Online Robot Dead Reckoning Localization Using Maximum Relative Entropy Optimization With Model Constraints

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Abstract. The principle of Maximum relative Entropy optimization was analyzed for dead reckoning localization of a rigid body when observation data of two attached accelerometers was collected. Model constraints were derived from the relationships between the sensors. The experiment's results confirmed that accelerometers each axis' noise can be successfully filtered utilizing dependency between channels and the dependency between time series data. Dependency between channels was used for *a priori* calculation, and *a posteriori* distribution was derived utilizing dependency between time series data. There was revisited data of autocalibration experiment by removing the initial assumption that instantaneous rotation axis of a rigid body was known. Performance results confirmed that such an approach could be used for online dead reckoning localization.

Keywords: Maximum relative Entropy, accelerometer, dead reckoning localization, sensor fusion
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INTRODUCTION

Due to the dynamic forces on a wheel of a single axis differential drive robot, the instantaneous center of rotation might not lie at the wheels' axis line. There is translational and rotational kinetic energy of the other wheel that has an instantaneous influence because of rigid body inertia. So the instantaneous center of rotation shifts away from the line of the mechanical wheels axis. It can be proved that the assumption of instantaneous center of rotation lying at mechanical axis of the wheels would lead to motion formulas implying no inertia of the rigid body. So such an assumption is not realistic. Consequently the instantaneous rotation axis of differential drive robot must be treated as not lying at the axis of differential drive, which leads to a general problem of dead reckoning of the rigid body. The robot localization problem will be analyzed as a localization problem of a rigid body.

There is one more assumption, which needs to be considered in the beginning. The discretization step of the accelerometers' observations is considered to be small enough to treat both translational and rotational accelerations as linear functions of time, allowing use of cubic spline constraints in the optimization process.

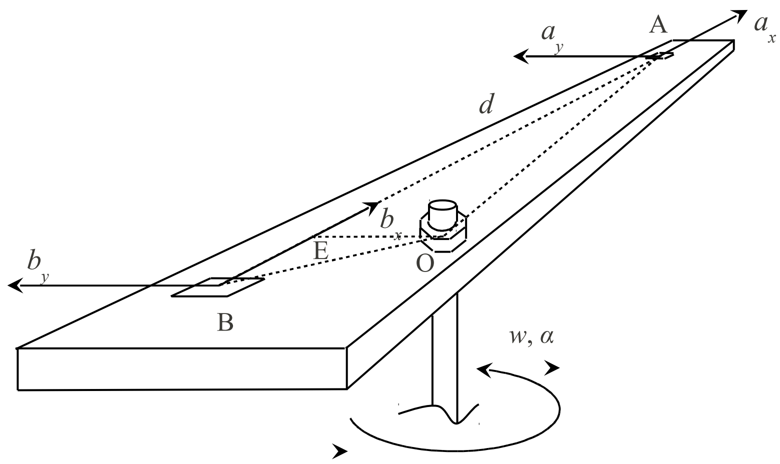

FIGURE 1. The scheme of autocalibration experiment with two accelerometers mounted on a rigid body.

This work was focused on the situation of two accelerometers firmly mounted on the rigid body, see Fig. 1. However, the same principles are extended to sensor fusion when having two accelerometers, gyroscope and two wheel encoders for a single axis differential drive robot.

Before going into details of the probability distribution calculation, it is necessary to speak about the optimization procedures taken in this work. The general entropy equation for the optimization process is as follows

$$S_{P,P_{old}} = -\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} P\left(x_{1,1},x_{2,1},\dots,x_{m,1},\dots,x_{1,i},x_{2,i},\dots,x_{m,i},\dots,x_{1,n},x_{2,n},\dots,x_{m,n}\right)$$

$$\mathrm{Ln}\left(\begin{array}{c} P\left(x_{1,1},x_{2,1},\dots,x_{m,1},\dots,x_{1,i},x_{2,i},\dots,x_{m,i},\dots,x_{1,n},x_{2,n},\dots,x_{m,n}\right) / \\ \prod_{j=1}^{n}\arg\max_{P_{old,j}}\left(\begin{array}{c} P_{old,j}\left(x'_{1,j},x'_{2,j},\dots,x'_{m,j}\right) \\ -\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} Ln\left(P_{old,j}\left(x'_{1,j},x'_{2,j},\dots,x'_{m,j}\right)\right) \\ \mathrm{d}x'_{1,j}\mathrm{d}x'_{2,j}\dots\mathrm{d}x'_{m,j}\end{array}\right)\end{array}\right)\mathrm{d}x_{1,1}\dots\mathrm{d}x_{m,n},$$

where x is an interval (from gyroscope or wheel encoder) or register (from accelerometer axis) observation at time $i \cdot \Delta t$ (in seconds) when there are m measurement channels. P is the *a posteriori* probability distribution and P_{old} is the *a priori* probability distribution.

There are inference flows during the whole optimization process:

a) inference of instantaneous center of rotation. Here *a priori* is selected as a joint instantaneous probability distribution functions,
b) inference of angular velocity and acceleration. Here *a priori* is selected as a joint function of marginal probability distributions of *x*.

In case of (a) model constraints and a Maximum Entropy (*ME*) solution enabled the use of a joint distribution function in the next inference step of Maximum relative Entropy (*MrE*). Meanwhile the *a priori* distribution of case (b) leads to exponential distribution solutions, so only marginal distributions were available for a further online optimization processes. Moreover, Eq.1 for case (b) is similar to mutual information entropy formula as in information theory. However, here it has a slightly different meaning: the purpose is to find the MrE solution so that all time series constraints are satisfied and resulting *a posteriori* distribution has the minimum divergence to prior knowledge, which was collected while observing the channels' measurements.

Both *a priori* and *a posteriori* probabilities could not have been updated simultaneously because of constraint commutativity (see a discussion on commutativity in work of A.Giffin and A. Caticha in [2]). One of the reasons for non-commutativity is the fact that when optimizing distribution with the constraints of observed values, one cannot apply another constraint that would contradict the observed values. This raises the need for performing an additional Bayesian updating step.

INFERENCE OF INSTANTANEOUS CENTER OF ROTATION

It must be noted that the probability distributions for P_{ax}, P_{ay}, P_{bx} and P_{by} have been already derived in work [1]. Naturally, it raises a question whether these formulae can be used for maximizing the likelihood for inferring an instantaneous center of rotation, which was incorporated into geometrical coefficients. The answer is *yes*. It is possible to use those equations for further inference. However, it would lead to nonlinear equations introducing the need for iterative solutions. The purpose of this paper was to concentrate on using Maximum relative Entropy optimization and prove that one can construct the solution differently, and simultaneously satisfy the requirements for high volume performance which is the key factor in practical online sensor fusion applications.

When two accelerometers are firmly mounted on the rigid body as in Fig.1, the following relationships are valid for any discrete set of accelerometers observations where c_{ax} is a measurement observed from axis a_x and c_{ay} from axis a_y at accelerometer A. Correspondingly c_{bx} is a measurement observed from axis b_x, and c_{by} from axis b_y at accelerometer B:

$$c_{oa} = \frac{d \cdot c_a}{\sqrt{c_a^2 + c_b^2 - 2 \cdot \left(c_{ax} \cdot c_{bx} + c_{ay} \cdot c_{by}\right)}}\,, \quad c_{ob} = \frac{d \cdot c_b}{\sqrt{c_a^2 + c_b^2 - 2 \cdot \left(c_{ax} \cdot c_{bx} + c_{ay} \cdot c_{by}\right)}}$$

where instantaneous accelerations c_a and c_b are calculated using measurements from axis as follows

$$c_a^2 = c_{ax}^2 + c_{ay}^2 \text{ and } c_b^2 = c_{ax}^2 + c_{cy}^2\,,$$

d is the distance between accelerometers, c_{oa} and c_{ob} are pseudo observation values for distances OA and OB which are calculated using Eq. [2]. Both x-axes lie on the same line, and during the autocalibration experiment the instantaneous center of rotation did not match the axis O because of vibrations and inertia forces which were swinging the mechanical O axis to the sides.

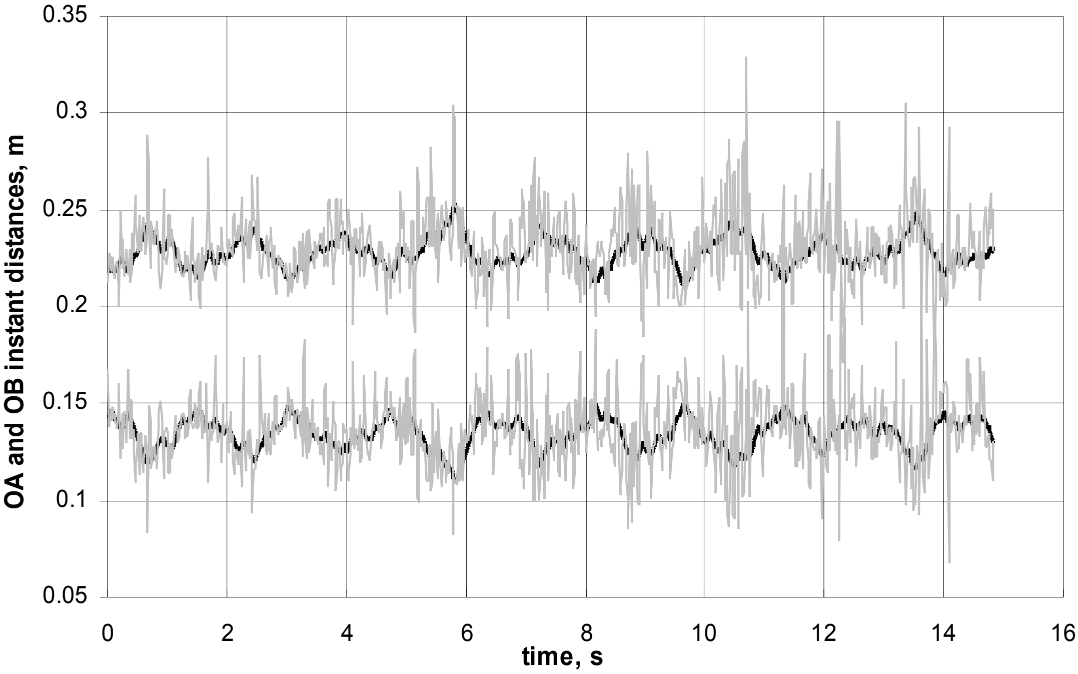

FIGURE 2. Instantaneous and filtered distances *OA* and *OB*

Figure 2 displays the instantaneous *OA* (upper curves) and *OB* (lower curves) distances calculated from observed data c_{ax}, c_{ay}, c_{bx} and c_{by}. The grey curve represents instantaneous values calculated using Eq.1 – 3. Bold curves represent filtered data after applying the online Maximum relative Entropy (*MrE*) optimization process. It can be seen that there are approximately 10 such swings when looking at Fig.1 that agreed with the number of rotations of the rigid body.

A Priori Maximum Entropy Distribution Based on Sensors Model Constraints

Similarly to previous work [1] *a priori* distributions were found using the ME formula

$$S_P = -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(o_a, o_b)\mathrm{Ln}(P(o_a, o_b))\mathrm{d}o_a \mathrm{d}o_b ,$$

and incorporating the following constraints

$$\int_{-\infty}^{+\infty} o_a P(o_a)\mathrm{d}o_a = c_{oa} \text{, and } \int_{-\infty}^{+\infty} o_b P(o_b)\mathrm{d}o_b = c_{ob} ,$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(o_a, o_b)\mathrm{d}o_a \mathrm{d}o_b = 1 ,$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{c_{oa}^{2}+c_{ob}^{2}-d^{2}}{c_{oa}c_{ob}}o_{a}\cdot o_{b}+d^{2}-o_{a}^{2}-o_{b}^{2}\right)P\left(o_{a},o_{b}\right)\mathrm{d}o_{a}\mathrm{d}o_{b}=0\,,$$

The solution of the Lagrangian constructed using Eq. 4 – 7 leads to the following joint distribution function

$$P\left(o_a,o_b\right)=\frac{\mathrm{Exp}\left[\dfrac{B\left(-\left(c_{ob}^{2}-d^{2}\right)o_a\left(c_{ob}-o_b\right)-c_{oa}^{3}o_b+c_{oa}^{2}o_a\left(c_{ob}+o_b\right)+c_{oa}\left(o_b\left(c_{ob}^{2}+d^{2}\right)-c_{ob}\left(o_a^{2}+o_b^{2}+d^{2}\right)\right)\right)}{c_{oa}c_{ob}}\right]}{c_{norm}},$$

where c_{norm} is the denominator normalizing constant as in Bayesian updating which is derived such that constraint Eq.6 is satisfied, and B is Lagrange multiplier for constraint Eq. 7.

After calculation of the marginal distributions of o_a and o_b, it can be seen that both marginal functions $P(o_a)$ and $P(o_b)$ represent normal distributions with the same variance, which is a function of Lagrange multiplier B. It is clear that the prior knowledge was not enough to infer the Lagrange multiplier B, which is in contrast with the work presented in [1], where the Lagrange multiplier was directly calculated using two second order probabilistic constraints.

To infer B the following formalism was taken into account:

Before making any observation there is no possibility to infer the variance of each measurement to be observed (assume no previous knowledge to do any prediction) by knowing only the model between sensors. In this specific situation even observations did not help to infer it. So by trying to confirm ME principles we need to assume that the variance of registers have a uniform distribution and that *the variance is spread evenly among distinct registers*. This allows us to infer B_i by knowing only the previous value, B_{i-1} . Moreover, when all *a priori* distributions P_{old} are known and they have a common B_1, it can be seen that *a posteriori* solution of MrE maximization does not depend on multiplier B_1. This literally means that in some real life situations it is not necessary to calculate the Lagrange multiplier and the *a posteriori* distribution can be calculated with partial knowledge.

A Posteriori Maximum Relative Entropy Distribution Based on Time Series Constraints

Based on previous discussion the Lagrange multipliers B have no meaning in their absolute values, but they carry information relative to each other. That is, any next B can be calculated using equality of their variances, which would lead to

$$B_i = B_1 \frac{\left(\left(c_{oa,1} - c_{ob,1}\right)^2 - d^2\right)\left(\left(c_{oa,1} + c_{ob,1}\right)^2 - d^2\right)}{\left(\left(c_{oa,i} - c_{ob,i}\right)^2 - d^2\right)\left(\left(c_{oa,i} + c_{ob,i}\right)^2 - d^2\right)} \frac{c_{oa,i}^2 c_{ob,i}^2}{c_{oa,1}^2 c_{ob,1}^2},$$

To maximize MrE entropy suggested in Eq.1 the following constraints were selected for time series

$$o_{a,i} = o_{a,i-1} \pm \Delta o_{a,i-1,i} \text{ and } o_{b,i} = o_{b,i-1} \pm \Delta o_{b,i-1,i},$$

These linear constraints assume a smooth change of the instantaneous center of rotation position and can be defined empirically. When set to values that are too large, they result in a bigger uncertainty in the calculated baseline for *OA* and *OB*. The physical meaning of $\Delta o_{a,i-1,i}$ and $\Delta o_{b,i-1,i}$ is of inertia. That is, when set to small values they would restrict *OA* and *OB* baseline to a straight line and would prevent exposing the swinging of the rotation center.

Moreover, Fig.1 shows uncertainty left in the calculated baselines of *OA* and *OB* (small fluctuation in **bold** curves). This uncertainty plays minimal role in the overall shape of baseline curves, but allows avoiding generalization of Karush-Kuhn-Tucker conditions on inequality constraints, which would add calculation overhead to the whole algorithm. In other words, instead of selecting a precise value effective to the current moment the maximum entropy value is being picked. Fluctuations in the curve are the "local" uncertainty that sums to zero asymptotically thus not effecting overall curve's shape.

If the constraints of Eq.10 were applied into maximization of Eq.1 where *a priori* distributions for every instantaneous set of registers were as follows

$$P_{\text{old}}\left(o_{a,1},..,o_{a,n},o_{b,1},..,o_{b,n}\right) = \prod_{i=1}^{n} P_{\text{old},i}\left(o_{a,i},o_{b,i}\right),$$

then an estimator of o_a and o_b would be derived as

$$o_{a,i} = \frac{\sum_{i=1}^{n} m_{bb,i} \cdot \sum_{i=1}^{n} m_{a,i} - \sum_{i=1}^{n} m_{ab,i} \cdot \sum_{i=1}^{n} m_{b,i}}{\sum_{i=1}^{n} m_{ab,i} \cdot \sum_{i=1}^{n} m_{ba,i} - \sum_{i=1}^{n} m_{aa,i} \cdot \sum_{i=1}^{n} m_{bb,i}},$$

$$o_{b,i} = \frac{\sum_{i=1}^{n} m_{aa,i} \cdot \sum_{i=1}^{n} m_{b,i} - \sum_{i=1}^{n} m_{ab,i} \cdot \sum_{i=1}^{n} m_{a,i}}{\sum_{i=1}^{n} m_{ab,i} \cdot \sum_{i=1}^{n} m_{ba,i} - \sum_{i=1}^{n} m_{aa,i} \cdot \sum_{i=1}^{n} m_{bb,i}},$$

where coefficients m are functions of $\Delta\varphi_{a,i-1,i}$, $\Delta\varphi_{b,i-1,i}$, $c_{ax,i}$, $c_{ay,i}$, $c_{bx,i}$, $c_{by,i}$ and d.

During revisiting of the autocalibration experiment, the data processing window was selected to be n=4 samples. Selecting higher numbers did not affect the overall shape of the baseline curves, but slowed down the calculation performance, cause the processing of local calculation window requires $2^n n$ iterations.

AUTOCALIBRATION EXPERIMENT REVISITED

After the baselines of the instantaneous centers of rotation were calculated, the estimation of the angular velocities and angular accelerations was improved by removing the assumption that instantaneous center of rotation was constant during the whole autocalibration experiment. Such an assumption was used in [1].

A Priori Maximum Entropy Distribution Based on Sensors Model Constraints

The *a priori* distribution is calculated as a joint function of marginal distributions P_{ax}, P_{ay}, P_{bx}, P_{by} and corresponds to the approach taken in [1] as follows

$$P_{\text{old}}\left(a_{x,1},..,a_{x,n},a_{y,1},..,a_{y,n},b_{x,1},..,b_{x,n},b_{y,1},..,b_{y,n}\right) = \prod_{i=1}^{n} P_{\text{old},i}\left(a_{x,i}\right)P_{\text{old},i}\left(a_{y,i}\right)P_{\text{old},i}\left(b_{x,i}\right)P_{\text{old},i}\left(b_{y,i}\right),$$

where a_x, a_y are acceleration x and y-axis values for accelerometer A, and b_x, b_y are x and y axis values for accelerometer B.

The difference is that the geometrical coefficients k_1 and k_2 are calculated for each instantaneous sample as

$$k_{1,i} = \frac{c_{oa,i}^2}{c_{ob,i}^2} \text{ and } k_{2,i} = \frac{\left(1+k_{1,i}\right)c_{oa,i}^2 - k_{1,i}d^2}{2c_{oa,i}^2}.$$

A Posteriori Maximum Relative Entropy Distribution Based on Time Series Constraints

A posteriori distribution is a result of optimization of Eq.1 as in the case of an inferring instantaneous center of rotation. The linear constraints were used as in [1], with the following difference: the next estimate of axis acceleration was expressed through the derived recurrent formula starting from the first sample of $a_{x,1}$.That is,

$$a_{x,i} = m_{ax,i} + m_{axa,i} a_{x,1},\ m_{ax,i} = f\left(m_{ax,i-1}, k_{1,i}, k_{2,i}, c_{3,i}, c_{4,i}\right) \text{ etc.,}$$

where coefficients $c_{3,i}$ and $c_{4,i}$ were derived from cubic spline relationships as in [1]. That is, the physical meaning of $c_{3,i}$ is the maximum angular acceleration constraint and $c_{4,i}$ is based on the maximum change in kinetic energy between the two subsequent samples. The change of kinetic energy is a function of maximum angular acceleration and the average angular velocity at the time the samples were observed.

FINAL NOTES

It was found that the smaller discretization interval, the better the results in the resulting baselines because the dependency between time series data was higher. The more dependency between discrete samples, the less uncertainty is left after the calculation of ME channel baselines. Moreover, higher discretization did not effect the online optimization filtering because asymptotic algorithm performance was O(n), where n was the number of instantaneous register sets observed.

The autocalibration experiment's observations were revisited with new MrE optimization process. It was found that 989 discrete samples of 4 channels (two accelerometers) were filtered over 953 milliseconds on a regular single core 2.4GHz processor where a prototype source code did not have full floating point optimizations implemented. Such optimization duration included *priori* and *posteriori* calculations of instantaneous center of rotation and *a priori* and *a posteriori* calculations of the rigid body's angular velocity and accelerations, i.e. in overall 4 Bayesian updating steps. The data processing window was selected as follows: 4 samples for instantaneous center of rotation, and 6 samples for angular velocities and accelerations.

If a discretization interval were one millisecond, this approach could still be implemented as an online method and the delay for the robot reaction to its position change would be 10 milliseconds.

The global optimization of MrE was not performed in this work and the results of online estimation of axis zero bias for each accelerometer did not give expected results as in [1]. This implies the need for further maximization of MrE when solving this task, and requires further investigation.

The main conclusion of this work is that the MrE approach can be used for online dead reckoning localization and that the dependent measurements reduce the overall entropy. The more dependency which is utilized, the less uncertainty is left in *a posteriori* distribution.

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