equation, whose non null solution is $h(x) = ax$ (Aczél) ; however it is an inconvenient solution because the h-mean is identical to the mean.

* Secondly we envisage the multiplicative decomposition $h(m_x + \Delta x_i) = k(m_x)\, g(\Delta x_i)$, now with $k(m_x) \sum g(\Delta x_i) = n\, k(m_x)\, g[h^{-1}(1/n \sum h(\Delta x_i))]$, that is to say the same result $g^{-1}(1/n \sum g(\Delta x_i)) = h^{-1}(1/n \sum h(\Delta x_i))$.
It results again $g = h$, then $h(m_x + \Delta x_i) = k(m_x)\, h(\Delta x_i)$ and by symmetry $h(m_x + \Delta x_i) = h(m_x)\, h(\Delta x_i)$. The solution of this other Cauchy functional equation is $h(x) = e^{cx}$, (Aczél), and this is a suitable solution, increasing and convex, when $c > 0$.

The chain of all these necessary conditions yields to the analytical solution $h^{-1}(1/n \sum h(x_i)) = m_x + 1/c \ln(1/n \sum e^{c\Delta x_i})$ with c and $1/c \ln(1/n \sum e^{c\Delta x_i})$ positive.

Reciprocally, the exponential function $h(x) = e^{cx}$ is sufficient so that the difference $T_2 = h^{-1}(1/n \sum h(x_i)) - m_x$ is only depending on Δx_i, with the expression $T_2(\Delta x_i) = 1/c \ln(1/n \sum e^{c\Delta x_i})$.

All of this entails the
<u>Theorem</u> : The means difference $h^{-1}(1/n \sum h(x_i)) - m_x$ only depends on $\Delta x_i = x_i - m_x$ and is non null if and only if the h function is an exponential function e^{cx}, $c > 0$.

Then it results that the $T_2(\Delta x_i)$ term has a second common property with variance, a structural one. Consequently this is an other way to characterize the dispersion of x_i data, and one may call it a “h-dispersion” (Maurin 2007), in the same way of the “h-mean” construction. However as this happens if and only if h is an exponential function, "expo-dispersion" is a better appellation.

IV - The analytical properties of the expo-dispersion

IV.1 - General properties with h increasing and convex
Property 1 : The $T_2(m, \Delta x_i)$ term is invariant under indices' permutation.

It comes immediately from the definition $T_2 = h^{-1}(1/n \sum h(x_i)) - m$. ♦

Coming from $x_i = m + \Delta x_i$ initial data, one poses $x_{i,t} = m + t\Delta x_i$, $t > 0$. The new $x_{i,t}$ have the same mean m than the x_i, and inside the $\sum \Delta x_i = 0$ hyperplane their position is homothetic to that of x_i following the ratio t, and let the function $\kappa(t) = T_2(x_{i,t}) = T_2(m, t\Delta x_i)$ which depends on t for every hyperplane point $\{\Delta x_i\}$.

Property 2 : The $\kappa(t)$ function in an increasing function.

Let $\varphi(t) = h(m + T_2(m, t\Delta x_i)) = h(m + \kappa(t)) = 1/n \sum h(x_{i,t}) = 1/n \sum h(m + t\Delta x_i)$, consequently φ are κ have the same monotony in t because h is increasing.
One has $\varphi(0) = h(m)$, $\varphi'(t) = 1/n \sum \Delta x_i\, h'(m + t\Delta x_i)$ and $\varphi''(t) = 1/n \sum \Delta x_i^2\, h''(m_x + t\Delta x_i)$.
Then we get $\varphi'(0) = 0$ and $\varphi''(t)$ is positive because of the convexity of h, it results that φ' is increasing and positive for $t \geq 0$, and that φ and then κ are both increasing versus t. ♦

corollary : $\kappa(0) = 0$ and $\kappa'(0) = 0$.

From $\varphi(t) = h(m + \kappa(t))$ and $\varphi(0) = h(m)$ one gets $\kappa(0) = 0$, and also $\varphi'(t) = h'(m + \kappa(t))\, \kappa'(t)$ and $\varphi'(0) = 0$; consequently $\kappa'(0) = 0$ because h is an increasing function.

This shows that every $T_2(m, \Delta x_i) = h^{-1}(1/n \sum h(x_i)) - m$ is null when all the differences Δx_i are null. This shows also that $T_2(m, t\Delta x_i)$ is increasing in t on $\sum \Delta x_i = 0$ hyperplane in relation with an homothety of t ratio and keeping the m mean constant ; in the same conditions we note that the variance varies as t^2.

IV.2 - Specific properties for the exponential function $h(x) = e^{cx}$
Property 3 : The expo-dispersion is invariant for every d translation.

It comes from $T_2(\Delta x_i) = 1/c \ln(1/n \sum e^{c\Delta x_i})$ when $h(x) = e^{cx}$. It comes also from the translativity property of exponential functions (Aczél), with which the h-mean for $h(x) = e^{cx}$ (as an "expo-mean") follows the same d translation as data x_i ; one may note that this property for the expo-mean is tritely followed by the usual arithmetic mean m. ♦

Property 4 : The expo-dispersion is convex in Δx_i.

First the h-mean I_x is convex in x_i, or in the x vector of x_i coordinates, when one has
$I_x(d\, x + (1-d)\, y) \leq d\, I_x(x) + (1-d)\, I_x(y)$ for $0 \leq d \leq 1$.
Because $I_x(x) = h^{-1}[1/n \sum h(x_i)]$ and when h is increasing, $I_x(x)$ is convex in x if one has the equivalent inequality $h[I_x(d\, x + (1-d)\, y)] \leq h[d\, I_x(x) + (1-d)\, I_x(y)]$, that is to say

$1/n \sum h(d\, x_i + (1\text{-}d)\, y_i) \leq h[d\, h^{-1}(1/n \sum h(x_i)) + (1\text{-}d)\, h^{-1}(1/n \sum h(y_i))]$.

With $h(x) = e^{cx}$ and $h^{-1}(u) = 1/c \ln u$ this yields
$1/n \sum e^{c(dx_i+(1-d)y_i)} \leq \exp\{c\, d/c \ln(1/n \sum e^{cx_i}) + c\, (1\text{-}d)/c \ln(1/n \sum e^{cy_i})\}$
or again
$\sum e^{cdx_i}\, e^{c(1-d)y_i} \leq (\sum e^{cx_i})^d\, (\sum e^{cy_i})^{1-d}$.
This is the Hölder inequality applied to e^{cx_i} and e^{cy_i} with d and 1-d parameters ; consequently the h-mean (and here expo-mean) I_x is convex in x vector.

Lastly when the n-uplets x_i and y_i have the same m mean and have different respective differences $\Delta x_i = x_i - m$ and $\Delta y_i = y_i - m$, $I_x(m, \Delta x_i) = I_x(x)$ is convex in Δx_i, and also $T_2(\Delta x_i) = I_x(m, \Delta x_i) - m$. ♦

<u>Property 5</u> : With exponential functions $h(x) = e^{cx}$ the κ function is convex in t.

Let the two data sets $x_i = m + t_1\Delta x_i$ et $y_i = m + t_2\Delta x_i$ with the same m mean, and the expo-dispersions $T_2(t_1\Delta x_i) = \kappa(t_1)$, $T_2(t_2\Delta x_i) = \kappa(t_2)$ and also $T_2(d\, t_1\Delta x_i + (1\text{-}d)\, t_2\Delta x_i) = \kappa(d\, t_1 + (1\text{-}d)\, t_2)$. The previous T_2 convexity on $\sum \Delta x_i = 0$ hyperplane entails the inequality
$T_2(d\, t_1\Delta x_i + (1\text{-}d)\, t_2\Delta x_i) \leq d\, T_2(t_1\Delta x_i) + (1\text{-}d)\, T_2(t_2\Delta x_i)$,
which means
$\kappa(d\, t_1 + (1\text{-}d)\, t_2) \leq d\, \kappa(t_1) + (1\text{-}d)\, \kappa(t_2)$. ♦

The set of exponential functions $h(x) = e^{cx}$ has a parameter c ;
<u>Property 6</u> : The expo-dispersion T_2 is increasing in c.

For these functions, T_2 is the h-mean of the Δx_i and we note it $T_2(\Delta x_i, c)$.
We recall the general property for the h-means $T_2(\Delta x_i, c_1) > T_2(\Delta x_i, c_2)$ whenever $h_{c1} o h_{c2}^{-1}$ is convex, (Hardy Littlewood & Pólya 1934). Here we have $h_{c1}(x) = e^{c_1 x}$ and $h_{c2}^{-1}(u) = 1/c_2 \ln u$, and consequently $h_{c1} o h_{c2}^{-1}(u) = u^{c_1/c_2}$ is convex for $c_1 > c_2$. ♦

IV.3 - A generalization with weighted means
With the same x_i data one may introduce a set of p_i probabilities $\sum p_i = 1$, the new average $m = \sum p_i\, x_i$, the differences $\Delta x_i = x_i - m$ on a new $\sum p_i\, \Delta x_i = 0$ hyperplane, and the new variance $\sigma^2 = \sum p_i\, \Delta x_i^2$. The h-mean definition $I_x = h^{-1}(\sum p_i\, h(x_i))$ is changed but the inequality $I_x \geq m$ is still verified with an increasing and convex h function (Hardy Littlewood & Pólya 1934) ; then the related term $T_2 = I_x - m$ is still positive.

As previously T_2 is only depending on Δx_i if and only if $h(x) = e^{cx}$, $c > 0$, and except for the analytical expression $T_2(\Delta x_i) = 1/c \ln(\sum p_i\, e^{c\Delta x_i})$, the hyperplane equation for Δx_i and the Property 1 related to invariance by permutation, T_2 statistic keeps all the previous properties in the case of uniform probabilities 1/n.
About the convexity of $T_2(\Delta x_i)$ in Δx_i, one merely needs to apply the Hölder inequality to the new positive quantities $p_i\, e^{cx_i}$ and $p_i\, e^{cy_i}$ with the same parameters d and 1-d.

V - The statistical properties of the expo-dispersion

These properties are close to variance's properties.

V.1 - The sum of two independent random variables

Property 7 : One gets $T_2(\mathbf{X}+\mathbf{Y}) = T_2(\mathbf{X}) + T_2(\mathbf{Y})$ for two independent random variables $\mathbf{X}$, $\mathbf{Y}$.

Because $T_2(\mathbf{X})$ is also $1/c \ln (E(e^{c(\mathbf{X}-m_x)}))$ one has $T_2(\mathbf{X}+\mathbf{Y}) = 1/c \ln [E(e^{c(\mathbf{X}-m_x + \mathbf{Y}-m_y)})]$
$= 1/c \ln [E(e^{c(\mathbf{X}-m_x)} e^{c(\mathbf{Y}-m_y)})]$
and then $1/c \ln [E(e^{c(\mathbf{X}-m_x)}) E(e^{c(\mathbf{Y}-m_y)})]$ with independency, that is to say
$= 1/c \ln [E(e^{c(\mathbf{X}-m_x)})] + 1/c \ln [E(e^{c(\mathbf{Y}-m_y)})]$.

Consequently T_2 has the same additivity property as variance under independency, as for the cumulants and the second characteristic function, (Kendall & Stuart 1963). ♦

V.2 - About the mixing of populations

Here we consider the mixing of populations or sub-populations of data x_{ij} with a new index j, i $= 1\ldots n_j$, $n_+ = \sum n_j$, of respective means m_j and weights $p_j = n_j/n_+$. For simplicity we keep the notations h and h^{-1} but of course it concerns exponential functions $h(x) = e^{cx}$, $c > 0$.

One has $I_j = h^{-1}(1/n_j \sum_i h(x_{ij}))$ and $T_{2j} = I_j - m_j$ for every sub-population, and following the mixing with the p_j weights there are the new related statistics mean $m_{mel} = \sum_j p_j m_j$, h-mean I_{mel} $= h^{-1}[1/n_+ \sum_j \sum_i h(x_{ij})] = h^{-1}[\sum_j p_j/n_j \sum_i h(x_{ij})] = h^{-1}(\sum_j p_j h(I_j))]$ and the expo-dispersion $T_{2mel} = I_{mel} - m_{mel}$, (subscript "mel" for French "mélange").

One has $T_{2mel} = h^{-1}(\sum_j p_j h(m_j + T_{2j})) - \sum p_j m_j$; and with the new differences $\Delta m_j = m_j - m_{mel}$ inside the $\sum p_j \Delta m_j = 0$ hyperplane, and using exponential for h, one has more precisely
$T_{2mel} = 1/c \ln (\sum p_j e^{c(m_{mel} + \Delta m_j + T_{2j})}) - \sum p_j m_j$
$= 1/c \ln (\sum p_j e^{c\Delta m_j} e^{cT_{2j}})$.

There are a lot of analytical properties for the global expo-dispersion T_{2mel} :

a) about the monotony : T_{2mel} is increasing in function of every T_{2j} and/or Δm_j ;

b) about the convexity : one has $\exp[cT_{2mel}(\Delta m_j)] = \sum p_j e^{c\Delta m_j} e^{cT_{2j}}$,
$\exp[cT_{2mel}(d\, \Delta m_{j1} + (1\text{-}d)\, \Delta m_{j2})] = \sum p_j e^{cT_{2j}} e^{c(d\, \Delta m_{j1} + (1\text{-}d)\, \Delta m_{j2})}$
$= \sum \{p_j e^{cT_{2j}} e^{c\Delta m_{j1}}\}^d \{p_j e^{cT_{2j}} e^{c\Delta m_{j2}}\}^{(1\text{-}d)}$.
This quantity is smaller than $(\sum p_j e^{cT_{2j}} e^{c\Delta m_{j1}})^d (\sum p_j e^{cT_{2j}} e^{c\Delta m_{j2}})^{(1\text{-}d)}$
$= (\exp[cT_{2mel}(\Delta m_{j1})])^d (\exp[cT_{2mel}(\Delta m_{j2})])^{(1\text{-}d)}$ from Hölder inequality,
and taking the logarithms of the two members one gets
$T_{2mel}(d\, \Delta m_{j1} + (1\text{-}d)\, \Delta m_{j2}) \leq d\, T_{2melj}(\Delta m_{j1}) + (1\text{-}d)\, T_{2mel}(\Delta m_{j2})$.
It results that the expo-dispersion of the mixing of sub-populations is a convex function of the differences $\Delta m_j = m_{mel} = m_j$ inside the $\sum p_j \Delta m_j = 0$ hyperplane.

V.3 - An algebraical decomposition for the expo-dispersion

We have also $T_{2mel} = I_{mel} - m_{mel} = I_{mel} - \sum_j p_j I_j + \sum_j p_j I_j - \sum_j p_j m_j$
$= (I_{mel} - \sum_j p_j I_j) + \sum_j p_j T_{2j}$ or in the reverse order for the terms

$T_{2mel} = \sum_j p_j T_{2j} + (I_{mel} - \sum_j p_j I_j)$.

The first term is the weighted mean of the expo-dispersions for each population, and the second, $h^{-1}(\sum_j p_j h(I_j)) - \sum_j p_j I_j$, is the expo-dispersion of I_j quantities weighted by p_j. Consequently

a) the second term is itself positive with an increasing convex h function, as for every h-mean inequality with uniform weights or not ;

b) the expo-dispersion of the mixing may be decomposed in two terms as follows :

expo-dispersion$_{mel}$ T_{2mel} = mean of **expo-dispersions** T_{2j}
+ **expo-dispersion** of h-means I_j.

This result is very close to the analogous and classical decomposition of the variance
variance$_{mel}$ = mean of **variances** + **variance** of means m_j,
as well as an analogous decomposition for the entropy of a mixing
entropy$_{mel}$ = mean of **entropies** + **entropy** of the weights p_j of populations
(Maurin 2000, 2000 b, note : the first member of formulae in II.2.c in 2000 b to correct).

Then, in the case of a mixing of discrete populations or sub-populations, it happens that the two expo-dispersion and entropy statistics have a common algebraic and structural decomposition, very close to the variance's one.
The variance, the entropy and lastly the h-dispersion are three statistics able to describe the dispersion of data (Maurin 1999 b, 2000, 2000 b, 2007), and for this occurence they share a common structural decomposition formula.

VI - Conclusions

The logarithm function is here the central figure of the story. Of course its properties are numerous and fruitful, several scientific domains take advantage of it, and here it renders possible the introduction of a new statistic called the expo-dispersion. This expo-dispersion is well suited for the noise levels $L_i = 10 \log p_i^2/p_0^2$ with the acoustic (sur-)pressure p_i because one has $p_i^2/p_0^2 = 10^{L_i/10} = e^{L_i/M}$ with $M = 10/\ln 10$, and physically the acoustic powers p_i^2 are additive for independent sounds (the general case in environmental acoustics). Then one gets expo-disp$(L_i) = 10 \log(1/n \sum 10^{L_i/10}) - 1/n \sum L_i$, and it may be an adequate tool to characterize the dispersion of noise levels (Maurin 2010), knowing that the quantity $10 \log (1/n \sum 10^{L_i/10})$ is the very classical equivalent noise level L_{eq} in acoustics, also sometimes called "energetic mean", (Beranek 1971, Liénard 1978).

As a dispersion statistic it has many common points with the variance, as does the Shannon entropy. These three statistics, variance, entropy, expo-dispersion also have a very noticeable common structural decomposition in relation with the mixing of populations of data ; this is an example of calculations which finely gather Statistics, Information theory and Acoustics.

All of this agrees with the opinion of their first table-maker John Napier, in connection with Henri Briggs, who chose a name because they allow a *logos* about *arithmos* (built with Greek terms), and who claimed about them "*Mirifici logarithmorum canonis descriptio*" (here in Latin) in a very general framework "*ejusque usus … in omni logistica mathematica, amplissimi, … explicatio*" (as reported by Dupuis, 1885).

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SFdS : Société Française de Statistique, French Statistical Society